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Comparative Study of First-Order Elastic and Second-Order Inelastic Analysis of Steel Structure

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Abstract

As IS 800:1984 has changed recommend actions from working stress to limit state design in year 2007, this required change in analysis from elastic to inelastic. Present commercial software's are not incorporating inelastic analysis *i.e.* plastic analysis. Hence the use of limit state design has been declined by structural designer. In this project study review of different methods of analysis of steel frame is made.

Keywords: First order elastic analysis • Second order elastic analysis • First order inelastic analysis • Second order inelastic analysis • Advance analysis

Introduction

Analysis of steel framed structure involves more complexity when compared with RC framed structure. The steel material behavior and its mechanical properties adds problems in analysis of steel structure to consider the effects of residual stresses, initial geometric imperfections and tendency of buckling, also steel structures are capable to undergo large deformation before collapse. This causes second order effects [1]. Steel possesses excellent ductility and post elastic strength. All these facts make the analysis of steel framed structure more complex especially when the design is required by limit state concept [2].

A brief review of advancement in different methods of analysis is presented in following text.

First-order elastic analysis

The first and common step in structural analysis is first-order elastic analysis, which is also called simply elastic analysis. In this case, deformations are assumed to be small so that the equation of equilibrium may be written with reference to un-deformed configuration of structure. Additionally, superposition is valid and any inelastic behavior of material is ignored [3]. This approach is used in the development of common analysis tools of profession, such as slope deflection method, moment distribution method that is found in most computer software.

Second-order elastic analysis

When equilibrium is expressed with reference to deformed shape of member as well as structure, the resulting analysis is a second order elastic analysis (Figure 1). It is a geometrical nonlinear analysis. Analysis includes member deformation *i.e.* P-δ effect and also sways *i.e.* P-Δ effect [4].

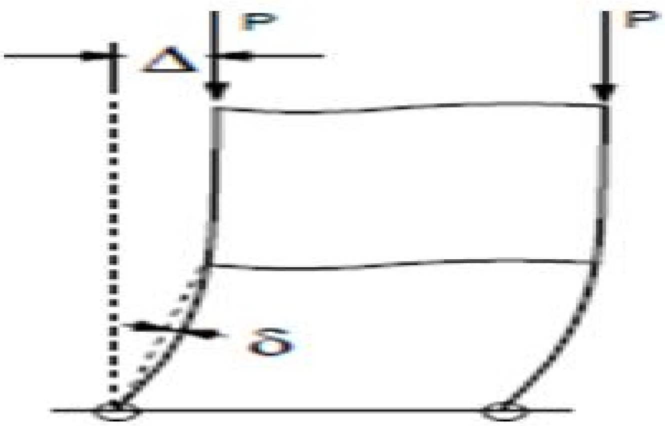

Figure 1. The P-Δ and P-δ effects.

Second order elastic analysis accounts for elastic stability effect but does not indicate limit strength. The load displacement history by this analysis may approach the critical buckling load obtained from the Eigen value solution which requires an iterative process. Hence this type of analysis is more complex than the first order elastic analysis.

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Materials and Methods

First order inelastic analysis

This analysis accounts for post elastic strength of member of the structure. Therefore it is also known as material non-linear analysis. In progressive loading and when elastic limit is crossed highly stressed section of the member yields completely and the section behaves a hinge known as plastic hinge. When it happens the particular section continues to resist plastic moment and undergoes large deformation. Progressive loading is continued till sufficient number of plastic hinges is developed and structure no longer resists any further additional load due to transformation of structure into a mechanism and hence it is said to be a plastic collapse. In this analysis, member deformations and sway effect of structure are not considered therefore the analysis does not reflect buckling and stability assessment [5].

Second order inelastic analysis

This analysis is nothing but the addition of effects of member deformation and drift effect of the structure in first order inelastic analysis. This gives complete, realistic and accurate analysis but makes the process complex. This analysis includes both geometric and material non-linearity and known as "advanced analysis". This advanced analysis is further classified into following categories.

Elastic-plastic hinge method: It is simple, approximate and efficient for representing inelasticity in frames. In this method, zero length plastic hinges are assumed to form at the ends of members, whereas other portions are assumed to remain elastic. Thus, it accounts for inelasticity but disregards the spread of yielding and residual stress effects between the plastic hinges.

The elastic–plastic hinge method can be first-order or second-order plastic analysis. The first-order elastic–plastic hinge methods, in which the non-linear geometric effects are neglected, predict the same ultimate load as conventional rigid-plastic analysis. In second-order elastic–plastic hinge analysis the deformed structural geometry is considered for formulating the stiffness equation.

Plastic zone method: In this method the cross section is subdivided in to small sub-elements, the residual stresses are considered constant within each sub-element. The stress state at each sub-element can be traced clearly and hence the gradual spread of yielding can be predicted. The plastic zone method eliminates the need for separate member capacity check, hence this method accepted to provide exact solution.

Refined plastichinge method: This approach is a refined version of elastic-plastic hinge approach. This method considers gradual stiffness degradation of plastic hinge section as well as gradual stiffness degradation of member between two plastic hinges [6].

To illustrate their theories following case studies are worked out.

Results and Discussion

A case study for validation of MASTAN software

In a warehouse, an area of 10 m × 40 m is to be covered by rectangular portals to be placed at 5 c\c if floor consist of 150 mm thick RCC slab with 50 mm thick finishing, design section of frame. The live load on frame is 5 kn/m^2. The effect of wind can be considering a working horizontal force of 12 kN at the floor level, which is 5 m above the base.

Solution

Load calculation

Self-weight of slab: 5 m × 0.15 m × 1 m × 25 kN/m^3=18.75 kN/m

Finishing: 5 m × 0.050 m × 1 m × 20 kN/m^3=5 kN/m

Self-weight of beam: 2 kN/m

Total dead load on frame: 25.75 kN/m=26 kN/m

Live load: 5 kN/m^2 × 5 m=25 kN/m

Elastic analysis

Design load combination: (Dead load+Live load)=(25+26)=51 kN/m

The analysis is explained in book "Limit state design in structural steel", Dr. M. R. Shiyekar, page no: 275-276 (Figure 2).

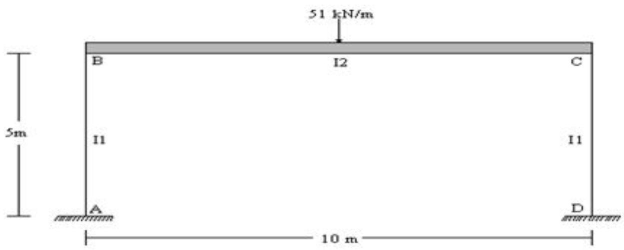

Figure 2. Limit state design in structural steel.

Results: Me=340 kNm.

Elastic analysis by using software

Section used for beam and column ISWB 600 (Tables 1 and 2).

Properties		
A	1.42E-02	m^2

Izz (Ixx)	7.43E-04	m^4
Iyy	4.59E-05	m^4
J	1.11E-06	m^4
Cw	3.25E-06	m^6
Zzz (Zp=Zxx)	3.11E-03	m^3
Zy	5.64E-04	m^3

Table 1. Properties.

Material properties		
E	2E+08	kN/m^2
v	0.3	-
Fy	250000	kN/m^2
wt. Density	77.04	kN/m^3

Table 2. Material properties.

Load applied on frame (UDL)=(DL+LL)=(26+25)=51 kN/m (Figure 3).

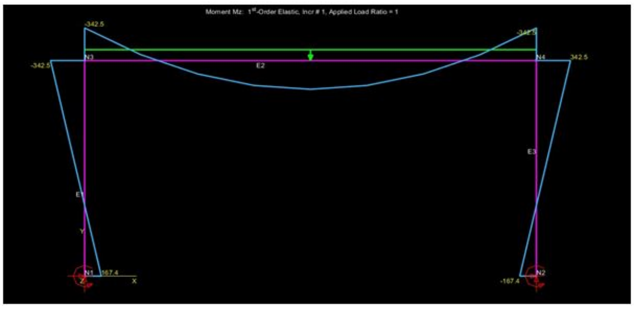

Figure 3. Output for elastic analysis of 1 bay 1 floor (DL+ LL).

Results: Max B. M. at Node 3 and 4=342.5 kNm.

Plastic analysis

Design load combination: 1.5(Dead load+Live load)=1.5(25 +26)=76.5 kN/m (Figure 4).

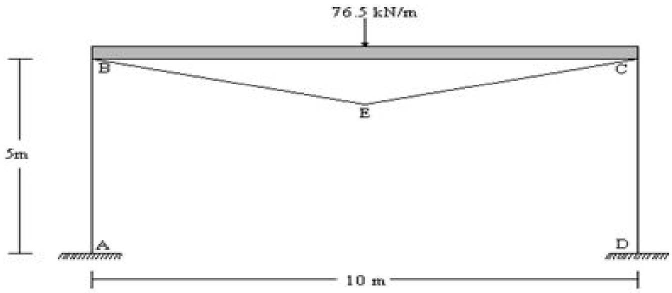

Figure 4. Plastic analysis.

Beam mechanism: Plastic hinges will develop at B and C in weaker section, *i.e.* in the column and at the center of beam E $Mp=wl^2/16=(76.5 \times 10^2)/16=478.125$ kNm (Tables 3 and 4).

Plastic analysis by using software: Section used for beam and column ISWB550.

Properties		
A	1.42E-02	m^2
Izz (Ixx)	7.43E-04	m^4
Iyy	4.59E-05	m^4
J	1.11E-06	m^4
Cw	3.25E-06	m^6
Zzz(Zp=Zxx)	3.11E-03	m^3
Zyy	5.64E-04	m^3

Table 3. Properties.

Material properties		
E	2E+08	kN/m^2
v	0.3	-
Fy	250000	kN/m^2
wt. Density	77.04	kN/m^3

Table 4. Material properties.

Load applied on frame (UDL)=1.5 × (DL+LL)=1.5 × (26+25)=76.5 kN/m (Figure 5).

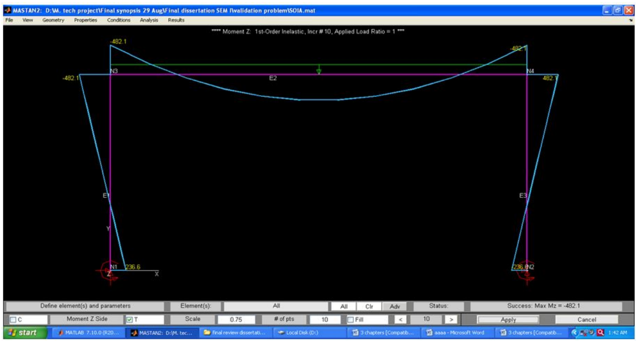

Figure 5. Output for inelastic analysis of 1 bay 1 floor (DL+LL).

Results: Max B. M. at Node 3 and 4=482.1 kNm.

Software analysis of multiday, multistory rectangular portal frame

Two bays, three bays and four bays up to five story portal frames are solved by FOEA and SOIA methods using software and its results are compared [7]. Out of these no of solution, 2 bay 3 story portal frame analysis for DL+LL combination are given as under (Tables 5 and 6).

Problem

Public building of 2 bay 3 story; distance between two frame c \c=4 m; thickness of slab=150 mm; live load intensity=4 kN/m^2; floor finish intensity=1.5 kN/m^2; design wind pressure=1 kN/m^2 (Figure 6).

Solution: DL+LL Load calculation

Self-weight of slab: 0.150 m × 25 kN/m^3=3.75 kN/m^2

Finishing: 1.5 kN/m^2

Total DL intensity=3.75+1.5=5.25 kN/m^2

Live load intensity=4 kN/m^2

Aspect ratio (β)=Ly/Lx

=5/4=1.25

Equivalent trapezoidal DL load on long beam

P=(W × lx)/2 (1-(1/(3 × β × β)))

P=8.264 × 2 kN/m

P=16.527 kN/m

Weight of brick wall per meter=0.23 m × 4 m × 20 kNm^3 =18.4 kN/m

Self-Weight of beam per meter=0.852 kN/m

Total DL on beam=16.527+18.4+0.852=35.78 kN/m

Equivalent trapezoidal LL load on long beam

P=(W × lx)/2 (1-(1/(3 × β × β)))

P=6.296 kN/m

Total LL on beam=2 × 6.29=12.592 kN/m

Total DL+LL on beam per meter=35.78+12.592=48.37 kN/m

For inelastic analysis factored load is consider=1.5 × 48.37 =72.56 kN/m

FOEA (First Order Elastic Analysis)

ISMB 350 beams and column

Properties		
A	6.58E-03	m^2
Izz (Ixx)	1.35E-04	m^4
Iyy	6.51E-06	m^4
J	3.24E-07	m^4
Cw	1.83E-07	m^6
Zzz(Zp=Zxx)	8.85E-04	m^3
Zyy	1.44E-04	m^3

Table 5. Properties.

Material properties		
E	2E+08	kN/m^2
v	0.3	-
Fy	250000	kN/m^2
wt. Density	77.04	kN/m^3

Table 6. Material properties.

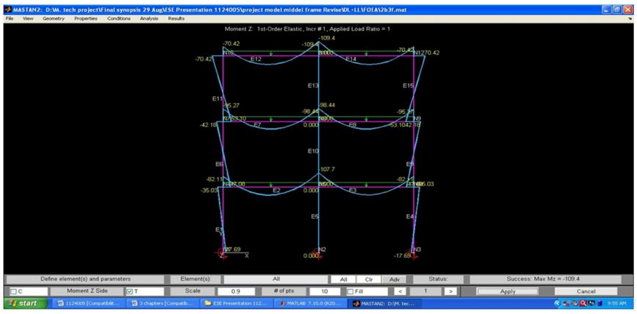

Figure 6. Output for first-order elastic analysis of 2 bay 3 story (DL+LL)

Result: Max B. M.=109.4 kNm.

SOIA (Second Order Inelastic Analysis), ISMB 300 beam and column (Tables 7 and 8).

Properties		
A	5.54E-03	m^2
Izz (Ixx)	8.49E-05	m^4
Iyy	5.68E-06	m^4
J	2.17E-07	m^4
Cw	1.17E-07	m^6
Zzz(Zp=Zxx)	6.51E-04	m^3
Zyy	1.25E-04	m^3

Table 7. Properties.

Material properties		
E	2E+08	kN/m^2
v	0.3	-
Fy	250000	kN/m^2
wt. Density	77.084	kN/m^3

Table 8. Material properties.

For inelastic analysis factored load is consider, 1.5(DL +LL)=72.56 kN/m (Figure 7).

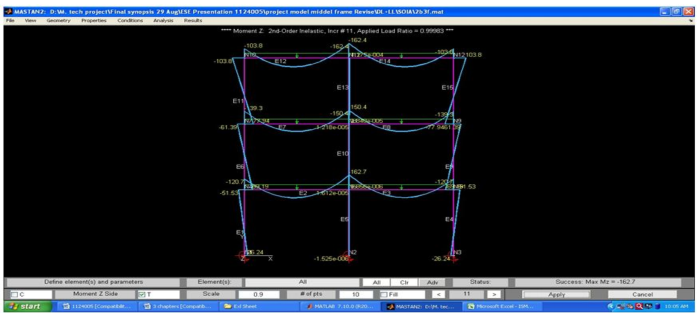

Figure 7. Output for second order Inelastic analysis of 2 bay 3 story (DL+LL).

Results Max B.M.=162.7 kNm

No. of plastic hinges develop: 4.

In previous chapter sample case studies are shown. Analysis of portal frames is performed for DL+LL, combinations by FOEA and SOIA methods [8-10]. Different parameters considered are number of bay, number of story, and different loading combinations. The results obtained from analysis are listed in tabular form are as follows, summary of results (Table 9).

Model	Section use for FOEA	Section use for SOIA	FOEA Max B.M (Me) kNm	SOIA Max B.M (Mp) kNm	No. of plastic hinges develop	Ratio Mp/Me
2 bay 1 story	ISMB 350	ISMB 300	122.2	162.5	2	1.329
2 bay 2 story	ISMB 350	ISMB 300	114.9	162.4	4	1.413
2 bay 3 story	ISMB 350	ISMB 300	109.4	162.4	4	1.484
2 bay 4 story	ISMB 350	ISMB 300	105.5	147.9	8	1.401
2 bay 5 story	ISMB 350	ISMB 350	110.4	208.7	2	1.89
3 bay 1 story	ISMB 350	ISMB 300	115.7	162.6	2	1.405
3 bay 2 story	ISMB 350	ISMB 300	111.4	162.4	2	1.457
3 bay 3 story	ISMB 350	ISMB 300	107.2	162.4	2	1.514
3 bay 4 story	ISMB 350	ISMB 300	104.6	148	12	1.414
3 bay 5 story	ISMB 350	ISLB 325+6p	109	116.2	2	1.066
4 bay 1 story	ISMB 350	ISMB 300	116.8	162.6	2	1.392
4 bay 2 story	ISMB 350	ISMB 300	112.2	162.4	2	1.447
4 bay 3 story	ISMB 350	ISMB 300	107.8	162.4	2	1.504
4 bay 4 story	ISMB 350	ISMB 300	104.7	148.2	8	1.415
4 bay 5 story	ISMB 350	ISWB 350	108.9	127.5	2	1.17

Table 9. Output for FOEA and SOIA for DL+LL combination.

Conclusions

- In elastic analysis working load in DL+LL combination is 48.37 kN/m while in inelastic analysis it is the factored load for DL +LL combination, 1.5 × 48.37=72.56 kN/m. It is interesting to note that maximum moment in inelastic analysis is less than 1.5 times that of elastic analysis.
- Software is validated and results of both FOEA and SOIA are found to be in close agreement with manual calculations
- As the degree of indeterminacy of structure increases the limit state of collapse by SOIA is due to formation of plastic hinges as well as buckling of member.
- In most cases design moment by FOEA and SOIA are found to be below 1.5. This results in economy in selection of section for frame.

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