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Automorphic Lie Algebras in Higher Dimensions

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Abstract

The study offers the entire classification of automorphic Lie algebras based on $sl_n(C)$, where $sl_n(C)$ contains no trivial summands, the poles are in any of the exceptional G-orbits in C and the symmetry group G acts on $sl_n(C)$ through inner automorphisms. The analysis of the algebras within the framework of traditional invariant theory is a crucial aspect of the categorization. This offers both a strong computational tool and raises new concerns from an algebraic standpoint (such as structure theory) that indicate to other uses for these algebras outside of the realm of integrable systems. The study demonstrates, in particular, that the class of automorphic Lie algebras connected to TOY groups (tetrahedral, octahedral and icosahedral groups) depend solely on the automorphic functions of the group, making them group independent Lie algebras. This may be proven by generalising the classical idea to the case of Lie algebras over a polynomial ring and creating a Chevalley normal form for these algebras.

Keywords: Automorphic lie algebras • Infinite-dimensional lie algebras • Chevalley normal forms

Introduction

In which automorphic algebras connected to finite groups were taken into consideration, the classification of ALiAs was introduced. These are the cyclic groups Z/N, dihedral groups DN, tetrahedral groups T, octahedral groups O and icosahedral groups Y from Klein's classification. Starting with the finite-dimensional algebra $sl_2(C)$, the authors of examine automorphic algebras connected to the dihedral group DN; instances of ALiAs based on $sl_3(C)$ were also covered. In, the authors provide a classification of inner action automorphic algebras with no trivial summands that are connected to the dihedral group DN. When the problem is uniformly defined using the notion of invariants, it represents a further, critical step towards the entire classification. As a result, finite group symmetric $sl_2(C)$-based ALiAs may be fully classified. The new methodology is what led to the current findings, but when considering higher-dimensional Lie algebras, it is no longer possible to make the simplification that the representations of G acting on the spectral parameter as well as on the natural representation V of the base Lie algebra are the same, as in [1,2].

Description

The purpose of this study is to give a comprehensive classification of automorphic Lie algebras for the situation $g=sl_n(C)$ with poles at exceptional G-orbits and an inner action on $sl_n(C)$ without trivial summands. Exceptional orbits are those that have less than |G| elements and are designated by the notation z=a,b,c, where a,b,c are the forms that have zeros at z. The analysis of these algebras within the framework of traditional invariant theory is a crucial aspect of this strategy. In a nutshell, the complex projective line CP1, which is made up of the quotients X/Y of two complex variables, is used to represent the Riemann sphere. Möbius transformations on are therefore equivalent to linear transformations by the same matrix on the vector (X, Y). The G-invariant subspaces of C[X,Y]-modules are then discovered using classical invariant theory, where C[X,Y] is the ring of polynomials in X and Y. A choice of multiplicative set of invariants is then used to localise these ring modules of invariants. This decision translates into choosing a G-orbit with z poles. The ALiA is produced by the set of elements in the localization of degree zero, or the set of elements that can be represented as functions of. Following computation, the algebra is converted into a Chevalley normal form in the manner of the conventional Chevalley basis; we think this is the most practical form for analysis. In the $sl_n(C)$ instance, the isomorphism question can now be resolved and a more precise isomorphism conjecture may be developed [3].

From the perspective of computations, classical invariant theory offers a potent instrument for analysis. A comprehensive categorization has, in fact, been hampered by computational issues. Inherent challenges resulting from the problem's larger dimensionality (going from $sl_2(C)$ to $sl_n(C)$, n>2) as well as the decision to use two separate group representations, which requires a higher degree ground form rather than degree two as in, were present.

The paper is structured as follows: in the following section, the computational challenges are presented and addressed in two ways (the challenges resulting from the growing dimensionality of the problem are discussed in Section 2 but ultimately addressed in Section 4): first, by using classical invariant theory, thus working with polynomials in X and Y, rather than rational functions of, until the very last stage when the Riemann sphere is identified with the commutative sphere The background information from representation theory of finite groups is reviewed in Section 2.2, with a focus on the TOY groups. Basic concepts from invariant theory, such as decompositions into irreducible representations and Molien series, are recalled in Sections 2.2 and 2.3. Transvection is used to compute invariant matrices. The notion of matrices of invariants is introduced in Sect., which then addresses the second significant computing hurdle of the issue and enables the definition of the Chevalley normal form for ALiAs. Sect. 5 provides normal forms for $sl_n(C)$-based ALiAs, while Sect. 6 introduces the idea of an automorphic Lie algebra's invariant. The key findings are covered in the conclusions, which also touch on the predictive potential of invariants [4,5].

They are defined by taking into account how invariant matrices multiply invariant vectors. When the invariant matrices are described in terms of this action, it results in very simplified matrices with G-invariant elements. The structural constants of the Lie algebra are preserved by the map to matrices of invariants. We stress that because the invariant matrices are represented in a -dependent basis, which trivialises the conjugation action on the matrices and only leaves the action on the spectral parameter, they are not invariant under the typical group action.

Conclusion

It is important to emphasise that the intersection of these two topics is

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what presents the research's main concerns and renders the topic intriguing and worthwhile of further investigation. A natural deformation of classical Lie theory is provided by the ALiAs theory, which may be of interest to physicists. In particular, it keeps the Cartan matrix, maintaining the classical theory's finitely produced nature.

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Conflict of Interest

No conflict of interest.

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