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The Variation-Difference Method of Design of Shells with Normal Unconjugated Coordinate System and Design of Some Type of Surfaces for Checking of the Correction of the Program

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Abstract

At the classical literature of the design of the shell there is used the coordinated system of the middle surfaces of principle curvatures or common non-orthogonal coordinate system. The system of equations of the shells of complex forms is complicated and it's difficult to receive the analytical decision. For analyses of the shells with non-orthogonal coordinate system and the most shells of complex form in the coordinate system of principle curvatures there is used usually finite element method which usually uses only the equation of the surface but don't take into account the geometrical characteristics of the middle surfaces of the shell. At the chair of strength of materials and constructions of Engineering Academy RUDN there is worked up the complex program VRMSHELL for analyses of the shells of complex forms by variation-difference method. The complex includes the library of curves on the base of which there are calculated the geometrical characteristics of the classes of the surfaces included at the program complex. The complex includes the classes of surfaces as cylindrical, surfaces of rotation, Joachimsthal's surfaces, Monge's surfaces. The coordinate system of surfaces of these classes is the coordinate system of principle curvatures.

Keywords: Normal cycle surfaces • Non-conjugated orthogonal coordinates • Thin-walled shells • Membrane theory of shells • Variation-difference method

Introduction

In this article we investigated normal cycle surfaces. The normal cycle surfaces there are formed by moving of the circle of change or constant radius at the normal plane of the directrix, which is a line of the centers of the circles. The coordinate system of normal cycle surfaces is orthogonal but isn't conjugated. The formulas of deformations of such type of shells differ of the analog formulas of shells with coordinate system of principle curvatures. On the base of investigation of the normal cycle surfaces there were made some modifications and class of normal cycle surfaces there was included at the program complex.

But the changing in complicated programs may put to some mistake and it's necessary to check up the correction of the work of the program. It's usually can be made by the comparison of the results received by the investigated program with the results received by analytical methods or by another program or the same program. In may be made by the same program but using another class of surfaces which gives the same form of shell [1-4].

At this work there are shown the peculiarity of design of shells with orthogonal non-conjugated system by variation-difference methods and there are given the example of design of stress-strain state of shells of rotation and tube shells. The shells of rotation are the normal cycle shell with strait line of centers and some decision may be received by analytical methods. The tube shells are the normal cycle shells with constant radius. Both of these types of shells enter to the class of Monge's surfaces and the some calculations there were made by both blocks of the program complex.

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Program Complex of VRMSHELL

The program complex VRMSHELL includes the library of curves. For every curve there are written the coordinate formulas and their derivation to third order. The new curves may be added at the library if it's necessary. On the base of the library of the curves there are formed the sections of the shells according given parameters and calculated the geometrical characteristics and their necessary derivations at any point of the surfaces. These blocs of program there are written for every class of surfaces included at the program complex. It's used the equations of directrix and generating curves of the concrete surface.

The variation-difference method there is based on the Lagrange's principle – principle of minimum of the total energy of deformation [5]

$$E=U-T; \quad \delta E=\delta U-\delta T=0, \tag{1}$$

E is a total energy of deformation; U is a potential energy of deformation; T is the work of external forces; δ is a symbol of variation of functional [6].

$$U=\frac{C}{2}\iint_{\Omega}\left[(\varepsilon_1+\nu\varepsilon_2)\varepsilon_1+(\varepsilon_2+\nu\varepsilon_1)\varepsilon_2+\frac{1-\nu}{2}S\varepsilon_{12}\right]d\Omega+\frac{D}{2}\iint_{\Omega}[(\chi_1+\nu\chi_2)\chi_1+(\chi_2+\nu\chi_1)\chi_2+(1-\nu)\chi_{12}]d\Omega; \quad ;$$

$$T=\iint_{\Omega}(q_1\cdot u_1+q_2\cdot u_2+q_3\cdot u_3)d\Omega, \quad C=\frac{Eh}{1-\nu^2}, \quad D=\frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

ε_i, χ_i – tangential and bending deformations of middle surface of the shell; N_i, M_i are tangential and bending forces; q_i are the projections of loads at the surface coordinate system of the shell, u_i are the displacements of middle surface; Ω – parameter of the area of the middle surface.

The section of the analyzed shell there is graduated by the net of equal or different paces. Using formulas of the numerical difference ratio with net displacements at the functional of total energy and minimizing it there is received the system of the algebraic equations. The decision of the equation gives web displacements at the nets of the shell. Using again the numerical difference ratios there are received the strains and further the tangential and bending forces and stresses. This bloc of the program is common but takes into account peculiarity of the class of the surface. The program there are realized the analyses of the sections of the shell with different condition of support and elastic supports and elastic foundation as well and on different types of loads. There may be analyzed shells with holes and shells with different contours.

Peculiarity of the normal cycle surfaces

The vector equation of the normal cycle surface with plane line of centers of generating curve (included at the program)

$$\boldsymbol{\rho}(u,v) = \boldsymbol{r}(u) + R(u)\boldsymbol{e}(u,v), \quad , \tag{3}$$

where $p(u, v)$ $\boldsymbol{\rho}(u,v)$ - radius-vector of the surface; $\boldsymbol{r}(u)$ - radius-vector of the directrix line of centers of generating circles; $R(u)$ - function of radius of the generating circles; $\boldsymbol{e}(u,v) = \boldsymbol{v}\cos v + \boldsymbol{\beta}\sin v$ - equation of the circle of unit radius at the normal plane of the line of the centers of the circles; $\boldsymbol{v}, \boldsymbol{\beta}$ - vectors of normal and bi-normal of the line of circles; v- polar angle at the normal plane of the of the line of the centers of circles.

The geometrical characteristics of the surface:

$$A = \sqrt{R'^2 + (s' - R k_s \cos v)^2}\ ; \quad B=R; \quad F = 0;$$

$$L = \frac{1}{A}\{R'[s'' - (2k_s \cdot R' + k_s' R)\cos v]n - (s' - R k_s \cos v)[(s' - R k_s \cos v)k_s \cos v + R'']\};$$

$$M = \frac{R R' k_s \sin v}{A}; \quad N = \frac{R}{A}(s' - R k_s \cos v),$$

$$k_1 = -\frac{L}{A^2}; \qquad k_2 = -\frac{N}{B_2} = -\frac{s' - R k_s \cos v}{AR}; \quad k_{12} = -\frac{N}{AB} = -\frac{R' k_s \sin v}{A^2} \tag{2}$$

$k_s = ks'$; k - curvature of the directrix line; $s' = |\boldsymbol{r}'|$; $\frac{\partial ..}{\partial u} = ..'$; k_1, k_2, k_{12} curvatures and twisting

curvature of the normal cycle surface.

The coordinate system of normal cycle surfaces is orthogonal but isn't conjugated so the generating circles aren't the coordinates of principle curvatures of the surfaces.

The orthogonal coordinate system allows using the common Hooke's law and the formula of potential energy (2). But at the equation of deformations must be used parameter of twisting curvatures k_{12} [7]:

$$\varepsilon_1 = \frac{1}{A_1}\frac{\partial u_1}{\alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}u_2 + k_1 u_3; \quad \varepsilon_2 = \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}u_1 + \frac{1}{A_2}\frac{\partial u_2}{\alpha_2} + k_2 u_3;$$

$$\varepsilon_{12} = \frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}\left(\frac{u_1}{A_1}\right) + \frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}\left(\frac{u_2}{A_2}\right) - \underline{2k_{12} \cdot u_3}. \quad \chi_1 = \frac{1}{A_1}\frac{\partial \vartheta_1}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}\vartheta_2 - \underline{k_{12}\omega_3};$$

$$\chi_2 = \frac{1}{A_2}\frac{\partial \vartheta_2}{\partial \alpha_2} + \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}\vartheta_1 + \underline{k_{12}\omega_3}; \quad \chi_{12} = \tau_2 + k_2\omega_1 + \underline{k_{12}\varepsilon_2} = \tau_1 + k_1\omega_2 + \underline{k_{12}\varepsilon_1}$$

$$\omega_1 = \frac{1}{A_1}\frac{\partial u_2}{\partial \alpha_1} - \frac{1}{A_1 A_2}u_1 - \underline{k_{12}u_3}; \quad \omega_2 = \frac{1}{A_2}\frac{\partial u_1}{\partial \alpha_2} - \frac{1}{A_1 A_2}u_2 - \underline{k_{12}u_3}; \quad \omega_3 = \frac{1}{2A_1 A_2}\left(\frac{\partial A_2 u_2}{\partial \alpha_1} - \frac{\partial A_1 u_1}{\partial \alpha_2}\right);$$

$$\tau_1 = \frac{1}{A_1}\frac{\partial \vartheta_2}{\partial \alpha_1} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_{21}}\vartheta_1; \quad \tau_1 = \frac{1}{A_2}\frac{\partial \vartheta_1}{\partial \alpha_2} - \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}\vartheta_2;$$

$$\vartheta_1 = k_1 u_1 - \underline{k_{12}u_2} - \frac{1}{A_1}\frac{\partial u_3}{\partial \alpha_1}; \qquad \vartheta_2 = -\underline{k_{12}u_1} + k_2 u_2 - \frac{1}{A_2}\frac{\partial u_3}{\partial \alpha_2} \tag{3}$$

Results and Discussion

All formulas of deformation of middle surface of shells with orthogonal not conjugated coordinate system differ from the formulas with system of principle curvatures only by components with geometrical coefficient k_{12} except deformation of twisting curvature χ_{12}. The formula of deformation χ_{12} usually used at the coordinate system of principle curvature of technical theory of shells differs from more common formula χ_{12} equations (3). But analyses of shells with coordinate system of principle curvatures don't show essential difference with using any of formulas.

The bloc of analyses of normal cycle shells there was included at the program complex.

After adding at the program complex some new blocs it's necessary to check the correction of their working. The correction of calculation of geometric characteristics of the middle surfaces of the shell may be checked up using the MathCad complex. The correction of results of analyses may be done if there are analytical decisions of some types of the shells of investigated class or types of the shells, which may be of another class of shells included in the program complex. Of course the results of analyses may be compared with analyses made by another program complex (for example by the finite elements method).

If the directrix of the normal cycle surface is a right line it will be the surface of revolution. The analyses of those shells may be received by analytical methods for membrane theory or by the VRMSHELL complex on the base of block of shells of revolution. If the radius of normal cycle surface is constant then it will be a tube surface. These types of surfaces incudes at the class of Monge's surfaces, which class of shells there are included in the complex.

The analyses of the cylindrical, cone and shell of paraboloid of revolution with the boundary condition of membrane shell theory shows the good coincidence with analytical decision so as with analyses made by bloc shell of revolution with complex boundary condition.

Lower there are shown the results of stress-strain analyses of tube shell with sine directing line (Figure 1). The analyses there were made on the blocs of normal cycle and Monge's surfaces. The results on the base of both program blocks with the same web shows difference not more 1 percent.

The parameters of the middle surface of the shell: direcnrix x=u; y=asin(πu/L); a=3 m; L=10 m; R=1 m; thickness of the shell h=0.1 m; module of elasticity E=3.5 $\cdot$ 10^8 kPa. The analyses was made on the own weight of the shell q=1 kPa.

At the torsion of the tube there is realized the swing-fixed support (Figure 2).

At Figure 2 there are shown the epures of the normal tangential forces received by using blocks of normal cycle surfaces and Monge's surfaces. The results of analyses with block of Monge's surfaces there are written at the brackets.

The results of analyses of sine tube of shell on own weight shows, that the circular tangential normal forces N_v at 7 and more times less of the longitudinal normal forces. The greatest normal longitudinal forces N_u there are at the upper part of the shell $v=\pi$ at the swing-fixed support,

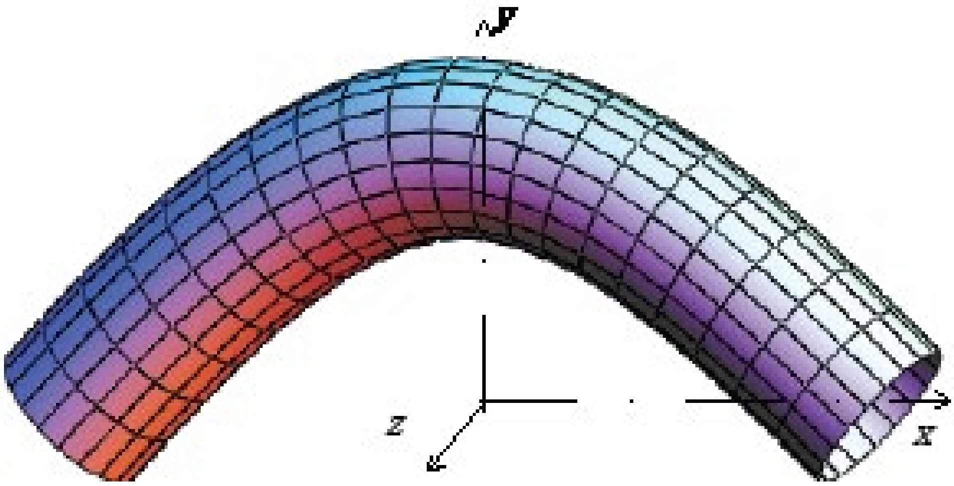

Figure 1. The sine tube surface.

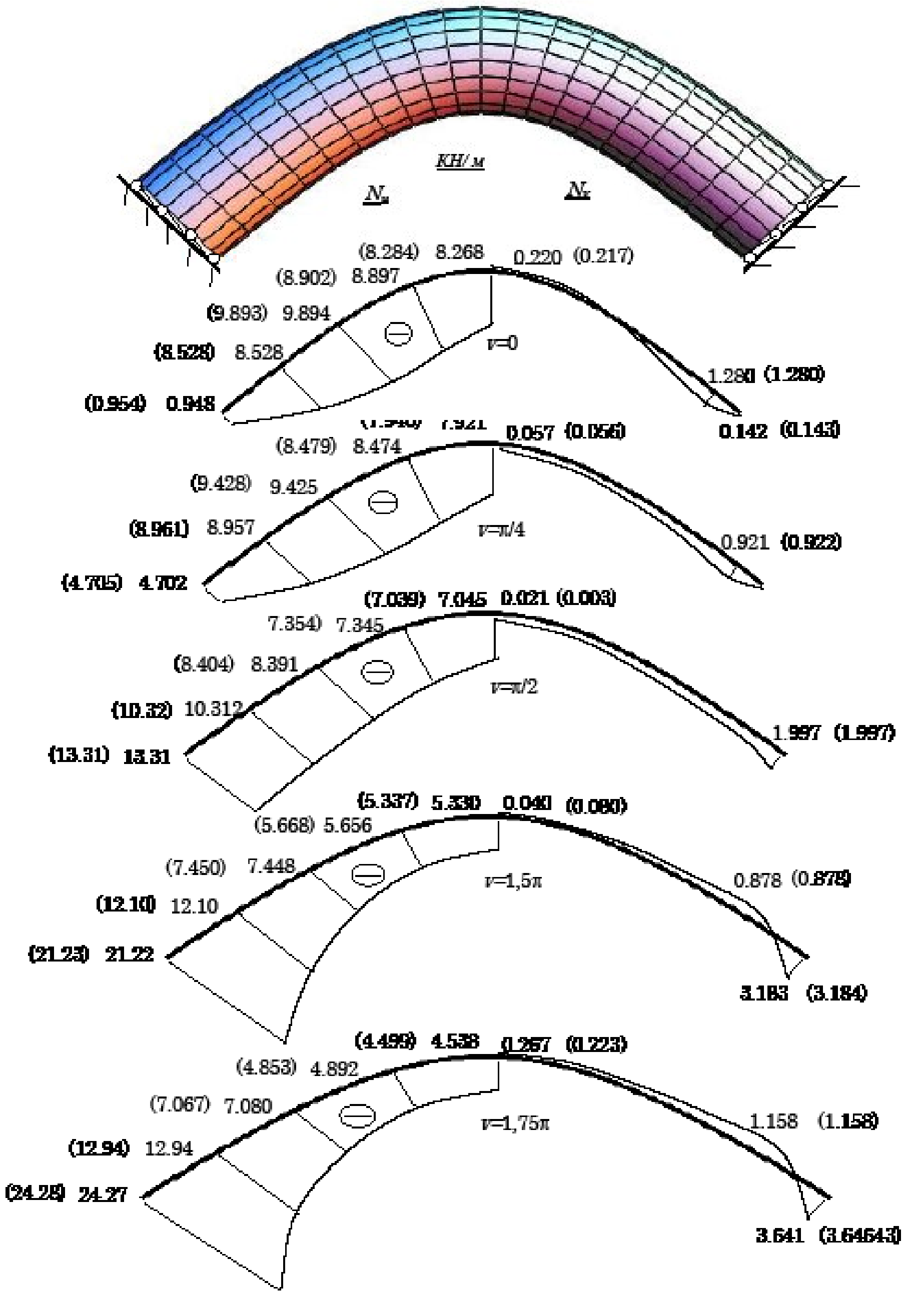

Figure 2. Tangential normal forces at sine tube.

$N_u^{max} = 24.27 kN/m$, $\sigma_{Nu}^{max} = 242.7 kPa$. At the lower zone of the tube shell *v*=0 the largest normal forces a at the middle zone of the half of the tube, $N_u^{max} = 9.89 kN/m$, $\sigma_{Nu}^{max} = 98.9 kPa$.

The largest normal circular N_v there are at the upper part of the shell *v*= at the swing-fixed support as well, $N_v^{max} = 3.64 kN/m$, $\sigma_{Nv}^{max} = 36.4 kPa$.

The moment bending forces there were calculated as well. But the stresses from bending forces there are not more 1, 2 percent of the tangential normal stresses, $\sigma_{Mu}^{max} = 2.66 kPa$, $\sigma_{Mv}^{max} = 2.37 kPa$. So, it may be said that it's realized the membrane theory of shells [8].

Conclusion

The analyses of stress-strain state of shell of revolution and tube shells on the base of block of normal cycle surfaces there has shown the near results with the results received by analytic analyses and analyses of the same shells made with the help of another blocks of the program complex. So this program block may be used for analyses of common types of normal cycle shells.

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