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Research Article

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Upper and Lower Weaky m_X - $\alpha\psi$ - Continuous Multifunctions

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Abstract

In this paper, we introduce the notion of m_X - $\alpha\psi$ -closed set and investigate some of its properties. We introduce upper/lower m_X - $\alpha\psi$ -continuous, weakly m_X - $\alpha\psi$ -continuous, slightly m_X - $\alpha\psi$ -continuous and almost m_X - $\alpha\psi$ -continuous multifunctions from a set satisfying certain minimal condition into a topological space and also we obtain their characterizations and properties of such multifunctions.

Keywords: Minimal structure; m_X - $\alpha\psi$ -closed set; m_X - $\alpha\psi$ -continuous functions in minimal structure spaces

AMS (2000) Subject Classification: 54A05, 54A20, 54C08, 54D10, 54C60.

Introduction

In 1961, Levine [1] introduced the notion of weakly continuous functions. Popa and Smithson [2,3] independently introduced the concept of weakly continuous multifunctions. Noiri [4] introduced the concept of minimal structure on a nonempty set. Also they introduced the notion of m_X -open set and m_X -closed set and characterize those sets using m_X -cl and m_X -int operators respectively. Further they introduced m-continuous functions [5] and studied some of its basic properties. Noiri and Popa [6] introduced and studied other forms of continuous multifunctions namely, slightly m-continuous multifunctions.

In this paper, we introduce m_X - $\alpha\psi$ -closed set and also we study some of the upper/lower m_X - $\alpha\psi$ -continuous multifunctions as the multifunctions are defined between a set satisfying certain minimal condition into a topological space. We obtain some characterizations and some properties of such multifunctions.

Preliminaries

In this section, we introduce the m-structure and define some important subsets associated to the m-structure and the relation between them.

Definition

Let X be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the power set of X. Where m_X is an m-structure (or a minimal structure) on X, if ϕ and X belong to m_X.

The members of the minimal structure m_X are called m_X -open sets, and the pair (X, m_X) is called an m -space. The complement of m_X -open set is said to be m_X -closed.

Definition

[7] Let X be a nonempty set and m_X an m -structure on X. For a subset A of X, m_X -closure of A and m_X -interior of A are defined as follows:

1. m_X - $Cl(A) = \cap\{F : A \subseteq F, X - F \in m_X\}$
2. m_X - $Int(A) = \cup\{F : U \subseteq A, U \in m_X\}$.

Lemma

[7] Let X be a nonempty set and m_X an m -structure on X. For subsets A and B of X, the following properties hold:

1. m_X - $Cl(X - A) = X - m_X$ - $Int(A)$ and m_X - $Int(X - A) = X - m_X)$ - $Cl(A)$.
2. If $(X - A) \in m_X$, then m_X - $cl(A) = A$ and if $A \in m_X$ then m_X - $int(A) = A$.
3. m_X - $Cl(\phi) = \phi$, m_X - $Cl(X) = X$, m_X - $int(\phi) = \phi$ and m_X - $int(X) = X$.
4. If $A \subseteq B$ then m_X - $Cl(A) \subseteq m_X$ - $Cl(B)$ and m_X - $int(A) \subseteq m_X$ - $int(B)$.
5. $A \subseteq m_X$ - $Cl(A)$ and m_X - $Int(A) \subseteq A$.
6. m_X - $Cl(m_X$ - $Cl(A)) = m_X$ - $Cl(A)$ and m_X - $Int(m_X$ - $Int(A)) = m_X$ - $Int(A)$.

Lemma

[5] Let (X, m_X) be an m -space and A a subset of X. Then $x \in m_X$ - $cl(A)$ if and only if $U \cap A \neq \phi$ for every $U \in m_X$ containing x.

Definition

[7] A minimal structure m_X on a nonempty set X is said to have the property β if the union of any family of subsets belonging to m_X belongs to m_X.

Remark

[8] A minimal structure m_X with the property β coincides with a generalized topology on the sense of Lugojan.

Lemma

[9] Let X be a nonempty set and m_X an m -structure on X satisfying the property β. For a subset A of X, the following property hold:

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1. $A \in m_X$ iff m_X - $int(A) = A$
2. $A \in m_X$ iff m_X - $cl(A) = A$
3. m_X - $int(A) \in m_X$ and m_X - $cl(A) \in m_X$

Definition

A subset A of an m-space (X, m_X) is called

1. an m_X-preopen set [10] if $A \subseteq U$ - $int(m_X$ - $cl(A))$ and a m_X-preclosed set if m_X - $cl(m_X$ - $int(A)) \subseteq A$,

2. an m_X-semiopen set [10] if $A \subseteq m_X$ - $cl(m_X$ - $int(A))$ and a m_X-semiclosed set if m_X - $int(m_X$ - $cl(A)) \subseteq A$,

3. an m_X-semi generalized-closed [10] (briefly m_X - sg -closed) set if m_X - $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X-semi-open in (X, m_X). The complement of an m_X - sg -closed set is called an m_X - sg -open set.

The m_X-pre closure (resp. m_X-semi closure, m_X - α -closure) of a subset A of an m-space (X, m_X) is the intersection of all m_X-pre closed (resp. m_X-semi closed, m_X - α -closed) sets that contain A and is denoted by m_X - $pcl(A)$ (resp. m_X - $scl(A)$, m_X - m_X).

m_X - $\alpha\psi$ -closed and m_X - $\alpha\psi$ -open sets

Definition

A subset A of an m-space (X, m_X) is called an

1. m_X - α -open set if $A \subseteq m_X$ - $int(m_X$ - $cl(m_X$ - $int(A)))$ and an m_X - α -closed set if m_X - $cl(m_X$ - $int(m_X$ - $cl(A))) \subseteq A$,

2. *m_X - ψ -closed set if m_X - $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X - sg -open in (X, m_X). The complement of an m_X - ψ -closed set is called an m_X - ψ -open set.*

3. *m_X - $\alpha\psi$ -closed set if m_X - $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X - α -open in (X, m_X). The complement of an m_X - $\alpha\psi$ -closed set is called an m_X - $\alpha\psi$ -open set.*

Notation

For an m-space (X, m_X), $O(X, m_X)$ (resp. $SO(X, m_X)$, $PO(X, m_X)$, $\alpha O(X, m_X)$, $SGO(X, m_X)$, $\psi O(X, m_X)$, $\alpha\psi$ - $O(X, m_X)$) denotes the class of all open (resp. m_X-semiopen, m_X-preopen, m_X - α -open, m_X-sg-open, m_X - ψ -open, m_X - $\alpha\psi$ -open) subsets of (X, m_X).

Definition

Let (X, m_X) be an m-space and let A be a subset of X. Then

1. *the intersection of all m_X - m_X -closed sets containing A is called the m_X - $\alpha\psi$ -closure of A and is denoted by m_X - $\alpha\psi$ - $cl(A)$.*

2. *the union of all m_X - $\alpha\psi$ -open sets that are contained in A is called the m_X - $\alpha\psi$ -interior of A and is denoted by m_X - $\alpha\psi$ - $int(A)$*

Example (1)

Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $m_X = \{\phi, X, \{a\}, \{b\}, \{a, b\}$.

Then $SO(X, m_X) = \{\phi, X, \{a\}, \{b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, c, d\}\}$,

$\alpha O(X, m_X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$ and

$\alpha\psi$ - $O(X, m_X) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}\}$.

Example (2)

Let $X = \{a, b, c\}$. Define the m-structure on X as follows: $m_X = \{\phi, X, \{a\}, \{b\}\}$.

Then $SO(X, m_X) = \{\phi, X, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$,

$\alpha O(X, m_X) = \{\phi, X, \{a\}, \{b\}\}$ and $\alpha\psi$ - $O(X, m_X) = P(X)$.

Example (3)

Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $m_X = \{\phi, X, \{a\}, \{b\}, \{a, b, c\}, \{a, b, d\}\}$.

Then $SO(X, m_X) = \{\phi, X, \{a\}, \{b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$

$\alpha O(X, m_X) = \{\phi, X, \{a\}, \{b\}, \{a, b, c\}, \{a, b, d\}\}$ and

$$\alpha\psi - \begin{matrix} O(X, m_X) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \\ \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\} \end{matrix}.$$

Definition

The intersection of all m_X - α -open subsets of (X, m_X) containing A is called the m_X - α -kernel of A (briefly, m_X - $\alpha ker(A)$) i.e. m_X - α $ker(A) = \cap\{G \in m_X$ - $\alpha O(X) : A \subseteq G\}$. And m_X - s $ker(A)$, m_X - $sgker(A)$, an m_X - ψ $ker(A)$ are defined similarly.

Theorem (1)

Let A be a subset of (X, m_X), then A is m_X - (α, ψ) -closed if and only if m_X - $\psi cl(A) \subseteq m_X$ - $\alpha ker(A)$.

Proof

Suppose that A is m_X - $\alpha\psi$ -closed and let $D = \{S : S \subseteq X, A \subseteq S : S is\ an\, m_X$ - α -open $\}$. Then m_X - $\alpha ker(A) = \bigcap_{S \in D} S$. Observe that $S \in D$ implies that $A \subseteq S$ follows m_X - $\psi cl(A) \subseteq S$ for all $S \in D$.

Conversely, if m_X - $\psi cl(A) \subseteq m_X$ - $aker(A)$, take $S \in \alpha O(X, m_X)$ such that $A \subseteq S$ then by hypothesis,

m_X - $\psi cl(A) \subseteq m_X$ - $\alpha ker(A) \subseteq S$.

This shows that A is m_X - $\alpha\psi$ -closed.

Theorem (2)

For subsets A and B of (X, m_X), the following properties hold:

1. *If A is m_X - ψ -closed, then A is m_X - $\alpha\psi$ -closed.*

2. *If m_X has the property β and A is m_X - $\alpha\psi$ -closed and m_X - α -open then A is m_X - ψ -closed.*

3. *If A is m_X - $\alpha\psi$ -closed and $A \subseteq B \subseteq \psi cl(A)$ then B is m_X - $\alpha\psi$ -closed.*

Proof

1. Let A be an m_X - ψ -closed set in (X, m_X). Let $A \subseteq U$, where U is m_X - α -open in (X, m_X). Since A is m_X - ψ -closed, m_X - $\psi cl(A) = A$, m_X - $\psi cl(A) \subseteq U$. Therefore, A is m_X - $\alpha\psi$ -closed.

2. Since A is m_X - α -open and m_X - $\alpha\psi$ -closed, we have m_X - $\psi cl(A) \subseteq A$. Therefore, A is m_X - ψ -closed

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3. Let U be an m_X - α -open set of (X,m_X) such that $B \subseteq U$, then $A \subseteq U$. Since A is m_X - $\alpha\psi$ -closed, m_X - $\psi cl(A) \subseteq U$. Now m_X - $\psi cl(B) \subseteq m_X$ - $\psi cl(m_X$ - $\psi cl(A)) \subseteq U$. Therefore, B is also an m_X - $\alpha\psi$ -closed set of (X,m_X).

Theorem (3)

Union of two m_X - $\alpha\psi$ -closed sets is m_X - $\alpha\psi$ -closed.

Proof

Let A and B be two m_X - $\alpha\psi$ -closed sets in (X,m_X). Let $A \cup B \subseteq U$, U is m_X - α -open. Since A and B are m_X - $\alpha\psi$ -closed sets, m_X - $\psi cl(A) \subseteq U$ and m_X - $\psi cl(B) \subseteq U$. This implies that m_X - $\psi cl(A \cup B) \subseteq m_X$ - $\psi cl(A)$ - $\psi cl(B) \subseteq U$ and so m_X - $\psi cl(A \cup B) \subseteq U$. Therefore $A \cup B$ is m_X - $\alpha\psi$ -closed.

Theorem (4)

Let m_X be an m -structure on X satisfying the property β and $A \subseteq X$. Then A is an m_X - $\alpha\psi$ -closed set if and only if there does not exist a nonempty m_X - α -closed set F such that $F \neq \phi$ and $F \subseteq m_X$ - $\psi cl(A) - A$.

Proof

Suppose that A is an m_X - $\alpha\psi$ -closed set and let $F \subseteq X$ be an m_X - α -closed set such that $F \subseteq m_X$ - $\psi cl(A) - A$. It follows that, $A \subseteq X - F$ and $X - F$ is an m_X - α -open set. Since A is an m_X - $\alpha\psi$ -closed set, we have that m_X - $\psi cl(A) \subseteq X - F$ and $F \subseteq X - m_X$ - $\psi cl(A)$. Follows that, $F^+(V) \subset m_X$ - $\psi cl(A) \cap (X - m_X$ - $\psi cl(A)) = \phi$, implying that $F = \phi$.

Conversely, if $A \subseteq U$ and U is an m_X - α -open set, then m_X - $\psi cl(A) \cap (X-U) \subseteq m_X$ - $\psi cl(A) \cap (X-A) = m_X$ - $\psi cl(A) - A$. Since m_X - $\psi cl(A) - A$ does not contain subsets m_X - α -closed sets different from the empty set, we obtain that m_X - $\psi cl(A) \cap (X-U) = \phi$ and this implies that m_X - $\psi cl(A) \subseteq U$ in consequence A is m_X - $\alpha\psi$ -closed.

We can observe that if in Theorem 3.11., the property β is omitted then the result can be false as we can see in the following example.

Example

Let $X = \{a,b,c,d\}$. The m -structure on X is defined as $m_X = \{\phi, X, \{a\}, \{b\}, \{a,c\}\}$. $\alpha C(X,m_X) = \{X, \phi, \{d\}, \{b,d\}, \{c,d\}, \{b,c,d\}, \{a,c,d\}\}$

and $\alpha\psi$ - $C(X,m_X) = \{X, \phi, \{b\}, \{c\}, \{d\}, \{a,c\}, \{a,d\}, \{b,d\}, \{b,c\}, \{c,d\}, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}\}$.

The set $\{a\}$ is not an m_X - $\alpha\psi$ -closed set and there does not exist m_X - α -closed set F such that $F \neq \phi$ and $F \subseteq m_X$ - $\psi cl(A) - A$.

Theorem (5)

Let (X,m_X) be an m -space and $A \subseteq X$, then A is m_X - $\alpha\psi$ -open if and only if $F \subset m_X$ - $\psi int(A)$ where F is m_X - α -closed and $F \subset A$.

Proof

Let A be an m_X - $\alpha\psi$ -open, F be m_X - α -closed set such that $F \subset A$. Then $X - A \subset X - F$, but $X - F$ is m_X - α -closed and $X - A$ is m_X - $\alpha\psi$ -closed implies that m_X - $\psi cl(X-A) \subset X - F$. Follows that $X - m_X$ - $\psi int(A) \subset X - F$. In consequence $F \subset m_X$ - $\psi int(A)$.

Conversely, if F is m_X - α -closed, $F \subset A$ and $F \subset m_X$ - $\psi int(A)$. Let $X - A \subset U$ where U is m_X - α -open, then $X - U \subset A$ and $X - U$ is m_X - α -closed. By hypothesis, $X - U \subset m_X$ - $\psi int(A)$. Follows $X - m_X$ - $\psi int(A) \subset U$ but it is equivalent to m_X - $\psi cl(X - A) \subset U$. Therefore, $X - A$ is m_X - $\alpha\psi$ -closed and hence A is m_X - $\alpha\psi$ -open.

Weak m_X - $\alpha\psi$ -continuous and almost m_X - $\alpha\psi$ -continuous multifunctions

Definition (1)

Let (X,m_X) be an m -space and (Y,σ) a topological space. A multifunction $F:(X,m_X) \to (Y,\sigma)$ is said to be

1. *upper m_X - $\alpha\psi$ -continuous (resp. upper almost m_X - $\alpha\psi$ -continuous, upper weakly m_X - $\alpha\psi$ -continuous) at a point x if for each open set V of Y containing $F(x)$, there exists an m_X - $\alpha\psi$ -open set U of m_X containing x such that $F(U) \subset V$ (resp. $F(U) \subset int(cl(V)), F(U) \subset cl(V)$),*

2. *lower m_X - $\alpha\psi$ -continuous (resp. lower almost m_X - $\alpha\psi$ -continuous, lower weakly m_X - $\alpha\psi$ -continuous) at a point $x \in X$ if for each open set V of Y such that $F(x) \cap V \neq \phi$, there exists an m_X - $\alpha\psi$ -open set U of m_X containing x such that $F(u) \cap V \neq \phi$ (resp. $F(u) \cap int(cl(V)) \neq \phi, F(u) \cap cl(V) \neq \phi$) for each $u \in U$,*

3. *upper/lower m_X - $\alpha\psi$ -continuous (resp. almost m_X - $\alpha\psi$ -continuous, weakly m_X - $\alpha\psi$ -continuous) if it has this property at each point $x \in X$.*

Definition (2)

A multifunction $F:(X,m_X) \to (Y,\sigma)$ is said to be almost m_X - $\alpha\psi$ -open if $F(U) \subset int(cl(F(U)))$ for every m_X - $\alpha\psi$ -open set U of m_X.

Theorem (1)

If a multifunction $F:(X,m_X) \to (Y,\sigma)$ is upper weakly m_X - $\alpha\psi$ -continuous and almost m_X - $\alpha\psi$ -open, then F is upper almost m_X - $\alpha\psi$ -continuous.

Proof

Let V be any open set in Y containing $F(x)$. Then there exists an m_X - $\alpha\psi$ -open set U of m_X containing x such that $F(U) \subset cl(V)$. Since $F(x)$ is almost m_X - $\alpha\psi$ -open, $F(U) \subset int(cl(F(U))) \subset int(cl(V))$. Therefore, F is upper almost m_X - $\alpha\psi$ -continuous.

Theorem (2)

Let $F:(X,m_X) \to (Y,\sigma)$ be a multifunction such that $F(x)$ is open in Y for each $x \in X$. Then, the following properties are equivalent:

1. *F is lower m_X - $\alpha\psi$ -continuous;*
2. *F is lower almost m_X - $\alpha\psi$ -continuous;*
3. *F is lower weakly m_X - $\alpha\psi$ -continuous.*

Proof

(i) $\Rightarrow$ (ii) and (ii) $\Rightarrow$ (iii): The proofs of these implications are obvious.

(iii) $\Rightarrow$ (i): Let $x \in X$ and V be any open set such that $F(x) \cap V \neq \phi$. There exists an m_X - $\alpha\psi$ -open set U of m_X such that $F(u) \cap cl(V) \neq \phi$ for each $u \in U$. Since $F(u)$ is open, $F(u) \cap V \neq \phi$ for each $u \in U$ and hence F is lower m_X - $\alpha\psi$ -continuous.

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Slightly m_X - $\alpha\psi$ -continuous Multifunctions

Definition

Let (X,m_X) be an m-space and (Y,σ) a topological space. A multifunction $F:(X,m_X)\to(Y,\sigma)$ is said to be

1. *upper slightly m_X - $\alpha\psi$ -continuous if for each $A\in m_X$ and each clopen set V of Y containing $F(x)$, there exists an m_X - $\alpha\psi$ -open set U of m_X containing x such that $F(U)\subset V$,*

2. *lower slightly m_X - $\alpha\psi$ -continuous if for each $x\in X$ and each clopen set V of Y such that $F(x)\cap V\neq\phi$, there exists an m_X - $\alpha\psi$ -open set U of m_X containing x such that $F(u)\cap V\neq\phi$ for each $u\in U$.*

Theorem (1)

For a multifunction $F:(X,m_X)\to(Y,\sigma)$, the following are equivalent:

1. *F is upper slightly m_X - $\alpha\psi$ -continuous;*
2. *$F^+(V)=m_X$ - $\alpha\psi$ - $int(F(V))$ for each $V\in CO(Y)$;*
3. *$F^-(V))$ - $\alpha\psi$ - $cl(F^-(V))$ for each $V\in CO(Y)$.*

Proof

(i) $\Rightarrow$ (ii): Let V be any clopen set of Y and $x\in F^+(V)$. Then $F(x)\in V$. There exists an m_x - $\alpha\psi$ -open set U of m_x containing x such that $F(U)\subset V$. Thus $x\in U\subset F^+(V)$ and hence $x\in m_X$ - $\alpha\psi$ - $int(F^+(V))$. Therefore, we have $F^+(V)\subset m_X$ - $\alpha\psi$ - $int(F^+(V))$. But, m_x - $\alpha\psi$ - $int(F^+(V))\subset F^+(V)$, we obtain $F^+(V)=m_X$ - $\alpha\psi$ - $int(F^+(V))$.

(ii) $\Rightarrow$ (iii): Let K be any clopen set of Y. Then $Y-K$ is clopen in Y. By (ii) and Lemma 2.3., we have $X-F^+(K)=(Y-K)=m_X$ - $\alpha\psi$ - $int(F^+(Y-K))=X-[m_X$ - $\alpha\psi$ - $cl(F^-(K))]$. Therefore, we obtain $F^-(K)=m_X$ - $\alpha\psi$ - $cl(F^-(K))$.

(iii) $\Rightarrow$ (ii): This follows from the fact that $F^-(Y-B)=F^+(B)$ for every subset B of Y.

(ii) $\Rightarrow$ (i): Let $x\in X$ and V be any clopen set of Y containing $F(x)$. Then $x\in F^+(V)=m_X$ - $\alpha\psi$ - $int(F^+(V))$. There exists an m_x - $\alpha\psi$ -open set U of m_x containing x such that $x\in U\subset F^+(V)$. Therefore, we have $x\in U$, U is an m_x - $\alpha\psi$ -open set of m_x and $f(U)\subset V$. Hence F is upper slightly m_x - $\alpha\psi$ -continuous.

Theorem (2)

For a multifunction $F:(X,m_X)\to(Y,\sigma)$, the following are equivalent:

1. *F is lower slightly m_x - $\alpha\psi$ -continuous;*
2. *$F^-(V)=m_X$ - $\alpha\psi$ - $int(F^-(V))$ for each $V\in CO(Y)$;*
3. *$F^+(V)=m_X$ - $\alpha\psi$ - $cl(F^+(V))$ for each $V\in CO(Y)$.*

Proof

(i) $\Rightarrow$ (ii): Let $V\in CO(Y)$ and $x\in F^-(V)$. Then $F(x)\cap V\neq\phi$ and by (i) there exists an m_x - $\alpha\psi$ -open set U of m_x containing x such that $F(u)\cap V\neq\phi$ for each $u\in U$. Therefore, we have $U\subset F^-(V)$ and hence $x\in U\subset m_X$ - $\alpha\psi$ - $int(F^-(V))$. Thus, we obtain $F^-(V)\subset m_X$ - $\alpha\psi$ - $int(F^-(V))$ and by Lemma 2.3., $F^-(V)=m_X$ - $\alpha\psi$ - $int(F^-(V))$.

(ii) $\Rightarrow$ (iii): Let $V\in CO(Y)$. Then $Y-V\in CO(Y)$ and by (ii) we have $X-F^+(V)=F^-(Y-V)=m_X$ - $\alpha\psi$ - $int(F^-(Y-V))=X-m_X$ - $\alpha\psi$ - $cl(F^+(V))$. Hence we obtain $F^+(V)=m_X$ - $\alpha\psi$ - $cl(F^+(V))$.

(iii) $\Rightarrow$ (i): Let x be any point of X and V any clopen set of Y such that $F(x)\cap V\neq\phi$. Then $x\in F^-(V)$ and $x\notin X-F^-(V)=F^+(Y-V)$. By (iii), we have $x\notin m_X$ - $\alpha\psi$ - $cl(F^+(Y-V))$. By Lemma 2.4., there exists an m_X - $\alpha\psi$ -open set of m_x containing x such that $U\cap F^+(Y-V)=\phi$, hence $U\subset F^-(V)$. Therefore, $F(u)\cap V\neq\phi$ for each $u\in U$ and F is lower slightly m_x - $\alpha\psi$ -continuous.

Corollary (1)

For a multifunction $F:(X,m_X)\to(Y,\sigma)$, where m_x has the property β, the following are equivalent:

1. *F is upper slightly m_x - $\alpha\psi$ -continuous;*
2. *$F^+(V)$ is m_x - $\alpha\psi$ -open in (X,m_X) for each $V\in CO(Y)$;*
3. *$F^-(V)$ is m_x - $\alpha\psi$ -closed in (X,m_X) for each $V\in CO(Y)$.*

Corollary (2)

For a multifunction $F:(X,m_X)\to(Y,\sigma)$, where m_x has the property β, the following are equivalent:

1. *F is lower slightly m_x - $\alpha\psi$ -continuous;*
2. *$F^-(V)$ is m_x - $\alpha\psi$ -open in (X,m_X) for each $V\in CO(Y)$;*
3. *$F^+(V)$ is m_x - $\alpha\psi$ -closed in (X,m_X) for each $V\in CO(Y)$.*

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