# Weak topological functors <sup>1</sup>

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#### Abstract

We introduce weak topological functors and show that they lift and preserve weak limits and weak colimits. We also show that if  $A \to B$  is a topological functor and J is a category, then the induced functor  $A^J \to B^J$  is topological. These results are applied to a generalization of Wyler's top categories and in particular to functor categories of fuzzy maps, fuzzy relations, fuzzy topological spaces and fuzzy measurable spaces.

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### 1 Introduction

Almost forty years ago, Zadeh [13] introduced the category of fuzzy sets. The objects in this category are maps from ordinary sets to the unit interval and the morphisms are ordinary nondecreasing maps with respect to the fuzzy sets. Two years later, Goguen [5] replaced the unit interval by an arbitrary complete ordered lattice. Since then a lot of work has been devoted to proposing different versions of what fuzzy algebras of distinct types may be, e.g. fuzzy semigroups, fuzzy groups, fuzzy rings, fuzzy ideals, fuzzy semirings, fuzzy near-rings (see [8] for an overview). Fuzziness has also been introduced in various topological settings, e.g. fuzzy topological spaces [4], fuzzy measurable spaces [9] and fuzzy topological groups [2]. Categories of fuzzy structures are often topological over their corresponding base categories, that is, the category of modules in the case of fuzzy modules [10], the category of groups in the case with fuzzy topological groups [3] and so on. This is an important fact since it implies that many properties of the top category, such as completeness and cocompleteness, are automatically inherited from the ground category. However, the theory of topological functors (see e.g. [1]) does not cover the lifting of weak limits and colimits (that is, the uniqueness property of the mediating morphism is dropped). To cover these cases, we introduce *weak topological functors* (see Definition 2.1) and show a lifting result for such functors (see Proposition 2.2). To simultaneously treat all fuzzy algebraical constructions, we show a result (see Proposition 2.3) concerning the topologicality of functors between functor categories. In the end of the article (see Section 3), these results are applied to a generalization (see Proposition 2.5) of Wyler [12] to prove (weak) completeness and cocompleteness results for functor categories of categories of fuzzy maps, fuzzy relations, fuzzy continuous maps and fuzzy measurable maps.

# 2 Topological functors

Let A be a category. The family of objects and the family of morphisms in A is denoted ob(A)and mor(A) respectively. The domain and codomain of a morphism  $\alpha$  in A is denoted  $d(\alpha)$  and  $c(\alpha)$  respectively. The composition of two morphisms  $\alpha$  and  $\beta$  in A with  $d(\alpha) = c(\beta)$  is denoted  $\alpha\beta$ . The identity morphism at  $a \in ob(A)$  is denoted  $id_a$ . We let  $hom_C(a, b)$  denote the collection of morphisms from a to b. Let Cat denote the category with small categories as objects and

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functors between such categories as morphisms. If B is another category, then let  $B^A$  denote the category with functors from A to B as objects and natural transformations between such functors as morphisms. If  $U \in ob(B^A)$  and  $b \in ob(B)$ , then we let  $U_b$  denote the corresponding fibre category in the sense of Grothendieck [6], that is, the subcategory of A having as objects all  $a \in ob(A)$  with U(a) = b and as morphisms all  $\alpha : a \to a'$  in mor(A) between such objects with  $U(\alpha) = id_b$ . A (simple) source in B is a pair  $S = (b, (\beta_i)_{i \in I})$  consisting of an object b in B and a family of morphisms  $\beta_i : b \to b_i$  in B indexed by some class I (with cardinality one). If  $b_i = U(a_i), i \in I$ , for some objects  $a_i \in ob(A)$ , then S is called U-structured. A U-lift of such a U-structured source S is a source  $\hat{S} = (a, (\alpha_i)_{i \in I})$  in A where  $\alpha_i : a \to a_i$  satisfy  $U(\alpha_i) = \beta_i$ ,  $i \in I$ . We say that such a U-lift of S is weakly initial if for any U-lift  $\hat{S}' = (a', (\alpha'_i)_{i \in I})$  of a U-structured source  $S' = (b', (b_i)_{i \in I})$ , equipped with a  $\gamma : b' \to b$  in B with the property that  $\beta'_i = \beta_i \gamma, i \in I$ , there is a  $\hat{\gamma} : a' \to a$  in A with  $U(\hat{\gamma}) = \gamma$  and  $\alpha'_i = \alpha_i \hat{\gamma}, i \in I$ . A weak initial U-lift is called initial if the morphism  $\hat{\gamma}$  is unique. Concepts dual to "initial" are called "final".

**Proposition 2.1.** Let  $U : A \to B$  be a functor. (a) If each U-structured source (sink) in B has a weak initial (final) U-lift, then U is faithful. (b) Each U-structured source in B has a weak initial U-lift if and only if each U-structured sink in B has a weak final U-lift. (c) Each U-structured source in B has a unique initial U-lift if and only if each U-structured sink in B has a weak final U-lift.

**Proof.** Adapt the proofs of Theorems 21.3 and 21.9 in [1] to the weak situation.  $\Box$ 

**Definition 2.1.** We call a functor satisfying either of the equivalent criteria in Proposition 2.1(c) (or (b)) (weakly) topological. Note that our definition of topological functor coincides with the one given by Herrlich [7].

**Proposition 2.2.** Let  $U : A \to B$  be a weakly topological functor. (a) If  $F : J \to A$  is a functor with weak (co)limit L, then U(L) is a weak (co)limit of UF; (b) If L is a weak (co)limit of UF, then there is a weak (co)limit  $\hat{L}$  of F such that  $U(\hat{L}) = L$ ; (c) A is weakly (co)complete if and only if B is weakly (co)complete; (d) If U is topological, then (a) and (b) hold with weakness removed. Furthermore, in that case, the (co)limit  $\hat{L}$  is unique subject to the condition  $U(\hat{L}) = L$ .

**Proof.** We only show the "limit" part of the result. The proof of the "colimit" part is dual and is therefore left to the reader. (a) Suppose that L is a limit of  $F: J \to A$  with morphisms  $p_i: L \to F(i), i \in ob(J)$ , such that  $p_j = \alpha p_i$  for all  $\alpha: i \to j$  in J. Suppose that there are  $p'_i: X \to UF(i), i \in ob(J)$ , with the property that  $p'_j = UF(\alpha)p'_i$ , for all  $\alpha: i \to j$  in J. By weak topologicality of U, there is a U-lift  $\widehat{X}$  of X and  $\hat{p'}_i: \widehat{X} \to F(i), i \in ob(J)$ , with  $U(p'_i) = p'_i, i \in J$ . By Proposition 2.1(a) U is faithful. Therefore the equality  $U(\alpha \hat{p'}_i) =$  $U(\alpha)U(\hat{p'}_i) = U(\alpha)p'_i = g_j = U(\hat{p'}_j)$  implies that  $\alpha \hat{p'}_i = \hat{p'}_j$ . Then there is  $m: \widehat{X} \to L$  with  $p_i m = p'_i, i \in ob(J)$ . This implies that  $U(m): X \to U(L)$  is the desired map. Hence U(L) is a weak limit of UF. Now we show (b). Suppose that L is a weak limit of UF with morphisms  $p_i: L \to UF(i)$  such that  $p_j = UF(\alpha)p_i$  for all  $\alpha: i \to j$  in J. Let  $(L, \hat{p}_i)$  be a weak initial U-lift of the U-structured source  $(L, p_i)$ . By U-faithfullness,  $\hat{p}_j = F(\alpha)\hat{p}_i$  for all morphisms  $\alpha : i \to j$ in J. Now suppose that there is an object X in A and morphisms  $q_i: X \to F(i), i \in ob(J)$ , such that  $q_i = F(\alpha)q_i$  for all morphisms  $\alpha : i \to j$ . Since L is a weak limit of UF, there is a morphism  $\gamma: U(X) \to L$  with  $U(q_i) = \gamma p_i$ . By weak initiality there is a morphism  $\gamma: X \to \widehat{L}$ in A such that  $q_i = \hat{p}_i \gamma$ , which implies that  $(\hat{L}, \hat{p}_i)$  is a weak limit of F. (c) follows directly from (a) and (b). (d) is Proposition 21.5 in [1]. 

**Proposition 2.3.** If J is a category and  $U: A \to B$  is a topological functor, then the induced functor  $U^J: A^J \to B^J$  is topological.

**Proof.** Let  $n^i: G \to G_i = U^J(F_i), i \in I$ , be a  $U^J$ -structured source in  $B^J$ . Associated to this source, we define a functor  $F: J \to A$  in the following way. For each  $j \in ob(J)$  let  $m_j^i$ :  $F(j) \to F_i(j)$  be the unique initial lift of the U-structured source  $n_j^i: G(j) \to G_i(j) = UF_i(j)$ ,  $i \in I$ . Take a morphism  $\alpha : j \to j'$  in J. Since,  $G_i(\alpha)n_j^i = n_{j'}^i G(\alpha), i \in I$ , there is, by initiality of the lift, a unique  $F(\alpha): F(j) \to F(j')$  with the property that  $F_i(\alpha)m_j^i = m_{j'}F(\alpha)$ ,  $i \in I$ . To show that F is a functor we need to show that F respects composition of morphisms. Take another morphism  $\alpha': j' \to j''$  in J. By uniqueness and initiality of  $F(\alpha'\alpha)$  and the fact that the following chain of equalities holds  $m_{i''}^i F(\alpha')F(\alpha) = F_i(\alpha')m_{i'}^i F(\alpha) = F_i(\alpha')F_i(\alpha)m_{i'}^i =$  $F_i(\alpha'\alpha)m_i^i = m_{i''}^i F(\alpha'\alpha), i \in ob(I)$ , we get that  $F(\alpha')F(\alpha) = F(\alpha'\alpha)$ . Also note that since U is topological, the functor F is unique subject to the condition that  $U^{J}(F) = G$ . Define a collection of natural transformations  $m^i: F \to F_i, i \in I$ , by the morphisms  $m^i_j: F(j) \to F_i(j), j \in ob(J)$ . By the construction of the  $m^i$ ,  $i \in I$ , we get that they are indeed natural transformations. Suppose that  $n^i: G' \to G_i, i \in I$ , and  $q: G' \to G$  are natural transformations satisfying  $n^i q = n^i, i \in I$ . Suppose that there are natural transformations  $m^i : F' \to F_i, i \in I$ , in  $A^J$  with  $U^{J}(m^{i}) = n^{i}, i \in I$ . Now we define a natural transformation  $p: F' \to F$  in  $A^{J}$ , with  $m^{i}p = m^{i}$ , in the following way. Since U is topological there is for each  $j \in ob(J)$  a unique morphism  $p_j: F'(j) \to F(j)$  in A subject to the conditions that  $m_i^i p_j = m_i^i, i \in I$ , and  $U(p_j) = q_j$ . Define p by the morphisms  $p_j, j \in ob(J)$ . Take a morphism  $\alpha : j \to j'$  in J. By the definition of F, we get that  $F_i(\alpha)m_i^i = m_{i'}^i F(\alpha), i \in I$ . Therefore,  $m_{i'}^i F(\alpha)p_i = F_i(\alpha)m_i^i p_i = F_i(\alpha)'m_i^i, i \in I$ . Since each  $m^i$  is a natural transformation, we get that  $F_i(\alpha)'m_i^i = m_i^i F'(\alpha)$  and hence that  $m_{j'}^i p_{j'} F'(\alpha) = m_{j'}^i F'(\alpha) = m_{j'}^i F(\alpha) p_j, i \in I.$  Since  $m_{j'}^i : F(j') \to F_i(j'), i \in I$ , is the unique initial lift of  $n_{j'}^i: G(j') \to G_i(j'), i \in I, r := m_{j'}^i p_{j'} F'(\alpha) = m_{j'}^i F(\alpha) p_j: F'(j) \to F_i(j'), i \in I,$ is a lift of  $n_{j'}^i q_{j'} G'(\alpha) = n_{j'}^i G(\alpha) q_j : G'(j) \to G_i(j'), i \in I, n^i q = n^i, i \in I$ , and the morphisms  $s_1 := p_{j'}F(\alpha)$  and  $s_2 := F(\alpha)p_j$  satisfy  $m_{j'}s_1 = r = m_{j'}s_2, i \in I$ , we get that  $s_1 = s_2$  and hence that p is natural transformation. 

Let  $U : A \to B$  be a functor. If U is weakly topological then each fibre of U has weak (co)products. In fact, every simple U-structured source in B has a weak U-initial lift. Take  $b \in ob(B)$  and suppose that we have a set  $a_i, i \in I$ , of objects in  $U_b$ . Then the identity morphisms  $id_b : b \to U(a_i), i \in I$ , considered as a U-structured source, has a weak initial U-lift  $\alpha_i : a \to a_i, i \in I$ . The weak initiality of this lift is equivalent to the condition that  $(a, (\alpha_i)_{i \in I})$ is a weak product of  $a_i, i \in I$ , in  $U_b$ . In the same way one can show that if U is topological, then the weak (co)product above makes each fibre of U a complete ordered lattice (see Proposition 21.11 in [1]). If we make two additional assumptions, then this condition is sufficient for U to be weakly topological:

**Proposition 2.4.** (a) A functor  $U : A \to B$  is weakly topological if the following three properties hold: (i) Each simple U-structured source in B has a weak initial lift; (ii) Each fibre of U has weak products; (iii) The weak products in the fibres of U are weak products in A. (b) A functor  $U : A \to B$  is topological if the following three properties hold: (i) Each simple U-structured source in B has a unique initial lift; (ii) Each fibre of U is a complete ordered lattice; (iii) The infima in the fibres of U are products in A.

**Proof.** Suppose that (i), (ii) and (iii) in (a) hold. Suppose that  $\beta_j : b \to b_j, j \in J$ , is a *U*-structured source with  $b_j = U(a_j), j \in J$ . For each  $j \in J$  let  $a^j$  be a weak *U*-initial lift of  $b_j$  with morphisms  $\alpha^j : a^j \to a_j$  of  $\beta_j, j \in J$ . Let *a* be a weak product in  $U_b$  of the  $a^j, j \in J$ , with morphisms  $p_j : a \to a^j$  in  $U_b$ . Put  $\alpha_j := \alpha^j p_j, j \in J$ . Suppose that  $\beta : b' \to b$  and  $\beta'_j : b' \to b_j$  satisfy  $\beta'_j \beta = \beta'_j, j \in J$ . Suppose that  $\alpha'_j : a' \to a_j$  is a *U*-lift of  $\beta'_j, j \in J$ . By simple *U*-initiality, there are morphisms  $\gamma_j : a' \to a^j$  with  $\alpha'_j = \alpha^j \gamma_j, j \in J$ . By (iii) there is a morphism  $\gamma : a' \to a$ 

in A such that  $p_j\gamma = \gamma_j$ ,  $j \in J$ . Therefore  $\alpha_j\gamma = \alpha^j p_j\gamma = \alpha^j \gamma_j = \alpha'_j$ ,  $j \in J$ . Therefore (a) holds. (b) is a modification of (a).

Now we recall a folkloristic generalization of Wyler's [12] construction of top categories. Let  $\mathcal{F} : A \to Cat$  denote a contravariant functor. The category  $\int_A \mathcal{F}$  has as objects all pairs  $(a, x), a \in ob(A), x \in ob(\mathcal{F}(a))$ , and as morphisms all pairs  $(\alpha, \beta) : (a, x) \to (a', x')$  where  $\alpha : a \to a'$  in A and  $\beta : x \to \mathcal{F}(\alpha)(x')$  in  $\mathcal{F}(a)$ . The composition of  $(\alpha, \beta) : (a, x) \to (a', x')$  and  $(\alpha', \beta') : (a', x') \to (a'', x'')$  is defined by  $(\alpha', \beta')(\alpha, \beta) = (\alpha'\alpha, (\mathcal{F}(\alpha)(\beta'))\beta)$ . The identity morphisms are defined by  $id_{(a,x)} = (id_a, id_x)$ . In the sequel, the forgetful functor  $\int_A \mathcal{F} \to A$  will be denoted U. By the discussion preceding Proposition 2.4, we get that if U is weakly topological, then each  $\mathcal{F}(a), a \in ob(A)$ , has weak (co)products and that if U is topological, then each  $\mathcal{F}(a)$ ,  $a \in ob(A)$ , is a complete ordered lattice. If we make an additional assumption, then this condition is sufficient for U to be weakly topological:

**Proposition 2.5.** (a) If each  $\mathcal{F}(a)$ ,  $a \in ob(A)$ , has weak products and each  $F(\alpha)$ ,  $\alpha \in mor(A)$ , respects weak products, then U is weakly topological. (b) If each  $\mathcal{F}(a)$ ,  $a \in ob(A)$ , is a complete ordered lattice and each  $\mathcal{F}(\alpha)$ ,  $\alpha \in mor(A)$ , respects infima, then U is topological.

**Proof.** (a) Conditions (ii) and (iii) of Proposition 2.4(a) are immediate by the assumptions. Now we show condition (i) of Proposition 2.4(a). Suppose that  $\alpha : a \to a'$  is a morphism in A and  $x' \in ob(\mathcal{F}(a'))$ . Clearly  $(\alpha, id_{F(\alpha)(x')}) : (a, F(\alpha)(x')) \to (a', x')$  is a U-lift of  $\alpha$ . Suppose now that  $\alpha' : a'' \to a'$  and  $\alpha'' : a'' \to a$  are morphisms in A satisfying  $\alpha' = \alpha \alpha''$ . Suppose that  $(\alpha', \beta') : (a'', x'') \to (a', x')$  and  $(\alpha'', \beta'') : (a'', x'') \to (a, F(\alpha)(x'))$  are lifts of  $\alpha'$  and  $\alpha''$  respectively, satisfying  $(\alpha, id_{F(\alpha)(x')})(\alpha'', \beta'') = (\alpha', \beta')$  Since the left hand side of this equation simplifies to  $(\alpha \alpha'', (F(\alpha'')(id_{F(\alpha)(x')}))\beta'') = (\alpha \alpha'', (F(\alpha'')(id_{F(\alpha)(x')})\beta'') = (\alpha \alpha'', (G(\alpha'')(id_{F(\alpha)(x')}))\beta'') = ($ 

## 3 Applications

The category CfzSet. Let X be a set and C a category. As a generalization of Goguen's [5] definition of fuzzy sets, we say that a C-fuzzy set on X is a function  $\mu: X \to ob(C)$ . Let  $\nu: Y \to ob(C)$  be another C-fuzzy set. We say that a fuzzy function from  $\mu$  to  $\nu$  is a pair  $(f, \alpha)$ where  $f: X \to Y$  is a function and  $\alpha = (\alpha_x)_{x \in X}, \ \alpha_x \in hom_C(\mu(x), \nu(f(x))), \ x \in X$ . This is indicated by writing  $(f, \alpha) : \mu \to \nu$ . Let  $\tau : Z \to ob(C)$  be a third fuzzy set and  $(g, \beta) : \nu \to \tau$ a second fuzzy function. Let the composition of  $(g,\beta)$  and  $(f,\alpha)$  be defined by  $(gf,\beta\circ\alpha)$ where  $(\beta \circ \alpha)_x = \beta_{f(x)} \alpha_x, x \in X$ . It is easy to check that the collection of C-fuzzy sets and C-fuzzy functions form a category which we denote CfzSet. Let  $U: CfzSet \rightarrow Set$  denote the forgetful functor. Now we wish to present  $f_{zs_C}(Set)$  as a top category. For each set X, let  $\mathcal{F}(X) := U_X$ . Note that the objects in  $\mathcal{F}(X)$  are the C-fuzzy sets on X and the morphisms in  $\mathcal{F}(X)$  are all ordered triples  $(\alpha, \mu, \nu)$  where  $\mu$  and  $\nu$  are C-fuzzy sets on X and  $\alpha = (\alpha_x)_{x \in X}$ ,  $\alpha_x \in hom_C(\mu(x), \nu(x)), x \in X$ . If  $f: X \to Y$  is a function, let  $\mathcal{F}(f): \mathcal{F}(Y) \to \mathcal{F}(X)$  be defined by  $\mathcal{F}(f)(\nu)(x) = \nu(f(x)), \ \nu \in \mathcal{F}(Y), \ x \in X \text{ and } \mathcal{F}(f)(\alpha, \mu, \nu) = (\alpha^f, \mathcal{F}(f)(\mu), \mathcal{F}(f)(\nu)), \text{ where } f(\mu) \in \mathcal{F}(Y)$  $\alpha_x^f = \alpha_{f(x)}, x \in X$ . The correspondence  $\mathcal{F}$  is a contravariant functor from Set to Cat and there is an isomorphism  $\int_{Set} \mathcal{F} \cong CfzSet$  of categories. Hence, by Proposition 2.5 and a straightforward argument, the forgetful functor  $U: Cf_z(Set) \to Set$  is (weakly) topological if C is (has weak products) a complete ordered lattice. Therefore, by Proposition 2.2, CfzSet is (weakly) complete and cocomplete if C is a complete ordered lattice (has products or coproducts respectively). Furthermore, if we assume that C is a complete ordered lattice and J is a category, then, by Proposition 2.3, the category  $CfzSet^J$  is (co)complete whenever  $Set^J$  is (co)complete.

The category LfzRel. For the rest of the article, let L be a complete ordered lattice. Suppose that  $\mu : X \to L$  and  $\nu : Y \to L$  are L-fuzzy sets. We say that a relation  $R : X \to Y$  is L-fuzzy if  $\mu(x) \leq \nu(y)$ ,  $(x, y) \in R$ . The collection of L-fuzzy sets and L-fuzzy relations form a category LfzRel. Define a functor  $\mathcal{F} : Rel \to Cat$  on objects in the same way as for LfzSetand on relations  $R : X \to Y$  by  $\mathcal{F}(R)(\nu)(x) = \wedge \nu(y), x \in X$ , where the infimum is taken over all  $y \in Y$  with  $(x, y) \in R$ . The correspondence  $\mathcal{F}$  is a contravariant functor and there is an isomorphism  $\int_{Rel} \mathcal{F} \cong LfzRel$  of categories. Therefore, by Proposition 2.5(b), the forgetful functor  $U : LfzRel \to Rel$  is topological. Hence, by Proposition 2.2(c), LfzRel is weakly complete and cocomplete.

The categories LT, LTop and LMeas. We define the category LT as the category with objects  $(X,\mu)$  where X is a set and  $\mu$  is a subset of  $L^X$ , the set of maps from X to L. A morphism  $(f, \mu, \nu) : (X, \mu) \to (Y, \nu)$  in LT is a function  $f : X \to Y$  satisfying  $\tau f \in \mu, \tau \in \nu$ . If  $(g,\nu,\rho)$  :  $(Y,\nu) \to (Z,\rho)$  is another morphism in LT, then  $(g,\nu,\rho)(f,\mu,\nu) := (gf,\mu,\rho)$ . Now we present LT as a top category. For each set X, let  $\mathcal{F}(X)$  be the collection of subsets of  $L^X$  ordered under reversed inclusion. If  $f: X \to Y$  is a function and  $\nu$  is a subset of  $L^{Y}$ , let  $\mathcal{F}(f)(\nu) = \{\tau f\}_{\tau \in \nu}$ . It is easy to see that there is an isomorphism  $\int_{Set} \mathcal{F} \cong LT$  of categories. Therefore, by Proposition 2.5(b), the forgetful functor  $U: LT \to Set$  is topological. Therefore, by Propositions 2.2 and 2.3,  $LT^{J}$  is (co)complete whenever  $Set^{J}$  is (co)complete, for any category J. The category LTop of L-topological spaces (Chang [4]) is the full subcategory of LT with objects  $(X,\mu)$  where  $\mu$  is an L-fuzzy topology on X, that is, subsets of  $L^X$  closed under finite pointwise infima and arbitrary pointwise suprema. If L is a complemented lattice, then the full subcategory LMeas of L-measurable spaces (Klement [9]) is defined as the full subcategory of LT with objects  $(X, \mu)$  where  $\mu$  is an L-sigma algebra, that is, a subset of  $L^X$  closed under countable pointwise suprema and pointwise complements. By following the argument above, it is a straightforward task to show that the forgetful functors  $U: LTop \rightarrow Set$ and  $U: LMeas \rightarrow Set$  are topological. Therefore, by Propositions 2.2 and 2.3,  $LTop^{J}$  and  $LMeas^{J}$  are (co)complete whenever  $Set^{J}$  is (co)complete, for any category J.

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