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Triple Quadratic Hom-Lie Systems

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Introduction

Reductive homogeneous spaces with some canonical connections were identified by K. Nomizu using the Bianchi identities, specifically some identities involving the torsion and the curvature. By treating the torsion and curvature tensors of Nomizu's canonical connection as bilinear and trilinear algebraic operations satisfying specific axioms, K. Yamaguti provided an algebraic interpretation of these identities and created what he called a general Lie triple system. According to his work, a Lie triple system consists of a finite-dimensional linear space L and a trilinear map [_, _, _]: L× L ×L⟶ L

Description

Physicists use the quadratic Lie triple systems can be found in a wide variety of settings. Let's talk about a couple of them. The Yang-Baxter equation, which is found in a variety of fields including statistical mechanics, exactly solvable two-dimensional field theory, the quantum group, Hopf algebra, and the braid group, can be solved using Lie triple systems [1,2].

Lie triple systems, on the other hand, provide Z/ 2Z -graded Lie algebras, which are precisely the Lie algebras connected to symmetric spaces. The relationship between completely geodesic submanifolds and Lie triple systems is discussed in as well as the relationship between Riemannian globally symmetric spaces and Lie triple systems. It turns out that the commutator bracket multiplication defined using the multiplication in Hom-associative triple systems naturally leads to Hom-Lie triple systems. The idea of Hom-Lie triple systems generalizes Lie triple systems to a case where the trilinear law is twisted by a linear map. A. Makhlouf and S.D. Silvestrov started the systematic study of hom-structures, and D. Yau provided a generic approach for building hom-type algebras starting from regular algebras and a twisting self-map. For details on the various hom-algebra kinds [3].

In order to provide an inductive explanation of quadratic Lie algebras, A. Medina and Ph. Revoy invented the concept of double extension. Later, utilizing some ideas related to double extensions of Lie algebras, I. Bajo et al. provided inductive descriptions of quadratic symplectic Lie algebras in particular. The idea of double extension has additional uses. The author and S. Hidri are responsible for expanding the idea of double extension to Lie triple systems, while S. Benayadi and A. Makhlouf are responsible for expanding this idea to Hom-Lie algebras. The main contribution is to provide an inductive description of quadratic Hom-Lie triple systems using the concept of double extension. The definitions and various important Hom-Lie triple system structures are first briefly discussed. The theory of Hom-Lie triple system representations, which includes adjoint and coadjoint representations, is then introduced. In addition, we characterize quadratic Hom-Lie triple systems and show how they relate to representation theory. In general, the evidence that has been gathered may not be sufficient to describe all quadratic Hom-Lie triple systems. We focus on the idea of generalized double extension of Hom-Lie triple systems eliciting other action on them to get over this problem [4].

Lie triple systems The field *k* will be algebraically closed. It will be taken for granted that *k* has a feature that is not equal to 2. All vector spaces, algebras, tensor products, etc., will be taken over *k* unless otherwise specified. We start by reviewing some fundamental definitions for the reader. A Lie triple system is a vector space T with a trilinear ternary product [a, b, c] that meets the conditions (a, b, c, x, y, z T) for all three of the following three properties:

[a, a, b] = 0;

[a, b, c]+[b, c, a]+[c, a, b] = 0;

[a, b, [x, y, z]] = [[a, b, x], y, z]+[x, [a, b, y], z]+[x, y, [a, b, z]].

A morphism φ: T ⟶ T ' of Lie triple systems is a linear map preserving the triple product. Let $\mathbf{LTS}_k$ denote the category of Lie triple systems over *k*.

There are well-known relationships between the restricted cohomology of g and the restricted nullcone N1 (g), consisting of all X g fulfilling X[p] = 0, given a restricted Lie algebra g over a field k of positive characteristic p > 2. All of the [p]-nilpotent elements in g make up the whole nullcone N (g), and N1 (g) is a closed subvariety of it. A closed, conical subvariety of N1(g), known as the support variety, includes information about the cohomology of each finite-dimensional, restricted g-module M. This variety is known as the support variety. Additionally, as demonstrated in, cohomological techniques offer effective tools for examining the structure of N1. Our attempts to apply this theory to the situation of constrained Lie triple systems served as the primary inspiration for this paper. Here, the circumstances where the Lie triple system T = p g = Lie (G) results from an involution on a reductive algebraic group G are the most intriguing. When p is a good prime for G, Richardson has given a lot of details about the structure of the unipotent cone in the related symmetric space that is isomorphic to N (p) [5].

Conclusion

The foundations of a cohomology theory for Lie triple systems and their restricted forms, even if there is still much work to be done before we can generalize Lie triple system theory as envisioned. To that purpose, it starts by going through and expanding on some earlier findings by Harris about cohomology in the category (Lu (T),)-Mod for a specific Lie triple system T. It seems like the cohomology idea for T -Mod is much more nuanced.

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Conflict of Interest

The author shows no conflict of interest towards this manuscript.

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