

Topological and Algebraic Perspectives on the Geometry of Physical Measurements

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Introduction

Quantale-based aspects of noncommutative topology are extended by our investigation of the mathematical structure of the concept of measurement space. The result is a geometric model of physical measurements that shows how measurements and classical information are interdependent as primitive concepts and is both realistic and practical. Bohr's divide between classical and quantum is mathematically represented by a derived concept of the classical observer. C^* -algebras and second-countable locally compact open sober topological groupoids, respectively, yield two significant classes of measurement spaces. The latter refer to Schwinger's concept of selective measurement and produce measurements of the classical type. We establish a correlation between the properties of the observer and those of the groupoid and demonstrate that the measurement space associated with the reduced C^* -algebra of any second-countable locally compact Hausdorff étale groupoid is canonically equipped with a classical observer [1].

Description

Mulvey has hypothesized that suitable lattices of closed linear subspaces of a C^* -algebra A equipped with the naturally induced multiplication may play the role of spectrum of A . This notion of spectrum is in line with pointfree topology which holds that the location of open sets in a space rather than the space of points itself is the primary subject of investigation. The Gelfand–Naimark representation theorem ensures that the locale contains all the information about A , if A is commutative. However, there is no reason why the multiplication of more general subspaces should coincide with intersection, and Mulvey's idea was more ambitious because it aimed to find a general definition of spectrum such that any C^* -algebra, not just commutative ones, can be fully reconstructed from its spectrum. This is an example of this Quantales were the name given to such multiplication-equipped lattices that generalize locations; for a general overview, see. Quantum physics, in particular the idea that the multiplication can be interpreted as a temporal structure, served as the basis for this terminology: Rather than reading the product ab as "b and then a," think of it as two consecutive quantum measurements [2].

During the early years of quantale theory, when special attention was paid to right-sided quantales, such as those of closed right ideals of C^* -algebras a related operational interpretation of quantales was picked up in computer science and then in a quantale-based construction of Connes' noncommutative space of Penrose tilings. However, for general C^* -algebras, these do not contain sufficient information. Mulvey made another attempt, inspired in part by Rosicky's quantum frames, and proposed addressing the much larger quantale $\text{Max } A$ of all the closed linear subspaces of a C^* -algebra A with the naturally induced involution. The interplay between the whole quantale and its right-sided part plays an important role because the latter carries the noncommutative topology

of Giles and Kummer. From this fact and from the classification of irreducible representations of Mulvey and Pelletier, the surprising fact that the quantale $\text{Max } A$ is a complete invariant for unital C^* -algebras follows [2].

In the sense of Connes, C^* -algebras are the fundamental objects of noncommutative geometry; however, other structures play an important role as well. These include locally compact groupoids, particularly Lie groupoids, which are used to model quotients of spaces, foliated manifolds, or spaces with local symmetries. From these groupoids, C^* -algebras are then obtained as convolution algebra completions. In a broader sense, C^* -algebras can be derived from convolution algebras of sections of Fell bundles on groupoids. Furthermore, the "diagonals" of the C^* -algebras of the Fell line bundles that arise from central circle extensions of étale groupoids provide important characterizations of these bundles. Because, for instance, a precise algebraic characterization of the quantale associated with an étale groupoid G exists which furthermore can be framed in terms of a (bi-)categorical equivalence quantales have a close relationship to groupoids as well. In fact, quantales are better understood than C^* -algebras. Quantales are also closely related to groupoids. Fell bundles on G also have a lot in common with quantal maps [3].

This paper aims to make a connection between the aforementioned advancements and the initial operational interpretation of quantales in the context of quantum measurements. The primary goal is to provide an account of measurements that is also free of the conceptual paradoxes that have plagued quantum mechanics since its inception, rather than just to develop a mathematical theory for its own sake. This necessitates a new way of thinking: While systems' states are typically thought of as the fundamental "elements of reality," even if implicitly so, the measurements themselves are taken to be the elements of reality in this paper (for more on this justification, see section 2). Understanding the mathematical structure of measurement spaces and their relationship to state spaces is necessary to accomplish this. In algebraic quantum mechanics and algebraic quantum field theory, for instance, systems are described by C^* -algebras that contain their observables. In this paper, the quantale $\text{Max } A$, which is itself equipped with a suitable topology, will be regarded as the space of measurements associated with the system described by A if A is obtained from a suitable groupoid G . The relationship between the measurement space $\text{Max } A$ and the classical state space, which is the space of groupoid units 0 with symmetries carried by the groupoid structure, is mathematically described [4,5].

Conclusion

In addition to being an abstract mathematical structure, the topology of a space of measurements $\text{Max } A$ carries its own conceptual significance: Open sets are regarded as physical properties that carry a limited amount of classical information about the system being observed and can be measured with a finite number of resources. In this concept and an abstract definition of measurement space were presented with the intention of demonstrating a theory that considers measurements and classical information to be primitive concepts and, in a sense, yielding a model similar to Wheeler's it from bit. The purpose of this paper is to add to this by providing both a solid introduction to the mathematics of measurement spaces and a comprehensive account of the main examples, occasionally delving back into the reasoning behind the chosen mathematical structures, such as demonstrating that some imposed axioms.

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Conflict of Interest

None.

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