Journal of
Applied & Computational Mathematics

Trowbridge, J Appl Computat Math 2013, 2:4
DOI: 10.4172/2168-9679.1000142

Research Article | Open Access

Forecasting the US Capital Gains Tax Rate Using Markov Chains

Brian Trowbridge*

California State University, Hayward, CA, USA

Abstract

Using Markov-chains (formally a Markov process or Markov system) we will show how to estimate the probabilities of a capital gains tax rate changing from its current rate to another rate within a certain number of years. It is important to recognize that capital gains rates in the United States have their origins in every elected administration of the government that had the power to influence tax policy. The outcome of government elections is dependent on chance factors such as periods of war and peace, economic recession and expansion, current fiscal and monetary policy, changes in population demographics, just to name a few. The delicate balance between the government and the dynamic society that shapes tax policy is difficult to predict using deterministic modeling. Therefore, we look to stochastic modeling using the Markov-Chain processes to see if it could be of use in creating a useful model for capital gains tax rates in the United States.

Keywords: Forecasting; Markov Chains

Introduction

Background and academic significance

It is common practice to calculate the present value (PV) or net present value (NPV) of a future cash inflow net of the current capital gains tax rate. Various finance, accounting, and investment texts ([1-3], the popular Wiley Finance Series, and many respected undergraduate finance textbooks such as those published by McGraw-Hill, Blackwell) assume or ignore without justification the current capital gains tax rate when reducing the cash flow that is to be discounted. However, using the government's current enacted rate as an estimate of what the future rate will be may lend itself to an inaccurate PV or NPV calculation. If the probability is that a future rate would be different than the current rate one could either use the current enacted rate or the estimated future rate, depending on the magnitude of the probability that the current rate would change to the future rate. Similarly, an investor might want to pay the minimum amount of tax and sell an investment when a tax rate is lower than the current rate, but do so when the investment value is at its highest value. To find the optimal time to sell the investment he would need to know what the future rate was likely to be. If the probability that the rate would be significantly different than the current rate several years from a given starting point, he might wish to use the future rate to accurately gauge the future cost. He could then use a method of numerical optimization to find the values of decision variables that maximize the profit function.

Brief discussion of mathematical tools used in the model

Throughout this paper, P denotes a transition matrix for a Markov process, such that $\sum_{j}^{n} p_{ij} = 1$ and $p_{ij} \geq 0$ for all i and j. Each entry $p_{ij} \in P$ is the probability of moving from state i to state j in one step. Steps are often a unit of time, a year, a day, a week, etc. For our purposes a step will represent one year. If P is a transition matrix for a Markov process, and v is a distribution vector with the property that $vP=v$, we call v a steady state vector. In general, to find the steady state vector for a Markov process we need to solve the system of equations given by $x+y+z+\ldots=1$, $[x\,y\,z\ldots]P=[x,y,z\ldots]$. A steady-state vector of a regular Markov chain is an eigenvector for the transition matrix corresponding to the eigenvalue of 1. It is a convenient fact that a regular transition matrix stabilizes to a steady state distribution and a regular Markov process has only one steady state vector. A regular transition matrix is one that contains only positive entries for some power of the matrix. It is well known that we can find the steady state distribution of a regular transition matrix by raising the matrix to some large power. Once the rows of the matrix are all equal to the desired accuracy upon raising the matrix to some power, the power is large enough. If we allow the initial state to be represented as a vector v, we call v a probability vector (or initial distribution vector). Given a transition matrix of a Markov process, the probability that the chain will be in state i after n steps is $v^{(n)} = vP^n$. We will define z to be the random variable representing the government's enacted capital gains tax rate in a particular year. The U.S. capital gains tax rate is a simple percentage, which is multiplied against the net gain from the sale or other disposition during a year of an asset held by an individual or entity, to arrive at the capital gains tax.

Data

We begin with a brief description and inspection of the historical capital gains tax rates in the U.S. The U.S. capital gains tax rate is a simple percentage that is multiplied by the gain from the sale or disposition of assets during a year by an individual, corporation, or any other entity of the United States. Thus, for any given year there is a single rate that applies to gain from the sale or disposition of an asset and the data in Table 1 "U.S. capital gains tax rates" below lists those rates for each year beginning in 1916 through 2013.

Model assumptions

Prior to laying out the framework of our model, we will present key assumptions necessary for the creation of our model. In some instances the assumptions are reasonable, and in other cases simplistic, but necessary. Firstly, our model assumes that z can be grouped into

***Corresponding author:** Brian C Trowbridge, Department of Mathematics, California State University, Hayward, USA, E-mail: nairb1111@yahoo.com

Received September 13, 2013; **Accepted** october 09, 2013; **Published** October 14, 2013

Citation: Trowbridge BC (2013) Forecasting the US Capital Gains Tax Rate Using Markov Chains. J Appl Computat Math 2: 142. doi:10.4172/2168-9679.1000142

Citation: Trowbridge BC (2013) Forecasting the US Capital Gains Tax Rate Using Markov Chains. J Appl Computat Math 2: 142. doi:10.4172/2168-9679.1000142

Year	Z	Year	Z	Year	Z	Year	z
1916	15	1947	25	1978	39	2009	15.4
1917	67	1948	25	1979	28	2010	15
1918	77	1949	25	1980	28	2011	15
1919	73	1950	25	1981	23.7	2012	15
1920	73.00	1951	25.00	1982	20.00	2013	25.00 (est.)
1921	73	1952	25	1983	20		
1922	12.5	1953	25	1984	20		
1923	12.5	1954	25	1985	20		
1924	12.5	1955	25	1986	20		
1925	12.5	1956	25	1987	28		
1926	12.5	1957	25	1988	30.5		
1927	12.5	1958	25	1989	30.5		
1928	12.5	1959	25	1990	30.5		
1929	12.5	1960	25	1991	28.9		
1930	12.5	1961	25	1992	28.9		
1931	12.5	1962	25	1993	29.2		
1932	12.5	1963	25	1994	29.2		
1933	12.5	1964	25	1995	29.2		
1934	31.5	1965	25	1996	29.2		
1935	31.5	1966	25	1997	21.2		
1936	39	1967	25	1998	21.2		
1937	39	1968	26.9	1999	21.2		
1938	30	1969	27.5	2000	21.2		
1939	30	1970	32.3	2001	21.2		
1940	30	1971	34.3	2002	21.2		
1941	30	1972	36.5	2003	16.1		
1942	25	1973	36.5	2004	16.1		
1943	25	1974	36.5	2005	16.1		
1944	25	1975	36.5	2006	15.7		
1945	25	1976	39.9	2007	15.7		
1946	25	1977	39.9	2008	15.4		

Table 1: U.S. capital gains tax rates (z) from 1916 though 2013[a].

[a]Historical data taken from various sources, including the following:
http://www.taxpolicycenter.org/taxfacts/displayafacet.cfm?Docid=161
http://www.forebestadvice.com/Money/Taxes/Federal-Tax-Rates/Historical_Historical_Federal_Capital_Gains
http://www.treasury.gov/resource-center/tax-policy/Documents/OTP-CG-Taxes-Paid

s_1………… 0.00 ≤ z < 0.15
s_2………… 0.15 ≤ z < 0.25
s_3…………0.25 ≤ z < 0.35
s_4………… 0.35 ≤ z < 0.45
s_5……….. 0.65 ≤ z < 0.75
s_6……… 0.75 ≤ z < 0.85

Table 2: Transition states and z classes.

classes, where each class represents the range of potential values for z, and each considered a state in our Markov process. As we will see later, this method, while simplistic, is an analogue to many of the analyses involving comparative statistics (such as those of Aldeman et al. [4]) and essential for the creation of the model.

We define six states, namely S1, S2,…, S6 , listed in Table 2 above. This seems a reasonable starting point since the enacted tax rate historically has moved within these ranges. However, inspecting Table 1, it is clear that z has not reached either of the extremes of 0 or 0.85. These have been arbitrarily defined to be the boundaries of our model, necessary to create a finite set of states. Smaller or larger intervals of z could be created as additional or fewer classes and states as the required detail in a particular analysis might vary.

Secondly, changes between these classes of rates can be considered a stochastic process, particularly a Markov process, with transition probabilities remaining constant in time and the probability of changing from one state to another a function only of the two states involved. This is an obvious simplification since we are ignoring the numerous factors that influence the movement of z, such as government surpluses and deficits, U.S. government's fiscal and monetary policy, and the US government's contemporary view toward taxation, periods of war and peace, economic recession and expansion, current fiscal and monetary policy, changes in population demographics, etc. Rather than attempt to study each factor's individual contribution to movements in z, we simply represent the outcome of all these forces by one random variable z, which we want to model. Even though this is a severe restriction, such a treatment is analogous to many analyses involving long-run comparative statistics ([4] for a comprehensive discussion of this). Nonetheless, utilizing these assumptions we can create a framework where we can analyze the movement of the random variable z over time.

Results

Let a_{ij} be the number of times z moved from class *i* to class *j*. Counting the movements (transitions) of z in Table 1 between the classes (states) defined in Table 2, we arrive at the relative frequency of times z started in a particular state and moved to each of the other states.

The transition probabilities are therefore $p_{ij} = a_{ij} / \sum_{n=1}^{6} a_{ij}$ the maximum likelihood estimates of the stationary probabilities p_{ij} *are* $\hat{p}_{ij} = a_{ij} / \sum_{n=1}^{6} a_{ij}$ (2) ([5] for a discussion of this method and the transition counts and estimated transition probabilities). To establish dependence of z against the test hypothesis that z is independent we used a chi square test, essentially treating a matrix of transition counts as if it were a contingency Table ([6] in particular under 2.6 and 2.7). Testing the data in Table 1 resulted in a chi square test of 285.94 and is significant at the 99.5000 percent confidence level. Confident that z is dependent we proceeded to use the estimated transition matrix from (2) $\hat{p}$ defined above for p_{ij}. We then define our regular transition matrix of the Markov process to be

$$P = \begin{pmatrix} 0.9200 & 0.0000 & 0.0800 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8700 & 0.0900 & 0.0000 & 0.0400 & 0.0000 \\ 0.0000 & 0.0400 & 0.920 & 0.0400 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.2200 & 0.7800 & 0.0000 & 0.0000 \\ 0.2500 & 0.0000 & 0.0000 & 0.0000 & 0.5000 & 0.2500 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{pmatrix}$$

Examples of Application

We will illustrate practical application of our model through some examples from finance.

Example 1

An NY city investment analyst wishes to calculate the net present value of an equity fund expected to be sold for $100 million seven years from now. The current enacted capital gains tax rate is 0.15, the discount rate is currently

Citation: Trowbridge BC (2013) Forecasting the US Capital Gains Tax Rate Using Markov Chains. J Appl Computat Math 2: 142. doi:10.4172/2168-9679.1000142

0.1 and the transition matrix after seven years (steps) is

$$P^7 = \begin{pmatrix} 0.5585 & 0.0415 & 0.3618 & 0.0355 & 0.0023 & 0.0004 \\ 0.0828 & 0.4205 & 0.3629 & 0.0369 & 0.0773 & 0.0196 \\ 0.0071 & 0.1550 & 0.7029 & 0.1187 & 0.0135 & 0.0028 \\ 0.0017 & 0.0887 & 0.6530 & 0.2505 & 0.0053 & 0.0009 \\ 0.5701 & 0.0141 & 0.1867 & 0.0127 & 0.1646 & 0.0509 \\ 0.5654 & 0.0093 & 0.1508 & 0.0086 & 0.2034 & 0.0625 \end{pmatrix}$$

We use the basic present value formula $PV = \frac{R_t}{(1+i)^t}$ ([1], p. 600), where t is the time of the cash flow, *i* is the discount rate, 1+ i>0 and *Rt* is the net cash inflow (+) and outflow (-). [Note the analysis could easily be modified for a time series of multiple incoming and outgoing cash flows subject to z using the formula

$$NPV = \sum_{t=0}^{N} \frac{R_t}{(1+i)^t}] \qquad [1].$$

Since the current enacted capital gains rate is currently in state s_2, let the initial probability distribution vector be $v = (010000)$. Seven steps out the new distribution $v^{(7)} = vP^7$ =(0.0828 0.4205 0.3629 0.0369 0.0773 0.0196).

This shows there is a probability of 0.4205 that the capital gains rate will be the same state 7 steps out. However, there is also a probability of 0.3629 that the rate will transition to the state S_3 ($0.25 \le z \le 0.35$) during that time.

The PV calculations for comparison are as follows:

For $PV(R_{t_{s_2}}): \frac{75}{(1+.10)^7} \le PV(R_{t_{s_2}}) < \frac{85}{(1+.10)^7}$, Note: R_t has been reduced by the maximum value (tax rate) of its z class. This result in $38.4869 \le PV(R_{t_{s_2}}) \le 43.6184$.

For $PV(R_{t_{s_3}}): \frac{65}{(1+.10)^7} \le PV(R_{t_{s_3}}) < \frac{75}{(1+.10)^7}$. This results in $33.3553 \le PV(R_{t_{s_3}}) < 38.4869$. The difference in the range of these present values is \$10.2631 million, a significant amount for planning purposes. Given the probability of the higher tax rate in the future of 0.3629, the analyst may wish to plan for the investment yield at the higher z value in class S_3.

Example 2

An ambitious Wall Street fund manager wishes to create a commercial real estate fund model whose investment life is to exceed 99 years. He would like to know whether the initial probability distribution of the enacted tax rate will have any effect on the long-term probability distribution of the model. He would also like to know if it is advantageous to start the fund when the rate is in a particular class so that the long-term probability distribution is favorable to his fund's investors. The short answer is no. Earlier we discussed that we could find the steady state transition matrix by raising a regular transition matrix to some large power. If we raise the matrix *P* to 100 we find that the matrix stabilizes for an accuracy of 4 decimal places. Once the matrix stabilizes, no matter what assumptions are made about the initial probability distribution, the resulting probability distribution is the same.

Given our steady state transition matrix P,

$$P^{100} = \begin{pmatrix} 0.0902 & 0.1805 & 0.5865 & 0.1066 & 0.0289 & 0.0072 \\ 0.0902 & 0.1805 & 0.5865 & 0.1066 & 0.0289 & 0.0072 \\ 0.0902 & 0.1805 & 0.5865 & 0.1066 & 0.0289 & 0.0072 \\ 0.0902 & 0.1805 & 0.5865 & 0.1066 & 0.0289 & 0.0072 \\ 0.0902 & 0.1805 & 0.5865 & 0.1066 & 0.0289 & 0.0072 \\ 0.0902 & 0.1805 & 0.5865 & 0.1066 & 0.0289 & 0.0072 \end{pmatrix}$$

The probability distribution will be the same no matter what *v* we choose when we multiply vP^{100} Thus, it does not matter what class the enacted tax rate is in when we begin the fund. Note this is the case because all the rows of the steady state matrix are the same. When we multiply any initial distribution (a row vector), *v* and P^n (a steady state transition matrix), by the rules of matrix multiplication ([7] or any other elementary linear algebra text), vP^n will be the same, namely (0.0902 0.1805 0.5865 0.1066 0.0289 0.0072) for our example.

Conclusion

The ability to forecast the probabilities of a future capital gains rate in the U.S. can be useful to the financial community. Since capital gains rates lend themselves better to stochastic modeling than deterministic modeling, we can look toward the Markov Chain as suitable mathematical tool to model them. It is common practice to calculate the present value (PV) or net present value (NPV) of a future cash inflow net of the current capital gains tax rate. Various finance, accounting, and investment texts use the current capital gains tax rate for purposes of making the calculation. However, using the government's current enacted rate as an estimate of what the future rate will be may lend itself to an inaccurate PV or NPV calculation. Our model allows the creation of long-term forecasts of capital gains tax rates to be used in such calculations, and many more applications where it would be useful to know the likelihood of a particular future capital gains tax rate. Some of the assumptions necessary to create our model, while simplistic have a well documented history of use in other areas of statistical modeling, reference to the published works of which have been made earlier in this paper. While these assumptions represent a simplification of the problem they may be overcome in time as more research is performed in this area. They may also be overcome as additional data (sample size of years to model for example) progresses and we are able to modify them. As we better understand the nature of these rates in the context of financial and economic modeling some of these assumptions may be considered further, and perhaps serve as the basis for future work.

References

1. Shapiro A (2010) Multinational Financial Management. John Wiley and Sons, Inc, USA.
2. Bragg SM (2002) Business Ratios and Formulas. John Wiley & Sons, Inc, USA.
3. Low RJ (2004) Accounting & Finance for Small Business Made Easy, Entrepreneur Press.
4. Aldeman IG (1958) A Stochastic Analysis of the Size of Distribution of Firms. Journal of the American Statistical Association 53: 893-904.
5. Anderson TW, Goodman LA (1957) Statistical Inference about Markov Chains. Annals of Mathematical Statistics 28: 89-110.
6. Billingsley P (1960) Statistical Methods in Markov Chains. RAND Corporation.
7. Barnett RA, Ziegler MR (1987) Linear algebra an introduction with applications. Dellen Publishing Company, USA.