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To the Solution for Beal Conjecture

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Abstract

Found general expressions for infinite series of equations $A^x + B^y = C^z$, where A, B, C, x, y and z are positive integers and x, y and z are all greater than 2, (A), (B) and (C) have a Common Prime Factor. However, these expressions also hold true for fractional and negative values of x, y, and z, and cover the range of numbers from the infinitely small to the infinitely large.

Keywords: Generalized equations; $A^x + B^y = C^z$ for Beal Conjecture

Andrew Beal has formulated a Conjecture in number theory [1]:

If $A^x + B^y = C^z$ (1)

where A, B, C, x, y and z are positive integers and x, y and z are all greater than 2, then A, B and C must have a common prime factor. Is it appropriate to seek a systematic solution for the sum of two numbers in the assumption that A and B have even numbers. Minimum an even number of A (and B) is 2. Then the equation (1) with a Common Prime Factor 2 can be written as

$$2^x + 2^y = 2^z \qquad (2a),$$

or for different A, B, C:

$$\left(2^n\right)^x + \left(2^m\right)^y = 2^z \qquad (2b)$$

If $my = nx$, then

$$A^x + B^y = 2\cdot\left(2^n\right)^x;\ \ C^z = 2^{nx+1} \qquad (2c)$$

By the condition of Beal Conjecture are x, y and z are positive integers and all are greater than 2. In order to be agree with this condition for a different positive integers A, B and C, minimum value for $nx(=my)$ should be 23 and increase by 2^2, i.e.

$$nx(=my) = 2^2(2+k), \qquad (3)$$

where k is a positive integer from continuous integer row, including zero (k= 0, 1,2, 3...).

Then the expression (2b) satisfying Beal Conjecture, may be represented by expression (4):

$$\left(2^n\right)^x + \left(2^m\right)^y = 2^{nx+1}, \qquad (4)$$

where $nx(=my) = 2^2(2+k)$; n is equal or unequal to m; n and m are equal or unequal 1; or (4a):

$$2^{3+k} + 2^{3+k} = 2^{4+k}, \qquad (4a)$$

where k is any rational number.

Note that in the case k = 0, for differing A, B and C there is only one solution eq. 4, where C ≠ 2 and z ≠ (nx+1):

k = 0, n = 1, x = 8, m = 2, y = 4, C = 8, z = 3 (28 + 44 = 83).

Similarly, the known individual equations in which A, B and C have a Common Prime Factor [1]:

$$3^3 + 6^3 = 3^5 \Rightarrow 3^3 + (2\cdot3) = 3^5 \qquad (5)$$

$$7^6 + 7^7 = 98^3 \Rightarrow 7^6 + 7^7 = \left(2\cdot7^2\right)^3 \qquad (6)$$

$$33^5 + 66^5 = 33^6 \Rightarrow (3\cdot11)^5 + (6\cdot11)^5 = (3.11)^6 \qquad (7)$$

$$34^5 + 51^4 = 85^4 \Rightarrow (2\cdot17)^5 + (3\cdot17)^5 = (5\cdot17)^4 \qquad (8)$$

$$19^4 + 38^3 = 57^3 \Rightarrow 19^4 + (2\cdot19)^3 = (3\cdot19)^3, \qquad (9)$$

were reviewed and the common expressions of equations, satisfying the conjecture of Beal's infinite series, are obtained. The exception is a family of equations with Common Prime Factor 5, y = 2 (eq. 11).

$$3^{3(k+1)} + \left(2\cdot3^{k+1}\right)^3 = 3^{3k+5}, \qquad (10)$$

$$5^{2(k+1)} + \left(2\cdot5^{k+1}\right)^2 = 5^{2k+3} \qquad (11)$$

$$7^{3(k+2)} + 7^{3k+7} = \left(2\cdot7^{k+2}\right)^3, \qquad (12)$$

$$\left(3\cdot11^{6k+1}\right)^5 + \left(6\cdot11^{6k+1}\right) = \left(3\cdot11^{5k+1}\right)^6 \Rightarrow 33^{5(k+1)} + \left(2\cdot33^{k+1}\right) = 33^{5k+6}, \qquad (13)$$

$$\left(2\cdot17^{4k+1}\right)^5 + \left(3\cdot17^{5k+1}\right)^4 = \left(5\cdot17^{5k+1}\right)^4, \qquad (14)$$

$$19^{3k+4} + \left(2\cdot19^{4k+1}\right)^3 = \left(3\cdot19^{4k+1}\right)^3, \qquad (15)$$

In equations (10-15) k-value takes any rational number.

Other well-known equations [1], having a Common Prime Factor not represented in eqs. 5-9, are special cases of common expressions. For Example:

$2^2 + 2^3 = 2^4$ (eq. 4: n, m=1, x=3, y=3 or eq. 4a: k=0),

$2^9 + 8^3 = 4^5 \Rightarrow 2^9 + \left(2^3\right)^3 = \left(2^2\right)^5$ (eq. 4: n=1, m=3, x=9, y=3 or eq. 4a: k=6),

$3^9 + 54^3 = 3^{11} \Rightarrow 3^9 + \left(2\cdot3^3\right)^3 = 3\cdot11$ (eq. 10, k=2),

$27^4 + 162^3 = 9^7 \Rightarrow \left(3^3\right)^4 + \left(2\cdot3^4\right)^3 = 3^{14}$ (eq. 10, k=3).

Using negative or fractional values k can get arithmetic addition of prime numbers. For example, if in eq. 11 take k = -1, the equation would be simplified to 1+4 = 5. If k = -2 in equation 12, the result will be 1+7 = 8. This may be of interest to number theory.

Thus, there are infinite rows of equations $A^x + B^y = C^z$, where A, B, C are positive integers and have a Common Prime Factor. At the

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same time, x, y, and z are positive integers, and more 2, only private representations of generalized expressions, that occupy region of numbers to positive infinity.

Reference

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