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An Overview of Riemannian Symmetric Space

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Description

A fundamental and obvious question in the study of mathematics is how geometry affects the analytical properties and the behavior of PDE solutions. The most fascinating and important manifolds in Riemannian geometry are symmetric spaces. Élie Cartan found and thoroughly categorized these areas. In many areas of mathematics, including differential geometry, algebraic topology, number theory, harmonic analysis, etc., they provide a rich and comprehensible class of manifolds. There are numerous ways to characterize symmetric spaces. A symmetric space is a manifold that extends the idea of central symmetry on geodesics, according to intuition. It has what is known as geodesic symmetry around every point. A symmetric space is defined by the Lie theory as the product of a connected Lie group G and a compact Lie subgroup K. Since the 1950s, the field of harmonic analysis on symmetric spaces—also known as spherical Fourier analysis—has grown significantly. Following the groundbreaking work on spherical functions by Israel Gelfand and Harish-Chandra, their disciples, including Sigurur Helgason, Ramesh Gangolli, Veeravalli S. Varadarajan, etc., further improved the basic analysis tools. Numerous important branches of mathematics, including number theory, representation theory, Fourier analysis, and PDE, are connected by the spherical Fourier analysis. It logically develops into a major branch of modern mathematics [1-3].

Harmonic analysis on symmetric spaces, often called spherical Fourier analysis, has developed greatly since the 1950s. Following Israel Gelfand and Harish-Chandra's and others' revolutionary work on spherical functions, their students, such as Sigurur Helgason, Ramesh Gangolli, Veeravalli S. Varadarajan, etc., further enhanced the fundamental analysis tools. The spherical Fourier analysis links a number of significant fields of mathematics, including number theory, representation theory, Fourier analysis, and PDE. It logically evolves into a significant area of contemporary mathematics. A Riemannian manifold, or more generally, a pseudo-Riemannian manifold, is defined as a symmetric space in mathematics if its set of symmetries contains an inversion symmetry for each point. This can be investigated algebraically using Lie theory, which enabled Cartan to provide a comprehensive classification, or geometrically using Riemannian geometry, with implications for the theory of holonomy. Differential geometry, representation theory, and harmonic analysis frequently use symmetric spaces. If and only if a full, simply linked Riemannian manifold's curvature tensor is invariant under parallel transport, it is said to be a symmetric space in terms of geometry. More specifically, a Riemannian manifold (M, g) is said to be symmetric if and only if there is an isometry of M acting on the tangent space T pMT pM as minus the identity for each point p of M, naturally, both descriptions can be expanded to apply to the context of pseudo-Riemannian manifolds [4].

A symmetric space is, from the perspective of Lie theory, the product of the connected Lie group G and the Lie subgroup H, where G is (a connected component of) the invariant group of an involution of H. When H is compact, this definition simplifies to the Riemannian definition but provides extra information. There are numerous instances of Riemannian symmetric spaces in both mathematics and physics. Marcel Berger made the discovery that they play a crucial part in the holonomy theory. In addition to differential geometry, they are significant research topics in harmonic analysis and representation theory. According to the Cartan-Ambrose-Hicks theorem, M is locally Riemannian symmetric if and only if its curvature tensor is covariantly constant. It also states that every simply linked, complete locally Riemannian symmetric space is genuinely Riemannian symmetric.Every M-dimensional Riemannian symmetric space is Riemannian homogeneous and complete. In reality, the isometry group's identity component already works transitively on M.

Non-Riemannian symmetric locally Riemannian symmetric spaces can be created as open subsets of (locally) Riemannian symmetric spaces and as discrete groups of isometries with no fixed points as quotients of Riemannian symmetric spaces. Euclidean space, spheres, projective spaces, and hyperbolic spaces are all fundamental instances of Riemannian symmetric spaces, each having their own set of Riemannian metrics. Compact, semi-simple Lie groups with a bi-invariant Riemannian metric offer more examples. Any compact Riemann surface with a genus greater than 1 and the standard metric of constant curvature 1 is a locally symmetric space but not a symmetric space. With the exception of L(2,1), which is symmetric, every lens space is locally symmetric but not symmetric. By a discrete isometry with no fixed points, the lens spaces are quotients of the 3-sphere. Anti-de Sitter space is an illustration of a non-Riemannian symmetric space.

Scheme for classification

If a simply connected Riemannian symmetric space is not the union of two or more Riemannian symmetric spaces, it is said to be irreducible. Then, it can be demonstrated that each Riemannian symmetric space with a single connection is a Riemannian product of irreducible ones. As a result, we can further limit our classification to the Riemannian symmetric spaces that are irreducible and simply connected [5].

The second step is to demonstrate which of the following three categories each irreducible, straightforwardly connected Riemannian symmetric space M belongs to:

Euclidean type: M is isometric to a Euclidean space since it has vanishing curvature.

Compact type: M has sectional curvature that is nonnegative but not exactly zero.

Non-compact type: M has a sectional curvature that is nonpositive (but not quite zero).

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Conflict of Interest

The author shows no conflict of interest towards this manuscript.

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