

## Using Fuzzy Set Theory in an Optimization Setting

M.K. Luhandjula\*

Department of Decision Sciences, University of South Africa, PO Box 392, Unisa, Pretoria 0003, South Africa

### Abstract

Fuzzy set theory has a strong track record of success in the field of Optimization under uncertainty. It offers a proper framework for coming to grips with situations where imprecision and complexities are in the state of affairs in an Optimization setting.

This paper presents my personal views on the descriptive and prescriptive power of Fuzzy set Theory in letting informational and intrinsic imprecision be taken into account in an Optimization model.

The paper is jam-packed with information on how and why I started doing research in this field, along with encouragements and inspiration I got from Professor L.A. Zadeh and H.J. Zimmermann.

### First Steps in Fuzzy Set Theory

My interest on Fuzzy set Theory dates back to early 1980's when I was completing a PhD at the Free University of Brussels. I was then fascinated by Zadeh's theory [1,2] that challenges traditional reliance on two-valued logic and classical set theory as a basis for scientific inquiry.

I quickly realized that Fuzzy set Theory was offering opportunities both as a language and a tool to cope with imprecision and complexity. I then was extremely keen to apply this theory to decision situations where imprecision plays a pivotal role. It was also clear to me that although probabilistic theories [3-5] claim to model decision making under uncertainty, there was a qualitatively different kind of imprecision which was not covered by these apparatus, that is: inexactness, ill-definedness, vagueness.

Correctness of statements and judgments degrees of credibility, plausibility has little to do with occurrence of events, the back-bone of Probability Theory.

I was comforted on the above mentioned views by Prof H.J. Zimmermann, the first Principal Editor of the Journal Fuzzy Sets and Systems.

He provided me with guidance and intuition on when Fuzzy set Theory venue may be most appropriate and ultimately successful in the field of Mathematical Programming under uncertainty.

Fruitful exchanges on many aspects related to Fuzzy Mathematical Programming namely, Flexible programming [6], Mathematical Programming with fuzzy number coefficients [7], Duality and sensitivity analysis, were a catalyst that kept me continuing work on this field.

Early 1990's, I had a golden opportunity to visit Prof Zadeh at University of California, Berkely. I was impressed by his breadth of knowledge about all facets of his theory. I then took advantage of this visit to learn more about Fuzzy Set Theory, its relation with Black's work on vagueness [8] and some philosophical related issues ranging from ontological to application levels via epistemological one [9].

### From Optimization to Fuzzy Optimization

Optimization is a very old and classical area which is of high concern to many disciplines.

Engineering as well as Management, Politics as well as Medicine,

Artificial Intelligence as well as Operations Research and many other fields are in one way or another concerned with Optimization of designs, decisions, structures or information processes.

In a deterministic environment using a single well defined criterion for evaluating potential alternatives, the optimal decision can be obtained through user-friendly mathematical programming software. Optimization procedure is, in this case a batch-type process assuming a closed model in which all information is available and in which the Decision Maker could provide and process all information simultaneously.

In a turbulent environment involving intrinsic or informational imprecision, the Optimization process is not that simple. Fuzzy set Theory may be of great help in representing and treating such imprecise information.

### Fuzzy Mathematical Programming

The language of Fuzzy set Theory has been exploited with good reasons [10] to tolerate some leeways in the formulation of goals and constraints of a mathematical program. The soft goals and constraints of the mathematical program are, in this case represented by appropriate fuzzy sets reflecting the viewpoints of the Decision maker. Making use of the Bellman-Zadeh's confluence principle [11] one may single out a satisfying solution in this context. Several variants of Zimmermann's model may be found in the literature [12-14]. In [15,16], Zimmermann's approach has been carefully adapted to deal with multi-objective linear and fractional problems.

Another situation where Fuzzy Set Theory enters into the dance in a mathematical programming setting is when considered parameters are not well-known and are modeled by fuzzy numbers.

\*Corresponding author: M.K. Luhandjula, Department of Decision Sciences, University of South Africa, PO Box 392, Unisa, Pretoria 0003, South Africa, E-mail: [luhanmk@unisa.ac.za](mailto:luhanmk@unisa.ac.za)

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Several methods are described to solve this problem [17].

The general principle behind these methods is to convert the original problem into deterministic terms, by sticking as well as possible to uncertainty principle.

Existing transformation strategies are based either on rules for comparing fuzzy members of exploration of possibility and necessity measures. It is worth pointing out that a large amount of applications of fuzzy mathematical programming exists supporting the efficiency and the effectiveness of fuzzy optimization technique. An interesting reader is referred to [18,19] for a sample of these applications.

### Fuzzy Stochastic Optimization

In some real-life problems one has to base decision on information which is both fuzzily imprecise and probabilistically uncertain. Although consistency indexes providing a union nexus between imprecision of possibilistics nature and uncertainty of stochastic type exist, there are no reliable ways of transforming one to another.

This calls for new paradigms for incorporating simultaneously the two kinds of indeterminacy into mathematical programming models. Fuzzy stochastic Optimization is an attempt to fulfill this need.

The general methodological approach for Fuzzy Stochastic Optimization problems consists of crafting an uncertainty processing that suits the particular characteristics of the problem at hand, exploiting to a great extent the available structure [20]. This processing should embody a device that interprets the original Fuzzy Stochastic Optimization problem from the probabilistic and possibilistic lenses and perform good conversions from both the standpoints of effectiveness and efficiency.

This methodological approach has given rise to different methods for solving Fuzzy Stochastic Optimization problems, see. e.g. [21-23]. These ideas have been successfully applied to Portfolio selection [24] and too many other concrete real-life problems [15].

### Concluding Remarks

Mathematical programming under fuzziness provides a corpus of scientific knowledge that permits to scope effectively with vagueness instead of merely thwarting, suppressing or downplaying it.

Freud told us that the history of science is the history of alienation. Since Copernicus we no longer live at the centre of the universe; since Darwin man is no longer different from other animals and since Freud himself, conscience is just the emerged part of a complex reality hidden from us. Paraphrasing Freud, we can say since Zadeh we are no longer forced to approximate real problems of the more-or-less type by yes-or-no type models.

This is crucial in this post-modern era characterized by fragmentation of the truth and ascendancy of approximate reasoning.

Among lines for further development in the field of mathematical programming under fuzziness we may mention the following.

- Extension of the Bellman principle to the fuzzy and fuzzy stochastic cases so as to develop Fuzzy and Fuzzy stochastic cases so as to develop Fuzzy and Fuzzy Stochastic dynamic programming.

- Deep comparison of Fuzzy and Fuzzy stochastic Optimization

techniques. This may help to design a user friendly Decision Support System able to help a Decision maker confronted with a problem of optimization under fuzziness.

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