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Matrix Representation of a Star

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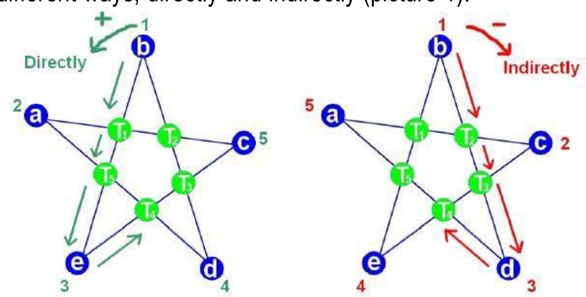
Abstract

In the present paper we define an oriented Star $\star_\alpha$ with α coefficient [1] and we further develop the procedure for finding eigenvalues and eigenvectors for an (5 × 5) Star-matrix directly or (5 × 5) Star-matrix indirectly. we give an overview of the methods to compute matrix-multiplication of a Star $\star_\alpha$ (generally square 5 × 5) with particular emphasis on the oriented matrix.

Keywords: Matrix orientation • Coefficient star-matrix

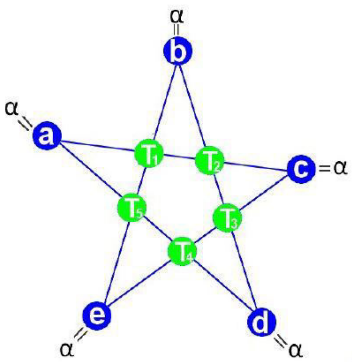
1.Introduction

In this article, we propose a matrix representation of an oriented star $\star_\alpha$ with α coefficient [1] and define two directions of orientation. It is therefore possible to orient a star $\star_\alpha$ with α coefficient in two different ways, directly and indirectly (picture 1).



$(Figure\ 1)$

A star with α coefficient is composed of five numbers outside a, b, c, d, e and five numbers inside T_1, T_2, T_3, T_4, T_5 These last five numbers are written in the form of 5-tuple (T_1, T_2, T_3, T_4, T_5) (Figure 2).



(Figure 2)

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In addition to having the sum α in each line.

The scalars α are called the star coefficient if α is a solution of equation $\alpha=T_1(\alpha)+T_2(\alpha)+T_3(\alpha)+T_4(\alpha)+T_5(\alpha)$ (Noted by $\alpha_\star$), a vector (T_1, T_2, T_3, T_4, T_5) is called a solution vector of this Star-System with α coefficient in five unknowns.

The present paper is organized as follows: In Section 2, we present some preliminary results and notations that will be useful in the sequel. In Section 3, we will use the convention here that the star $\star_\alpha$ has a positive or a negative (picture 1) orientation besides orientation of a star $\star_\alpha$. Another way to think of a positive orientation is that as we traverse the path following the positive orientation the star $\star_\alpha$ must always be on the left (that is, one may also speak of orientation of a (5 × 5) matrix, polynomial of degree 5, etc.). We present some examples of Star-matrix directly or indirectly.

2. Some Basic Definitions and Notations

In this section, we introduce some notations and star-system with coefficient **α** defined [1].

2.1. A star-system with α coefficient:

Dfinition 1 : Let a, b, c, d, e and **α** be real numbers, and let T_1, T_2, T_3, T_4, T_5 be unknowns (also called variables or indeterminates).

Then a system of the form

$$\begin{cases} T1+T2=\alpha-a-c \\ T2+T3=\alpha-b-d \\ T3+T4=\alpha-c-e \\ T4+T5=\alpha-a-d \\ T5+T1=\alpha-b-e \end{cases}$$

is called a star-system with α coefficient in five unknowns. We have also noted $\bigstar[a; b; c; d; e; \alpha] = \alpha$. The scalars a, b, c, d, e are called the coefficients of the unknowns, and α is called the constant "Chaff" of the star-system in five unknowns.

A vector (T_1, T_2, T_3, T_4, T_5) in R^5 is called a star-solution vector of this star-system if and only if $\bigstar[a; b; c; d; e; \alpha] = \alpha$. if and only if $\bigstar[a; b; c; d; e; \alpha] = \alpha$.
The solution of a Star-system is the set of values for T_1, T_2, T_3, T_4 and T_5 that satisfies five equations simultaneously.

2.2. A star-element:

A star-element [1] is a term of the five-tuple (T_1, T_2, T_3, T_4, T_5)

solution of a star-system ★[a; b; c; d; e; α] = α, wher (T_1, T_2, T_3, T_4, T_5) in R^5.

2.3. Star-Coefficient or Constant "Chaff":

The star-Coefficient or Constant "Chaff" [1] is also noted by $\alpha^{\star}$ and is a solution of equation : $\alpha = T_1(\alpha)+T_2(\alpha)+T_3(\alpha)+T_4(\alpha)+T_5(\alpha)$, wher ($T_1$, T_2, T_3, T_4, T_5) is solution of a star-system : ★[a; b; c; d; e; α] = α.

2.4. Star-Matrix:

The star-system with α coefficient [1] can be written in matrix form

$M_{\star}T = C_\alpha$

Where

$$M_{\star} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix},$$

vector T = (T_1; T_2; T_3; T_4; T_5) and

$$C_\alpha = \begin{pmatrix} \alpha - a - c \\ \alpha - b - d \\ \alpha - c - e \\ \alpha - a - d \\ \alpha - b - e \end{pmatrix}$$

$M_{\star}$ or M_{Staris} called the star-Matrix of the star-system with α coefficient (★[a; b; c; d; e; α] = α).

$M_{\star}$ a matrix is said to be of dimension 5×5. A value called the determinant of $M_{\star}$, that we denote by $|M_{\star}|$ or $|M_{Staris}|$, corresponds to square matrix $M_{\star}$.

Consequently, the determinant of $M_{\star}$ is $|M_{\star}| = 2$.

2.5. Set-Star:

The set-star is constructed from the solution set of linear star-system with α coefficient (★[a; b; c; d; e; α] = α). The Set-star will be noted by $S_{\star}$.

2.6. Star-System equivalent:

Equivalent Star-Systems [1] are those systems having exactly same solution, i.e. Two star-systems are equivalent if solution of on star-system is the solution of other, and vice-versa.

2.7. Parametrized Curves:

A parametrized differentiable curve is simply a specific subset of R^5 with which certain aspects of differntial calculus can be applied.

Definition 2 : A parametrized differentiable curve is a differentiable map $\alpha : I \rightarrow R^5$ of an open interval I = (a; b) of the real line R in to R^5.

2.8. Regular Curves:

A parametrized differentiable curve $\alpha : I \rightarrow R^5$, We call any point that satisfies $\alpha'(t) = 0$ a singular point and we will ristrict our study to curves without singular points.

Definition 3 : A parametrized differentiable curve is a differentiable $\alpha : I \rightarrow R^5$ is said to be regular if $\alpha'(t) \neq 0$ for all $t \in I$ (see [6],[7]).

2.9. Parametric Arclength:

Generalized, a parametric arclength starts with a parametric curve in R^5. This is given by some parametric equations $T_1(t)$; $T_2(t)$; $T_3(t)$; $T_4(t)$; $T_5(t)$, where the parameter t ranges over some given interval. The following formula computes the length of the arc between two points a, b.

Lemma 1 : Consider a parametric curve ($T_1(t)$; $T_2(t)$; $T_3(t)$; $T_4(t)$; $T_5(t)$), where $t \in (a; b)$. The length of the arc traced by the curve (see [4], [5]), as t ranges overt (a; b) is

$$L=\int_a^b \sqrt{(T'1(t))^2 + (T'2(t))^2 + (T'3(t))^2 + (T'4(t))^2 + (T'5(t))^2}dt$$

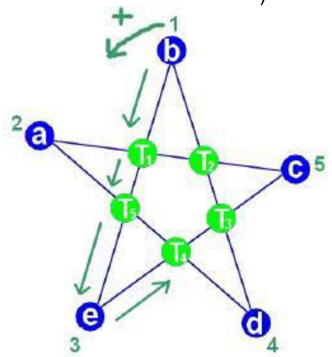
Thereafter we start with several examples with detailed solutions are presented.

3. Orientation of a Star with Coefficient A and Matrix Multiplication

3.1 Orientation of a Star

In this section, We choose two directions of travel on this Star with ∝ coefficient can be classified as negatively oriented (clockwise), positively oriented (counterclock-wise).

Where ★+ is a star oriented countreclockwise (positively oriented):

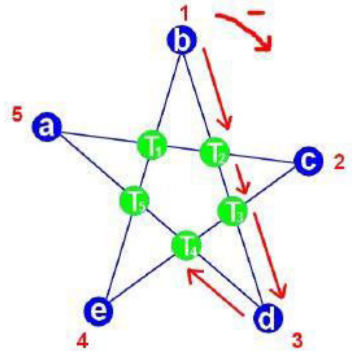


(*Figure* 3)

In the first case, one obtains a new matrix noted $M^{\star}{}_{+}$, called the star-matrix directly.

$$M^{\star}{}_{+} = \begin{pmatrix} b & a & e & d & c \\ T1 & T5 & T4 & T3 & T2 \\ T5 & T4 & T3 & T2 & T1 \\ e & d & c & b & a \\ T4 & T3 & T2 & T1 & T5 \end{pmatrix}$$

We note ★- is a star oriented clockwise (negatively oriented):



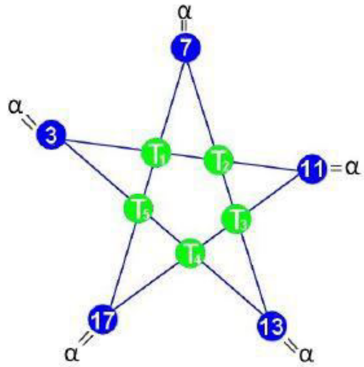
(*Figure* 4)

In the second case, one obtains a new matrix noted $M^{\star}{}_{-}$, called the star-matrix indirectly.

$$M^{\star}{}_{-} = \begin{pmatrix} b & c & d & e & a \\ T2 & T3 & T4 & T5 & T1 \\ T3 & T4 & T5 & T1 & T2 \\ e & d & c & b & a \\ T4 & T5 & T1 & T2 & T3 \end{pmatrix}$$

3.2. Image of a five prime numbers.

Exeample 1. We consider the Star-System with ∝ coefficient ★[3, 7, 11, 13, 17; ∝] = ∝ of Linear Equations, we can easily solve this using Star-Matrix $M_{\star}$.



(Figure 5)

⇔The star-systems of linear equation:

$$\begin{cases} T1 + T2 = \alpha - 14 \\ T2 + T3 = \alpha - 20 \\ T3 + T4 = \alpha - 28 \\ T4 + T5 = \alpha - 16 \\ T5 + T1 = \alpha - 24 \end{cases}$$

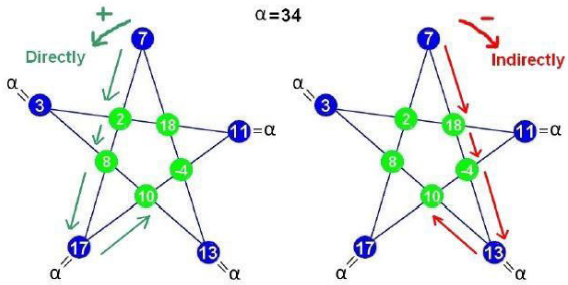
The solution of the Star-system ★[3; 7; 11; 13; 17; α] = α is therefore $T_1 = \frac{\alpha}{2} - 15$, $T_2 = \frac{\alpha}{2} + 1$, $T_3 = \frac{\alpha}{2} - 21$, $T_4 = \frac{\alpha}{2} - 7$ and $T_5 = \frac{\alpha}{2} - 9$.

So the overall solution is the star-set:

$S_\star = \{(\frac{\alpha}{2} - 15, \frac{\alpha}{2} + 1, \frac{\alpha}{2} - 21, \frac{\alpha}{2} - 7, \frac{\alpha}{2} - 9), \alpha \in R\}$.

In a particular case if $\alpha = \frac{\alpha}{2} - 15 + \frac{\alpha}{2} + 1 + \frac{\alpha}{2} - 21 + \frac{\alpha}{2} - 7 + \frac{\alpha}{2} - 9$ Then

- The Star-coefficient: $\alpha^\star = 34$.
- The star-element is (2; 18; −4; 10; 8).



(Figure 6

The star matrix directly

$$M_\star^{\star+} = \begin{pmatrix} 7 & 3 & 17 & 13 & 11 \\ 2 & 8 & 10 & -4 & 18 \\ 8 & 10 & -4 & 18 & 2 \\ 17 & 13 & 11 & 7 & 3 \\ 10 & -4 & 18 & 2 & 8 \end{pmatrix}.$$

and the star matrix indirectly

$$M_\star^{\star-} = \begin{pmatrix} 7 & 11 & 13 & 17 & 3 \\ 18 & -4 & 10 & 8 & 2 \\ -4 & 10 & 8 & 2 & 18 \\ 13 & 17 & 3 & 7 & 11 \\ 10 & 8 & 2 & 18 & -4 \end{pmatrix}$$

The determinant [2] of a matrix $M^{\star+}$ is denoted $\det(M^{\star+})$, after numerous calculations :

$$\det(M^{\star+}) = \det(M^{\star-}) = 642464 = 2^5 \times 17 \times 1181,$$

1181 is a prime number because it has only two distinct divisors: 1 and itself (1181).

The characteristic polynomial [3] of a matrix $M^{\star+}$, noted $P^{\star+}$:

$$P^{\star+}(\lambda) = -\lambda^5 + 26\lambda^4 + 472\lambda^3 + 3748\lambda^2 + 92704\lambda + 642464.$$

The characteristic polynomial of a matrix $M^{\star-}$, noted $P^{\star-}$:

$$P^{\star-}(\lambda) = -\lambda^5 + 14\lambda^4 + 888\lambda^3 + 9828\lambda^2 + 65504\lambda + 642464.$$

The following product was obtained from the two matices $M^{\star+}$ and $M^{\star-}$:

$$\mathbf{M^{\star+} \times C^{\alpha\star} = M^{\star-}},$$

Where

$$C^{\alpha\star} = \begin{pmatrix} \frac{341}{1181} & \frac{458}{1181} & \frac{-694}{1181} & \frac{474}{1181} & \frac{602}{1181} \\ \frac{602}{1181} & \frac{341}{1181} & \frac{458}{1181} & \frac{-694}{1181} & \frac{474}{1181} \\ \frac{474}{1181} & \frac{602}{1181} & \frac{341}{1181} & \frac{458}{1181} & \frac{-694}{1181} \\ \frac{-694}{1181} & \frac{474}{1181} & \frac{602}{1181} & \frac{341}{1181} & \frac{458}{1181} \\ \frac{458}{1181} & \frac{-694}{1181} & \frac{474}{1181} & \frac{602}{1181} & \frac{341}{1181} \end{pmatrix}$$

Called the matrix multiplication or chaffar-matrix. These results verified:

1) $|C^{\alpha\star}| = 1$,
2) ${}^tC^{\alpha\star} = (C^{\alpha\star})^{-1}$,
3) $M^{\star-} \times {}^tC^{\alpha\star} = M^{\star+}$,

So

$$\begin{vmatrix} 341 & 458 & -694 & 474 & 602 \\ 602 & 341 & 458 & -694 & 474 \\ 474 & 602 & 341 & 458 & -694 \\ -694 & 474 & 602 & 341 & 458 \\ 458 & -694 & 474 & 602 & 341 \end{vmatrix} = 1181^5.$$

A Surprise Result

1181=	341	458	−694	474	602
1181=	602	341	458	−694	474
1181=	474	602	341	458	−694
1181=	−694	474	602	341	458
1181=	458	−694	474	602	341
1181 //	‖	‖	‖	‖	‖
	1181	1181	1181	1181	1181

Now we look at matrix $M^{\star+}$ where one of the eigenvalues is repeated noted $\alpha^{+\star1}$ We shall see that this.

Eigenvalues:

$\alpha^{+\star1} = \alpha^{+\star2} = 2{,}49606814601474$, $\alpha^{+\star3} = 41{,}2085820470714$,

$\alpha^{+\star4} = -8{,}91508947299477$, $\alpha^{+\star5} = -11{,}2856288661062$

from where

$P^{\star+}(\lambda) = -\lambda^5 + 26\lambda^4 + 472\lambda^3 + 3748\lambda^2 + 92704\lambda + 642464$

$= -(\lambda - \alpha^{+\star1}) \times (\lambda - \alpha^{+\star3}) \times (\lambda - \alpha^{+\star4}) \times (\lambda - \alpha^{+\star5})$.

Investigate carefully the eigenvectors associated with the repeated eigenvalue $\alpha^{+\star1} = \alpha^{+\star2} = 2{,}49606814601474$:

$$V^{+\star1} = \begin{pmatrix} 397355942247 \\ -1169646488307 \\ 87713143092 \\ 67337161100 \\ 279731486034 \end{pmatrix},$$

$$V^{+\star2} = \begin{pmatrix} 768960900 \\ 0 \\ -1510526654 \\ -1309659309 \\ 2458111677 \end{pmatrix}.$$

The eigenvectors associated with the eigenvalue $\alpha^{+\star3} = 41{,}2085820470714$:

$$V^{+\star3} = \begin{pmatrix} 22737819930 \\ 12036188180 \\ 16332372660 \\ 22493851905 \\ 15604512348 \end{pmatrix}.$$

The eigenvectors associated with the eigenvalue

$\alpha^{+\star 4}$ = -8,91508947299477 :

$$V^{+\star 4}=\begin{pmatrix}354374696808\\-87613810200\\-537072656526\\-95530533\\341310400963\end{pmatrix}.$$

The eigenvectors associated with the eigenvalue $\alpha^{+\star 5}$ = -11,2856288661062 :

$$V^{+\star 5}=\begin{pmatrix}370244424180\\-120109067901\\-1005791381610\\231740412760\\697820107500\end{pmatrix}.$$

The conclusion is that since $\mathbf{M}^{\star +}$ is 5×5 and we can obtain five linearly independent eigenvectors then $\mathbf{M}^{\star +}$ be diagonalized.

$$\mathbf{M}^{\star +}=(\mathbf{P}^{\star +})\times\begin{pmatrix}\alpha\ddot{\star}1&0&0&0&0\\0&\alpha\ddot{\star}2&0&0&0\\0&0&\alpha\ddot{\star}3&0&0\\0&0&0&\alpha\ddot{\star}4&0\\0&0&0&0&\alpha\ddot{\star}5\end{pmatrix}\times(\mathbf{P}^{\star +})^{-1}$$

The Eigenvalues of matrix $M^{\star -}$:

$\alpha^{-\star 1}$ = 0,12957478575019, $\alpha^{-\star 2}$ = 0,12957478575019,
$\alpha^{-\star 3}$ = 41,8947762503062, $\alpha^{-\star 4}$ =14,0769629109033,
$\alpha^{-\star 5}$ =14,0769629109033

The eigenvectors associated with the repeated eigenvalue $\alpha^{-\star 1}$ = $\alpha^{-\star 2}$ = 0,12957478575019 :

$$V^{-\star 1}=\begin{pmatrix}3711681350\\6826536600\\101829170950\\-57560278737\\-41096493650\end{pmatrix},$$

$$V^{-\star 2}=\begin{pmatrix}96211507\\153385005\\0\\-112669810\\170312295\end{pmatrix}.$$

The eigenvectors associated with the eigenvalue $\alpha^{-\star 3}$ = 41,8947762503062 :

$$V^{-\star 3}=\begin{pmatrix}3023855534700\\2194340858325\\1704618233175\\3073845049275\\2321218746301\end{pmatrix}.$$

The eigenvectors associated with the eigenvalue $\alpha^{-\star 4}$ = $\alpha^{-\star 5}$ = -14:0769629109033 :

$$V^{-\star 4}=\begin{pmatrix}5436549800\\-17910960432\\3314727840\\4988359594\\4787944995\end{pmatrix},$$

$$V^{-\star 5}=\begin{pmatrix}1483518127\\0\\-781571280\\-1638184730\\1382412525\end{pmatrix}.$$

The conclusion is that since $\mathbf{M}^{\star -}$ is 5×5 and we can obtain five linearly independent eigenvectors then $\mathbf{M}^{\star -}$ be diagonalized.

$$\mathbf{M}^{\star -}=(\mathbf{P}^{\star -})\times\begin{pmatrix}\alpha\ddot{\star}1&0&0&0&0\\0&\alpha\ddot{\star}2&0&0&0\\0&0&\alpha\ddot{\star}3&0&0\\0&0&0&\alpha\ddot{\star}4&0\\0&0&0&0&\alpha\ddot{\star}5\end{pmatrix}\times(\mathbf{P}^{\star -})^{-1}$$

On the other hand

$$\mathbf{M}^{\star +}=\begin{pmatrix}1&0&0&0&0\\2/7&1&0&0&0\\8/7&23/25&1&0&0\\17/7&4/5&215/176&1&0\\10/7&-29/25&1/88&141/170&1\end{pmatrix}\times\begin{pmatrix}7&3&17&13&11\\0&50/7&36/7&-54/7&104/7\\0&0&-704/25&256/25&-606/25\\0&0&0&-340/11&-527/88\\0&0&0&0&1181/80\end{pmatrix}$$

$$\mathbf{M}^{\star -}=\begin{pmatrix}1&0&0&0&0\\18/7&1&0&0&0\\-4/7&-57/113&1&0&0\\13/7&12/113&-31/6&1&0\\10/7&27/113&-155/51&431/1360&1\end{pmatrix}\times\begin{pmatrix}7&11&13&17&3\\0&-226/7&-164/7&-250/7&40/7\\0&0&408/113&-712/113&1902/113\\0&0&0&-160/3&93\\0&0&0&0&1181/80\end{pmatrix}$$

3.3. Matrix multiplication and Star-function.

As we will see in the next subsection, matrix multiplication exactly corresponds to the composition of the corresponding linear transformations.

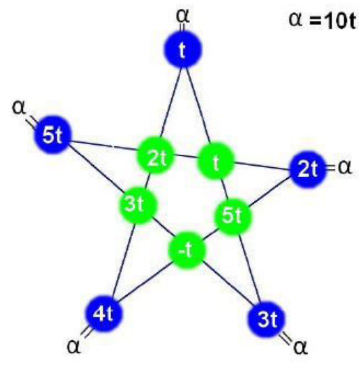
Exeample 1. Let $\alpha\in R$, for all $t\in R$ the star-system with α coefficient ★[t, 2t, 3t, 4t, 5t ; α] = α of linear equations, has a unique solution $(\frac{\alpha}{2}-4t, \frac{\alpha}{2}, \frac{\alpha}{2}-6t, \frac{\alpha}{2}-2t, \frac{\alpha}{2}-3t)$.

So the overall solution is the star-set:

$S_{\star}=\{(\frac{\alpha}{2}-4t, \frac{\alpha}{2}, \frac{\alpha}{2}-6t, \frac{\alpha}{2}-2t, \frac{\alpha}{2}-3t), \alpha\in R\}$.

In a particular case if $\alpha=\frac{\alpha}{2}-4t+\frac{\alpha}{2}+\frac{\alpha}{2}-6t+\frac{\alpha}{2}-2t+\frac{\alpha}{2}-3t$ Then

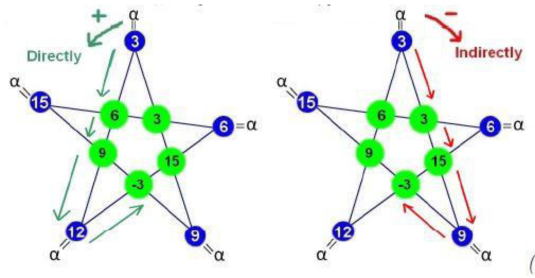
- The Star-coefficient: $\alpha^{\star}$ = 10t.
- The star-element is (t, 5t, -t, 3t, 2t) , for all t∈R.



(Figure 7)

In the special case t=3

- The Constant "Chaff": $\alpha^{\star}$= 30.
- The Star-set : $S_{\star}$ = {(3, 15, -3, 9, 6)}.



(Figure 8)

The star matrix directly

$$M_{\bigstar}^{\bigstar+} = \begin{pmatrix} 3 & 15 & 12 & 9 & 6 \\ 6 & 9 & -3 & 15 & 3 \\ 9 & -3 & 15 & 3 & 6 \\ 12 & 9 & 6 & 3 & 15 \\ -3 & 15 & 3 & 6 & 9 \end{pmatrix}.$$

The star matrix indirectly

$$M_{\bigstar}^{\bigstar-} = \begin{pmatrix} 3 & 6 & 9 & 12 & 15 \\ 3 & 15 & -3 & 9 & 6 \\ 15 & -3 & 9 & 6 & 3 \\ 9 & 12 & 15 & 3 & 6 \\ -3 & 9 & 6 & 3 & 15 \end{pmatrix}.$$

After numerous calculations :

$$\mathbf{det(M^{\bigstar+}) = det(M^{\bigstar-}) = 303750 = 2\times 5^4 \times 243,}$$

243 is a prime number.

The characteristic polynomial [3] of a matrix $M^{\bigstar+}$, noted $P^{\bigstar+}$:

$P^{\bigstar+}(\lambda) = -\lambda^5 + 39\lambda^4 + 45\lambda^3 - 4860\lambda^2 - 48600\lambda + 303750.$

The characteristic polynomial of a matrix $M^{\bigstar-}$, noted $P^{\bigstar-}$:

$P^{\bigstar-}(\lambda) = -\lambda^5 + 45\lambda^4 - 225\lambda^3 - 4590\lambda^2 - 8100\lambda + 303750.$

The following product was obtained from the two matices $M^{\bigstar+}$ and $M^{\bigstar-}$:

$$\mathbf{M^{\bigstar+} \times C^{\alpha\bigstar} = M^{\bigstar-},}$$

we get the following matrix : Called Chaffar-matrix

$$\mathbf{C^{\alpha\bigstar}} = \begin{pmatrix} \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{-3}{5} \\ -3 & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-3}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{-3}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{-3}{5} & \frac{2}{5} \end{pmatrix}$$

It should be hard to believe that our complicated formula for matrix multiplication actually means something intuitive such as chaining two transformations together.

Let $f_{\bigstar+} : R^5 \rightarrow R^5$ and $C_{\bigstar+} : R^5 \rightarrow R^5$ be linear transformations, and let $M^{\bigstar+}$ and $C^{\alpha\bigstar}$ be their standard matrices, respectively, so $M^{\bigstar+}$ is an 5×5 matrix and $C^{\alpha\bigstar}$ is an 5×5 matrix.

Then $f_{\bigstar+} \circ C_{\bigstar+} : R^5 \rightarrow R^5$ is a linear transformation, and its standard matrix is the product $M^{\bigstar-}$.

That is to say

$$\boldsymbol{f_{\bigstar+} \circ C_{\bigstar+} = f_{\bigstar-}}.$$

We have

1) $|C^{\alpha\bigstar}| = 1$,
2) ${}^{t}C\alpha^{\bigstar} = (C\alpha^{\bigstar})^{-1}$,
3) $M^{\bigstar-} \times {}^{t}C\alpha^{\bigstar} = M^{\bigstar+}$,

$$\begin{vmatrix} 2 & 2 & 2 & 2 & -3 \\ -3 & 2 & 2 & 2 & 2 \\ 2 & -3 & 2 & 2 & 2 \\ 2 & 2 & -3 & 2 & 2 \\ 2 & 2 & 2 & -3 & 2 \end{vmatrix} = 5^5.$$

A Surprise Result

5 =	2	2	2	2	-3
5 =	-3	2	2	2	2
5 =	2	-3	2	2	2
5 =	2	2	-3	2	2
5 =	2	2	2	-3	2
5 //	‖ 5	‖ 5	‖ 5	‖ 5	‖ 5

Eigenvalues of Matrix $\mathbf{M^{\bigstar+}}$:

$\alpha^{+\bigstar 1} = \alpha^{+\bigstar 2}$=-8,63341670127017, $\alpha^{+\bigstar 3}$ = 35,5216112793007, $\alpha^{+\bigstar 4}$ = 4,56303152617058, $\alpha^{+\bigstar 5}$ = 16,1821905970691,

The eigenvectors associated with the repeated eigenvalue $\alpha^{+\bigstar 1} = \alpha^{+\bigstar 2}$=-8,63341670127017 :

$$V^{+\bigstar 1} = \begin{pmatrix} 98344574847 \\ 112013121600 \\ -25550692626 \\ -231587128908 \\ 43946621216 \end{pmatrix},$$

$$V^{+\bigstar 2} = \begin{pmatrix} 792946375 \\ -796362144 \\ -552702912 \\ 0 \\ 789365352 \end{pmatrix}.$$

The eigenvectors associated with the eigenvalue $\alpha^{+\bigstar 3}$ = 35,5216112793007 :

$$V^{+\bigstar 3} = \begin{pmatrix} 876472270685 \\ 692417688314 \\ 573935035140 \\ 877293706740 \\ 555864138720 \end{pmatrix}.$$

The eigenvectors associated with the eigenvalue $\alpha^{+\bigstar 4}$ =4,56303152617058 :

$$V^{+\bigstar 4} = \begin{pmatrix} 616551396128 \\ 41545103600 \\ 10379739083 \\ -18801719040 \\ -80359270320 \end{pmatrix}.$$

The eigenvectors associated with the eigenvalue $\alpha^{+\bigstar 5}$ = 16,1821905970691 :

$$V^{+\bigstar 5} = \begin{pmatrix} 4698063285928 \\ -5265867438200 \\ 15114855363992 \\ -41372779281 \\ -6681220825688 \end{pmatrix}.$$

$\mathbf{M^{\bigstar+}}$ is 5×5 and we can obtain five linearly independent eigenvectors then $\mathbf{M^{\bigstar+}}$ be diagonalized.

Eigenvalues of Matrix $\mathbf{M^{\bigstar-}}$:

$\alpha^{-\bigstar 1} = \alpha^{-\bigstar 2}$ = -6,82337727172845, $\alpha^{-\bigstar 3}$ = 34,7324078557174, $\alpha^{-\bigstar 4}$ = 16,2578297114104, $\alpha^{-\bigstar 5}$ = 7,65651697632905,

The eigenvectors associated with the repeated eigenvalue $\alpha^{-\bigstar 1} = \alpha^{-\bigstar 2}$ = -6,82337727172845 :

$$V^{-\star 1} = \begin{pmatrix} 29986905705520 \\ 13314954236180 \\ -2893303898420 \\ -40499075163740 \\ 1929286651501 \end{pmatrix},$$

$$V^{-\star 2} = \begin{pmatrix} 432255005 \\ -144454440 \\ -504849748 \\ 0 \\ 266131202 \end{pmatrix}.$$

The eigenvectors associated with the eigenvalue $\alpha^{-\star 3}$ = 34,7324078557174 :

$$V^{-\star 3} = \begin{pmatrix} 469945893258 \\ 310960665501 \\ 379583371002 \\ 479217930612 \\ 258658839280 \end{pmatrix}.$$

The eigenvectors associated with the eigenvalue $\alpha^{-\star 4}$ = 16,2578297114104 :

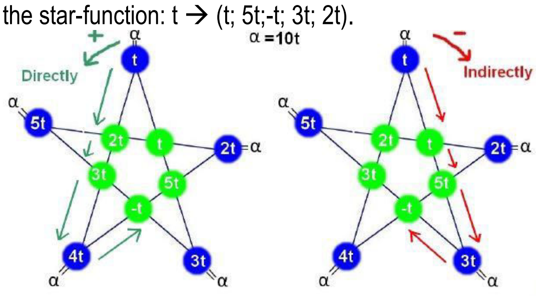
$$V^{-\star 4} = \begin{pmatrix} 87087650364 \\ -277650466212 \\ 356707497198 \\ 146564990520 \\ -143243581381 \end{pmatrix}.$$

The eigenvectors associated with the eigenvalue $\alpha^{-\star 5}$ = 7,65651697632905 :

$$V^{-\star 5} = \begin{pmatrix} 924275240 \\ -1812125436 \\ -5030544673 \\ -6842313940 \\ 9503960620 \end{pmatrix}.$$

Same *as* previously, the matrix $\mathbf{M}^{\star -}$ is 5×5 and we can obtain five linearly independent eigenvectors then $\mathbf{M}^{\star -}$ be diagonalized.

More generally,
For all t∈R the star-system ★[t, 2t, 3t, 4t, 5t ; `10t`] = `10t` has a unique solution (t, 5t,-t; 3t, 2t), the Star-coefficient $\alpha^{-\star}$ = 10t and the star-function: t → (t; 5t;-t; 3t; 2t).



(Figure 9)

The star matrix directly

$$M_{\star}^{\star +}(t) = \begin{pmatrix} t & 5t & 4t & 3t & 2t \\ 2t & 3t & -t & 5t & t \\ 3t & -t & 5t & t & 2t \\ 4t & 3t & 2t & t & 5t \\ -t & 5t & t & 2t & 3t \end{pmatrix}.$$

For all t∈R-{0}, det($\mathbf{M}^{\star +}$**(t)**)= $1250t^5$

The characteristic polynomial [3] of a matrix $M^{\star +}$, noted $P^{\star +}$:

For all (t, λ)∈R-{0}×R,

$P^{\star +}(t,\lambda) = -\lambda^5 + 13t\times\lambda^4 + 5t^2\times\lambda^3 - 180t^3\times\lambda^2 - 600t^4\times\lambda + 1250t^5$.

$$\mathbf{M}^{\star +}(\mathbf{t}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 16/7 & 1 & 0 & 0 \\ 4 & 17/7 & 11/19 & 1 & 0 \\ -1 & -10/7 & -11/19 & -1/20 & 1 \end{pmatrix} \times \begin{pmatrix} t & 5t & 4t & 3t & 2t \\ 0 & -7t & -9t & -t & -3t \\ 0 & 0 & -95t/7 & -40t/7 & 20t/7 \\ 0 & 0 & 0 & -100/19 & 50t/19 \\ 0 & 0 & 0 & 0 & 5t/2 \end{pmatrix}$$

We get the star matrix indirectly

$$M_{\star}^{\star -}(t) = \begin{pmatrix} t & 2t & 3t & 4t & 5t \\ t & 5t & -t & 3t & 2t \\ 5t & -t & 3t & 2t & t \\ 3t & 4t & 5t & t & 2t \\ -t & 3t & 2t & t & 5t \end{pmatrix}.$$

For all t∈R-{0}, det($\mathbf{M}^{\star -}$**(t)**)= $1250t^5$

The characteristic polynomial [3] of a matrix $M^{\star -}$, noted $P^{\star -}$:

For all (t, λ)∈R-{0}×R,

$P^{\star -}(t,\lambda) = -\lambda^5 + 15t\times\lambda^4 - 25t^2\times\lambda^3 - 170t^3\times\lambda^2 - 100t^4\times\lambda + 1250t^5$.

$$\mathbf{M}^{\star -}(\mathbf{t}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 5 & -11/3 & 1 & 0 & 0 \\ 3 & -2/3 & 1/4 & 1 & 0 \\ -1 & 5/3 & -7/16 & 9/20 & 1 \end{pmatrix} \times \begin{pmatrix} t & 2t & 3t & 4t & 5t \\ 0 & 3t & -4t & -t & -3t \\ 0 & 0 & -80t/3 & -65t/3 & -35t \\ 0 & 0 & 0 & -25/4 & -25t/4 \\ 0 & 0 & 0 & 0 & 5t/2 \end{pmatrix}$$

The following product was obtained from the two matices $M^{\star +}(t)$ and $M^{\star -}(t)$:

$$\mathbf{M}^{\star +}(\mathbf{t}) \times \mathbf{C}^{\alpha\star} = \mathbf{M}^{\star -}(\mathbf{t}),$$

Cha_ar-Matrix $\mathbf{C}^{\alpha\star}$ is a matrix independent of t

$$\mathbf{C}^{\alpha\star} = \begin{pmatrix} \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{-3}{5} \\ -3 & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-3}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{-3}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{-3}{5} & \frac{2}{5} \end{pmatrix}$$

The following results are obtained:

1) $|C^{\alpha\star}| = 1$,
2) ${}^{t}C^{\alpha\star} = (C^{\alpha\star})^{-1}$,
3) $M^{\star -}(t) \times {}^{t}C^{\alpha\star} = M^{\star +}(t)$,

The relationship between two matrices $M^{\star -}(t)$ and $M^{\star +}(t)$ is a convenient method of visualizing relationships quickly and definitively. One way of looking at it is that the result of matrix multiplication is important in research afterwards.

References

1. Chaffar, Mohamed Moktar. "Star with coefficient α in the set of real numbers." *J Comput Appl Math* (2020).
2. Muir, Thomas. "A treatise on the theory of determinants." Revised and enlarged by William H. Metzler, New York, NY: Dover. (1960) 1933.
3. Baley G. "Price Some identities in the theory of determinants." *Am Math Mon*. (1947) 75-90.

4. Ron Larson, Bruce H Edwards , Calculus, Brooks/Col, Cengage Learning (2009).
5. Van Heuraet and Hendrik, Epistola. "de transmutatione curvarum linearum in rectas Amsterdam: Louis Daniel Elzevir." (1659): 517-520.
6. Farouki, Rida T. "Curves from motion. motion from curves In Laurent"(1999).
7. Schumaker, LL. "Curve and Surface Design: Saint-Malo 1999." Vanerbilt Univ. Press. 63-90.