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Generalized Lie Theory and Applications

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Lie Symmetry Analysis and Exact Solutions

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Introduction

Fractional differential equations have been employed in applied sciences including fluid flow, chemistry, physics, biology, and other fields in recent decades to describe a variety of natural phenomena, for modelling various events which may depend on the past as well as the present. Additionally, fractional differential equations are seen to be more effective for examining the development of scientific phenomena under complex irregular situations. In order to find precise and numerical solutions, a variety of effective methods have been developed, including the variational iteration method, transformation method, finite-difference method, Exp-function method, homotopy analysis method, Adomian decomposition method, the first integral method, and sine-cosine method. In the early nineteenth century, the Norwegian mathematician Sophis Lie promoted Lie symmetry analysis, one of the most effective techniques for analysing nonlinear partial differential equations and locating their analytical solutions. It can be used for many different things, including as linearizing some nonlinear equations, creating novel solutions out of simple ones, creating integrator factors, reducing order, and reducing the independent variables [1]. The first thorough studies of symmetries admitted by fractional differential equations, concentrating on Riemann-Liouville and Caputo derivatives, were initiated by Gazizov et al. Wang and Xu looked at the symmetry characteristics of the time-fractional KDV equation. It is important to note that Yusuf examined the fluid flow equation.

Description

It is relatively new to apply the Lie symmetry analysis to FPDEs. We only find a small number of studies in the literature. For example, the authors of took the time fractional linear wave-diffusion equation into consideration and came up with a collection of dilations. Invariant solutions were built using dilation symmetries. The Lie symmetry analysis has been attempted to be applied to FPDEs. This approach has also been used to solve the Korteweg-de Vries and time fractional generalised Burgers equations [2]. In symmetry reductions and group-invariant solutions were achieved using group-analysis of the time-fractional Harry-Dym equation with Riemann-Liouville derivative. The Lie group analysis method was used by the authors of to examine the invariance characteristics of the fractional Sharma-Tasso-Olver equation. Conversely, conservation rules are crucial tools in the study of differential equations from both a mathematical and physical perspective. Integrability is quite likely if the underlying system is subject to conservation laws. If a Noether symmetry linked with a Lagrangian is known for Euler-Lagrange equations, the Noether theorem gives us a systematic technique for discovering PDE conservation rules. However, certain methods for determining the PDEs' conservation laws that lack a Lagrangian are described in the literature [3].

Advanced mathematics includes the concept of Lie point symmetry. Sophus Lie developed the concept of the Lie group toward the close of the nineteenth century in order to research the solutions of ordinary differential equations (ODEs). He demonstrated the main property that if an ordinary differential equation is invariant under one-parameter Lie group of point transformations, it can have its order decreased by one. The strategies for available integration were harmonised and expanded by this observation. The remainder of Lie's mathematical career was devoted to creating these continuous groups, which are today influential in many branches of the mathematical sciences. Lie and Emmy Noether laid the groundwork for and later Élie Cartan promoted the use of Lie groups in differential systems. Lie's approach produces group-invariant solutions and conservation rules for partial differential equations (PDEs). PDEs can be categorised into equivalence classes and new solutions can be deduced from existing ones by taking advantage of their symmetry. Furthermore, explicit solutions can be used as benchmarks in the design, accuracy testing, and comparison of numerical methods, and group-invariant solutions derived by Lie's method may shed light on the physical models themselves [4].

Lie's original theories have a tremendous deal of potential to have a significant impact on differential equation research on physically significant systems. The use of Lie group methods on actual physical systems, however, necessitates time-consuming computations. If done with a pencil, even calculating the continuous symmetry group of a small system of differential equations is prone to error. The space of independent variables, state variables (dependent variables), and derivatives of the state variables can all naturally be "stretched" by Lie groups and, consequently, by their infinitesimal generators, up to any finite order [5]. There are numerous further types of symmetries. For instance, contact transformations allow the infinitesimal generator's coefficients to rely on the coordinates' first derivatives. It is possible for them to incorporate derivatives of any order using Lie-Bäcklund transformations. Noether acknowledged the possibility of such symmetry existing. The exact invariant solutions, dynamical wave structures of multiple solitons, and three phases of similarity reductions of a (3+1)-dimensional generalised

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BKP-Boussinesq (gBKP-B) equation are all obtained using the Lie symmetry technique. We find infinitesimal vectors of the gBKP-B equation, and each of these infinitesimals is dependent on a set of Lie algebras made up of five independent arbitrary functions and two parameters. Following that, the one-dimensional optimum system of symmetry subalgebras and the studied vector fields' commutative and adjoint tables are formed to the original equation. The Lie symmetry technique converts the gBKP-B equation into numerous nonlinear ordinary differential equations based on each of the symmetry subalgebras [6].

Conclusion

Differential equations that appear as mathematical models in financial issues are subject to Lie group theory. Starting with the one-dimensional Black-Scholes model's complete symmetry analysis, we demonstrate that this equation falls under the category of linear second-order partial differential equations with two independent variables as defined by Sophus Lie. As a result, Lie's equivalence transformation formulae directly lead to the Black-Scholes transformation of this model into the heat transfer equation. Lie Differential equation symmetry analysis gives us a simple yet incredibly effective tool to obtain the conservation laws of the corresponding system represented by the (ordinary or partial) differential equations. The concept of their symmetry analysis is mainly driven by the differential equations' invariance under transformation of both the dependent and independent variables involved. By constructing diffeomorphism on the space of independent and dependent variables and mapping the solutions of the particular differential equation to the other solutions, this transformation creates a local group of point transformations.

Conflict of Interest

None.

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