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The Unification of Coulomb's Electrostatic Law with Newton's Gravitational Law: A Generalized Model

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Abstract

Our search for the unification of electrostatic force and gravity is one of the most pressing research areas. Sir Newton's universal gravitational constant G is and has been the key constant in the calculations of classical mechanics for the gravitational potential and force of attraction between two masses, as well as the motion in the solar system. Recent research work on gravity focused on finding low-frequency gravitational waves. In this paper it is shown that, Newton's gravitational law and Coulomb's electrostatic law are manifestations of the same fundamental interactions. G depends on the quantum physical composition of matter, being the atomic number/protons (Z) to atomic mass number (A) ratio. All planets orbiting the sun yield, within statistical significance, the same G. However, the reference frame of atomic nuclei is distinctly different for each element and from that of the solar/planetary system. In addition, the definition of what Newton called "gravity" is rooted in the relation of all orbital motion to Kepler's third law. Kepler's third law ($\alpha=R^3/T^2$) and Sir Newton's law of gravitational attraction ($F=-GMm/R^2$) are fundamental references for orbital motion. After the full derivation, it is also shown that the coulomb force of attraction ($F= -q^2/(4\pi\varepsilon_o R^2)$) in the hydrogen atom yields a significantly same result as the Newtonian force of attraction between the proton and electron in the hydrogen atom, with a gravitational constant of 7.55×10^{28} $N.m^2.kg^{-2}$. It is shown that the unifying gravitational constant for all matter of nature is G=Z/A {1.525 1892 × 10^{29}} $N.m^2.kg^{-2}$. It is further hypothesised, based on the outcome of the theoretical derivation and correlation of the results between the coulomb and gravitational forces that gravity is electrostatic in nature and that they are reciprocally special cases of the general formula derived and presented in this paper.
The conclusions drawn from the results are supported by the analyses of information, using existing solar system/planetary data and atomic physics data. The results were correlated and confirm the hypotheses.

Keywords: Relativity • Gravitational constant • Newton's gravitational law • Coulomb's electrostatic law • Classical mechanics • Quantum mechanics • Schrodinger's equation • Solar system • Astrophysics • Planetary motion • Periodic table of elements • Gravitational constants of elements and planetary masses • Atomic interactions • Space time • Kepler's laws • Mean planetary distance from the Sun • Orbital radius • Period of orbital revolution • Dimensionless numbers of nature • Large numbers of nature • Gauss' theorem • Gaussian surface

Abbrevation: A: Relative Atomic Mass Number, r.a.m; amu: atomic mass unit, 1.660 538 921 x 10^{-27} kg; AU: Astronomical Unit, based on the mean distance between the sun and earth; CoM: Centre of Mass; E: Energy eigenvalue of the state of the quantum mechanical wave function, ψ (r, t); G: Newton's gravitational constant, 6.674 28 × 10^{-11} $N.m^2.kg^{-2}$; G_f: Relative gravitational constant, $(Z/A)^*(1.5251892 \times 10^{+29})$ $N.m^2.kg^{-2}$; H: Hamiltonian in classical mechanics=T+V=(Kinetic Energy) + (Potential Energy) $\hat{H}$: Hamiltonian operator in quantum mechanics; M_i: Mass of the central body orbited by other bodies of mass mi (e.g. Sun, Earth or Nucleus); M_n: Molar mass of substance; m_i: Mass of planet i; m_e: Mass of electron, 9.109 382 91 × 10^{-31} kg; m_n: Mass of neutron, 1.674 927 351 × 10^{-27} kg; m_p: Mass of proton, 1.672 621 777 × 10^{-27} kg; N_A: Avogadro's Constant, N_A=6.022 141 × 10^{23} $atoms.mol^{-1}$; n: Principal quantum number; nm: number of moles of an element/substance=m/M_n; p: Momentum; q: C; R: Radial centre-to-centre distance from the central body M_i (CoM) to the orbiting body, m_i (CoM); R: Radial vector (x, y, z); r_i: Radius of planet i (x, y, z); r: radial vector of planet i, ((x, y, z); T_i: Orbital period of planet or orbiting mass, mi around the central body, M_i; Z Atomic number; ω_i: Angular speed of Planet mi around the central body; α:used to define Kepler's 3rd Law as $\alpha=R_3/T_2$; β: used to express the value of the relation between G=β ($4\pi\varepsilon_o$); δ: used to express the value of the relation between G= δ ($q2/4\pi\varepsilon_o$); ∇^2: Del-squared or Laplacian Operator; ε_o: permittivity of free space; π: 3.14159265359; θ angle in radians or degrees, as specified; λ wavelength; ρ: density of matter; ψ; wave function, ψ (r, t);

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Introduction

Johannes Kepler empirically derived three laws from the data initially compiled by Tycho Brahe. Brahe made his observations without the use of a telescope [1]. Kepler's third law which states that: The ratio of the cube of the orbital radius from the centre of the revolving object to the centre of the central body divided by the square of the corresponding orbital period is constant, herein denoted as $\alpha=R^3/T^2$, is fundamental to gravitational postulations, derivations and calculations.

Based on Kepler's laws, sir Newton followed with principia mathematica, including the derivation of F=ma (mass × acceleration) and developed the inverse square law of gravitational attraction, taking into account the centripetal force (mv^2/R or m ω^2R) and the central force of gravity ($F=-GMm/R^2$), which led to the empirical determination of the gravitational constant, G. Up to the writing of this paper, we used G as the universal constant for the determination of the gravitational potential and force of attraction between two objects of mass, M_i and m_j and a centre to centre distance R apart.

For over 330 years, since Newton's derivation and naming the phenomenon "gravity", scientists have been searching for the origin of gravity and the unification of the four fundamental forces of nature, being: the strong and weak forces, and the electromagnetic forces and gravity [2]. A study of Newtonian gravity measurements which imposed constraints on unification theories, found that such constraints were not viable [3]. Furthermore, using the gauge theory and "standard model", the unification of electromagnetic and weak forces has been treated [4]. However, the unification of gravity with electromagnetism/electrostatics remained unresolved up to the presentation of the hypothesis and derivations in this paper.

During the 20th century Einstein (1916) introduced General Relativity (GR), as a theory for us to describe gravity in terms of the curvature of space time. In addition to the famous $E^2=m^2c^4 + p^2c^2$ equation, Einstein developed the well-known GR equation.

To date, that GR equation is applied to describe gravity as the curvature of space-time. The proposition in this paper is that G is not constant in all frames of reference, but is relative to the centricity of orbital motion. Furthermore, it will be shown in this paper that gravity is electrostatic in nature and related to electromagnetic fields as also supported in. While Coulomb's law is an experimentally determined result, a fundamental theoretical model will be derived in this paper, which will also account for Newton's law of gravitation [5].

In this paper I take cognizance of the scientific principles necessary to present information and findings. In order not to clutter the derivations with vector notations and center of mass provisions in the derivations, I will present the derivations in scalar format and treat the mass of the heaviest body in the analysis as the CoM. It is also the purpose of this paper to present the hypotheses which are intended to lead us closer to a better understanding of how the universe works, by showing the relative nature of the gravitational constant, gravitational field, potential and force of attraction. It is also shown that it correlates with Coulomb force of interaction, not only in terms of similarity of principles or formulae but with regard to the actual natural phenomenon, the formulae and the calculated results. It will become evident that what we historically applied as the universal gravitational constant in all frames of reference, is in fact relative and applicable to a frame of reference in which $4\pi^2R^3/MT^2$ is constant and equivalent, allowing for statistical deviations of the calculated mean constant, G, for large Masses (M). The results will show that G is more specific in quantum mechanics to atomic nuclei, for each periodic table element and that the proton is a key gravitational particle. The question as to whether G is universal or not was already raised but remained unresolved [6].

Hypotheses

The following hypotheses are formulated to test the validity of the competing claims. The H_0 (Null Hypothesis) being the status quo and which may be rejected when H_1, H_2, H_3 (Alternative Hypotheses) are tested and provide supporting results to reject H_0:

H_0: Is the gravitational constant, G, universal and applicable to all frames of reference, where bodies, m_i, are in orbital motion and revolving a central body, M, and to determine the gravitation potential and force of attraction between the two bodies, M and m?

Is $G=6{,}674 \times 10^{-11}$ ($N.m^2.kg^{-2}$) universal, because the ratio R^3/T^2 is constant and equal in all frames of reference?

H_1: Is the gravitational constant, G, speci ic to the respective frame of reference of the central body, M, orbited by the other bodies, mi, in the particular orbital system of the central body?

Is the ratio R^3/T^2 constant in a speci ic frame of reference, but not equal across all frames of reference. Hence, is $G \neq 6{,}674 \times 10^{-11}$ ($N.m^2.kg^{-2}$) in all reference frames?

H_2: Is there a relation between Kepler's 3rd law R^3/T^2 and the quantum composition of matter, such that the central body, M, can be inferred by applying the gravitational constant formula for $M=4\pi^2R^3/GT^2$ based on the bodies, m_i, revolving the central body, M?

H_3: Is the gravitational force of attraction an electrostatic/ electromagnetic force, based on the intrinsic (constituted of protons, neutrons and electrons) nature of all matter? Is it expected that there is a relation between G and $q^2/4\pi\varepsilon_0$, such that $G=\delta q^2/4\pi\varepsilon_0$ where: δ a value to be determined by way of derivation and computation, from fundamental principles of physics? ε_0 is the permittivity of free space [7].

Materials and Methods

In order to develop, determine and test the validity and applicability of the respective hypotheses, it is important for us to revert to the original Newtonian definition and derivation of the gravity law and the resulting gravitational constant based on Kepler's 3rd law. The results should be congruent with existing laws and theories and produce testable predictions of the various interactions from planetary bodies to quantum mechanics.

During February 2022, I released a media statement which specified the result for the relative gravitational constant at atomic level, for hydrogen (^{2}H-Deuterium), to be 7.55×10^{28} $N.m^2.kg^{-2}$ by applying the readily available empirical data for the ^{2}H atom. To advance the significance of that result, it is also essential to derive and show the general result which is applicable to all elements of the periodic table, as well as the planetary system. Therefore, the derivations of the following key equations for planetary and atomic interactions were analysed: a) R^3/T^2; b) $4\pi^2R^3/MT^2$; and c) $\delta\ q^2/4\pi\varepsilon_0$. EXCEL Spreadsheets and readily available planetary and atomic data were used for the calculations. The source data for the planets was obtained from tables in Walker and Spiegel for atomic data from the Particle Physics Booklet and tables in Wong. The computed results were compared to the current Newtonian gravitational constant, G=6.674 28 × 10^{-11}. The three frames of reference, namely:

- The sun as the central body, M;
- The Hydrogen (Deuterium ^2H) nucleus as the central body, M;
- The earth as the central body, M for the moon orbiting the earth, forms the basis of the hypotheses tests.

It is worthwhile to note that, at the time of the development and presentation of the gravitational law of attraction in 1687, Sir Newton did not have access to some of the subsequent laws and theories such as those of Coulomb's inverse square law of electrostatic attraction and repulsion, as well as other theoretical developments from Dalton, Avogadro, Thomson, Millikan, Rutherford and Bohr, Chadwick and many other theories of atomic physics, nuclear physics, electromagnetism and quantum mechanics. Similarly, the model of the atom was not fully developed when Einstein published the general theory of relativity. Newton named the observed phenomenon "gravity".

Assumptions and data analyses of planets, the moon and the hydrogen (^2H) atom

The following main assumptions are considered to ensure that the consistency of analyses, validity, rigour and vigour are not compromised:

That the data utilized is related to the extent of planetary, near earth and atomic data available and significant enough to test the hypotheses, so as to draw material conclusions; that the information and data obtained from long established and utilized physics textbooks are accurate enough for the purpose of the hypotheses to be tested, without limitation to any other reliable source data that may be used by the reader; that the variations in data between the sources are not significant; that significant numbers are not emphasised, but are only distinguished as far as indicating the principle which needs to be reconfirmed or fundamentally revised; that based on the data sample sizes, where applicable and less then n=50, the t-distribution will be utilised for calculations of descriptive statistics and significance.

That for purposes of simplified algebra the respective central masses, M_i, being orbited are treated as the CoM, as each is more than 98% of the total mass of the respective system: Sun (99.8%), earth (98.7%) and deuterium-^2H nucleus (99.9%) [8].

Table 1 below contains the data used to calculate, analyse and test the alternate hypotheses, H_1, H_2 and H_3.

- Planetary data is for planets orbiting the sun, including the dwarf planet Pluto.
- Atomic (Hydrogen Atom) data, as the hydrogen atom forms the basis for many of our insights into atomic and nuclear mechanics.
- Moon (orbiting the Earth) data, as the closest permanent companion of the
  Earth, which also provides us with invaluable clues into the planetary system.

A: Planetary						
#	**M_i**	**R_i km x 10^6**	**T_i (earth yr)**	**V_i (km/s)**	**R^3/T^2**	**$4\pi^2R^3/MT^2$**
1	Mercury	58.20	0.241	47.90	3.4329E+18	6.8137E-11
2	Venus	108.00	0.615	35.10	3.3686E+18	6.6861E-11
3	Earth	150.00	1.00	29.80	3.4135E+18	6.7752E-11
4	Mars	227.00	1.88	24.10	3.3472E+18	6.6437E-11
5	Jupiter	778.00	11.86	13.10	3.3860E+18	6.7208E-11
6	Saturn	1427.00	29.46	9.65	3.3863E+18	6.7213E-11
7	Uranus	2871.00	84.00	6.80	3.3921E+18	6.7327E-11
8	Neptune	4 498.00	164.80	5.44	3.3890E+18	6.7266E-11
9	Pluto	5910.00	248.40	4.75	3.3836E+18	6.7160E-11
				Average, μ	3.3888E+18	6.7262E-11
				STD, σ	2.4343E+16	4.8318E-13
B: Atomatic (Hydrogen Atom)						
#	m_i	R_i m x 10^{-12}	T_i (S)	V_i (km/s)	R^3/T^2	$4\pi^2R^3/MT^2$
10	Hydrogen	52.8	1.52E-16	2187.279	6.3986	7.5460E+28
C: Moon (Orbiting the Earth)						
#	m_i	R_i m x 10^6	T_i (earth yr)	V_i (km/s)	R^3/T^2	$4\pi^2R^3/MT^2$
11	Moon	0.384	0.07479	1.02	1.02371E+13	6..7583E-11

Table 1. Planetary, Moon and Hydrogen atom data.

Results and Discussion

The computed results in Table 1, p.10, reveal the following important information.

For H_1: In congruence with Kepler's third law, the ratio R^3/T^2 for each planet in the solar system, including the dwarf planet Pluto which also orbits the sun is equal to $3.3888 \times 10^{18} \pm 0.024343 \times 10^{18}$; for the ^{2}H-Deuterium atom, the ratio, R^3/T^2=6.39; for the earth orbited by the moon, the ratio $R^3/T^2=1.023 \times 10^{13}$; based on the available sample data utilised, the Keplerian ratios are specific to their frames of reference and signi icantly different. Thus, reject H_0 and accept H_1.

For H_2: The ratio R^3/T^2 for the sun (as central body, M) is not equal to the respective ratios of the moon orbiting the earth (as central body M), as well as the electron orbiting the Hydrogen nucleus (as central body M); The ratio R^3/T^2 changes significantly from Helio-centric orbits to geo-centric orbits to the nuclei-centric orbits. The values of the ratios between the three (3) categories of Table 2 being:

A (Planets)	C (Moon)	B (^{2}H Atom)
3.3888×10^{18}	1.02371×10^{13}	6.398562
5.296×10^{17}	1.5999×10^{12}	1

Table 2. The values of the ratios between the three (3) categories.

In further support of sir Newton's law of gravitational attraction, the average G in the solar system is calculated to be $6.7262 \times 10^{-11} \pm 0.048318 \times 10^{-11}$. This constant, as formulated by sir Newton, is within material range (± 0.77309 %) of the internationally accepted (SI) value of $G=6.6742 \times 10^{-11}$.

The gravitational constant of attraction of the earth (as central body, M) on the moon is calculated from the data to be $G_{earth}=6.7583 \times 10^{-11}$. This value of G is also within material range (± 1.2600 %) of the internationally accepted (SI) value of $G=6.6742 \times 10^{-11}$.

However, the gravitational constant of attraction, using the data in Table 1, for atomic nuclei, in this case ^{2}H-Deuterium-Hydrogen (Z=1, $M=m_p+m_n$), was found to be $G_{atomic}=7.55 \times 10^{28}$ which is materially different from the G for the sun or the earth [9].

Based on the available sample data utilized, the gravitational constant is different in different frames of reference. Thus, reject H_0 and accept H_2.

For H_3: Lastly, to test H_3, the relation between $G=\delta q^2/4\pi\varepsilon_0$. Initially, $G=\beta\ (4\pi\varepsilon_0)$ was calculated and found to be β=0.599721, such that $G/4\pi\varepsilon_0=0.599721\simeq3/5$. This result interestingly resembled the factor sued in the coulomb calculations in nuclear physics, except that in this case β is not dimensionless, but merely providing a much reduced relational number, which may be interpreted and understood in future work.

From 5.7 above and the coulomb energy $V_c = 3/5\ (1/4\pi\varepsilon_0)\ (Ze^2/R)$ for a spherical nucleus and assuming that the charge is distributed spherically. Nature gives us the clue of the likely relation between gravitational potential and electrostatic potential following the observation that the same factor appears in the coulomb energy for the nucleus. The relation between the gravitational law and the electrostatic law was also investigated and the resulting "… gravitational constant G is expressed exactly through the elementary charge e and the electromagnetic vacuum constants ξ, ε_0 and μ_0", and the resulting dimensionless constant in that derivation, η_{ij} ($=4.39 \times 10^{-40}$), is speci ic to magnetic moment of interacting particles. Further generalisation is required to account for the quantum physical composition of different matter, from subatomic particles to large masses and constellations.

In addition to the relation asserted, I derive the following expanded relation to show that in the coulomb experiment, certain variables were implicitly incorporated in the experimentally determined law. The formulation is as follows:

Original coulomb's law: $F=Q_1Q_2/4\pi\varepsilon_0R^2$

Additions to the above equation:

The charges Q_1 and Q_2 are either added or removed from objects of mass m_1 and m_2, respectively to attain the net charge on each object.

The masses m_1 and m_2 are constituted of their respective atoms (protons, neutrons and electrons) as:

$m_1=\{A_1 \times (a.m.u.) + Z_1m_e\} \times (N_A \times n_{m1})$;

$m_2=\{A_2 \times (a.m.u.) + Z_2m_e\}\times (N_A \times n_{m2})$;

Z_i=The number of protons in the nuclei of the atoms of each mass;

$Z \times q_p=Z \times q_e$ Charge conservation;

(A_i-Z_i)=Neutrons in respective mass;

Applying Gauss' Theorem (closed surface) on neutral mass, then:

For mass m_1:

$\iint E_1.dS =(+Z_1q_p-Z_1q_e) \times NA_1 \times nm_1/\varepsilon_0=0$

For mass m_2:

$\iint E_2.dS = (+Z_2q_p-Z_2q_e) \times NA_2 \times nm_2/\varepsilon_1=0$

Applying Gauss' Theorem (closed surface) when charge has been added or removed:

For mass m_1: example of $^+$ve charge: (X number of $^-$ve charges)

For an object to be $^+$vely charged, electrons are removed. Thus, $\iint E.dS=(+Z_1q_p-Z_1q_e) \times NA_1 \times nm_1-(-X_1q_e)/\ \varepsilon_0={}^+X_1q_p/\ \varepsilon_0$

The new mass $(m_1)' = \{A_1 \times (a.m.u.) + Z1m_e\} \times (N_A \times n_{m1}) - X_1m_e$; For mass m_2: example of -ve charge: (Y number of -ve charges)

For an object to be -vely charged, electrons are added. Thus, $\iint E.dS=((+Z_2q_p-Z_2q_e) \times NA_2 \times nm_2-(-Y_2q_e)/\ \varepsilon_0=Y_2q_p/\ \varepsilon_0$

The new mass is $(m_2)' = \{A_2 \times (a.m.u.) + Z_2me\}\times (NA \times nm_2) + Y_2m_e$;

Applying "specific charge" relation for a negative and positive charges to quantized mass in relation to charges:

For an electron: $^{-}q_e/m_e$

For a proton: $^{+}q_p/m_p={}^{+}q_p/A$ (a.m.u)

Having introduced the above context of the relation of specific charge (charge and mass), I now revert to Coulomb's Law (in this example for m^1 $^{+}$vely charged and m^2 $^{-}$vely charged, but can be generalized) and apply lines 5.8 to 5.12 as follows:

$F=Q_1Q_2/4\pi\varepsilon_0R^2 = (+X_1qp)\ (-Y_2qe)/4\pi\varepsilon_0R^2$

$F= (^{+}X_1\ m_p\ (q_p/m_p))\ (-Y_2m_e\ (q_e/m_e))/4\pi\varepsilon_0R^2$

$F=(^{+}X_1\ m_p\ (q_p/A\ (a.m.u.))\ (-Y_2m_e\ (q_e/m_e))/4\pi\varepsilon_0R^2$

Rearranging the above equation, as follows:

$F=(q_p/A\ (a.m.u))\ (q_e/m_e)/4\pi\varepsilon_0\ R^2\ (+X_1mp)\ (-Y^2me)$

$F=(1)/A\ (1)/4^{\pi}\varepsilon_0R^2(qp)(qe)/\ a.m.u.)\ (me)\ (+X_1mp)(-Y_2me)$

$F=(1)/A(1)/4\pi\varepsilon_0R^2(qp)(qe)/\ a.m.u.)\ (me)\ (+X_1mp)(-Y_2me)/R^2$

This equation includes the Coulomb variables of charge and radial distance, as well as the Newtonian variables of mass and radial distance.

Based on the above derivation and correlation of coulomb's electrostatic law with Newton's gravitational law, there is a relation between the two fundamental laws of nature. Thus, reject H_0 and accept H_3.

Following the above generalized formulation of coulomb's law and the observation that Coulomb's law and Newton's law are fundamentally connected, except that for gravity the large masses are attracting each other, as opposed to quantum particles which can attract or repel each other, depending on the net Gaussian charge. Based on this important connection, the derivation of the relative gravitation constant, $G=4\pi^2R^3/MT^2$, which is applicable to all bodies from planetary, solar to atomic level (^{2}H-Deuterium) atom is provided as follows, before generalization:

From:

Force (F)=mass (m) × acceleration (a)=mv^2/R

$F=Zq^2/4\pi\varepsilon_0R^2$

m=mass of the electron in orbit.

$m_ev^2/R=Zq^2/4\pi\varepsilon_0\ R^2$

$m_e\ (2\pi R/T)^2/R=Zq^2/4\pi\varepsilon_0\ R^2$

$m_evR_n=nh/2\pi$ (Bohr Postulate) and $p=m_ev=h/2\pi R_n/n=h/\lambda$ (de Broglie Postulate)

After substitution and rearrangement, we have:

n=Principal quantum number;

$R_n=n^2h^2\varepsilon_0/Zq^2m\pi=n^2/Z\ R_B$; where RB is the Bohr Radius.

Then:

From the Bohr quantum condition and de Broglie condition for angular momentum, L: $L=R_n \times p_n=mv_nR_n\sin(90^o)=nh/2\pi$

We find v: $v_n=nh/2\pi mR$

Using v_n and R_n, m=me we ind the orbital period, T: $T_n=2\pi R/v$

Substituting v_n and R_n, we ind the orbital period,

T_n: $T_n=2\pi R_n/v_n=2\pi R/nh/2\pi m_eR_n=(2\pi R/1)\ (2\pi m_eR_n/nh)=4\pi^2\ m_e(R_n)^2)/nh$

Then from the semi-classical approach using Kepler's 3rd Law we find R^3/T^2 by substituting T with T_n and R with R_n, the result is:

$(R_n)^3/T^2=((n^2\ h^2\varepsilon_0/Zq^2m_e\pi)^3/(4\pi^2\ m_e(R_n)^2)/nh)^2=(n^8h^8(\varepsilon_0)^3/(16Z^3q^6(me)^5(\pi)^7(R_n)^4$

$=(n^8h^8\ (\varepsilon_0)^3/\ (16Z^3q^6\ (me)^5(\pi)^7(n^2h^2\varepsilon_0/Zq^2me\pi)^4=Zq^2/16me\pi^3\varepsilon_0$

=Z (6.414095)

$R^3/T^2=Z\ (q^2/16m\pi^3\varepsilon_0$

Furthermore, it is fundamental for us to apply Sir Newton's derivation and definition to determine the gravitational constant, G: $G=4\pi^2R^3/MT^2$

M is the mass of the atomic nucleus, based on the similar principal to the mass of the central body which is being orbited. The nuclear mass constitutes 99.9% of the mass of an atom. In the case of atomic nuclei, the mass, M is:

M=Relative atomic mass number (A) x atomic mass unit (a.m.u)

M=A (a.m.u.)

After substituting the results of R^3/T^2 and M in $G=4\pi^2R^3/MT^2$, we ind (the "-"sign is added to indicate the attractive nature of $(-q^2)=(+q)^*\ (-q)$):

$G=-Zq^2/A\ (a.m.u.)\ m4\pi\varepsilon_0$

After rearranging the above equation of G and will herein forward referred to as relative gravitational constant G_f, or Gatomic we find:

$Gf=-Z/A\{1/4\pi\varepsilon_0\}\{q/a.m.u.)\}\{q/m\}$

$=-Z/A\ \{1.525\ 189\ 2 \times 10^{29}\}\ N.m^2.kg^{-2}$

The above result for Gf clearly shows that gravity is: electrostatic in nature based on the interactions of charges, ^{+}q and ^{-}q. Hence, the reference to gravitational field; relates to the specific charges of the electron $\{^{-}q/m\}$ and nucleon $\{^{+}q/a.m.u)\}$ depends on the ratio of **Atomic Number (Z):** Relative Atomic Mass Number (A). Each periodic table element has a gravitational constant based on its respective nucleon composition, as given in Appendix A. The ratio of Z: A gives a likely correlation to nuclear stability, since the relative gravitation constant reduces with decreasing Z/A factor, as depicted.

The above results indicate that Gf is not constant across all frames of reference. The aforementioned derivations, results and observations are significant, as they unify Coulomb's law and Newton's gravitational law as interactions of the same fundamental phenomenon [10].

Following the above derivation of Gf, the Newtonian gravitational potential and Coulomb electrostatic potential are unified into one fundamental equation as:

$V(R)=-Zq^2/A\ (a.m.u.)\ m4\pi\varepsilon_0\ Mm/R$

As a result of the above generalised potential ield, the Coulomb potential energy equation in the Schrodinger equation is unified with the Newtonian gravitational energy equation, such that the Hamiltonians in the TDSE (Time Dependent Schrodinger Equation) and TISE (Time Independent Schrodinger Equation) can be substituted with the generalised potential equation:

TDSE: $-\hbar^2/2m\nabla^2 \hbar\,(r,t)+V\,(r,t)\,\psi\,(r,t) = i\hbar\,\partial/\partial t\,\psi\,(r,t)$

TISE: $-\hbar^2/2m\nabla^2 u\,(r) + V\,(r)\,u\,(r) = Eu(r)$

Such that:

$\hat{H} \stackrel{\text{def}}{=} -\hbar^2/2m\nabla^2+V\,(r)$

$E=Es \stackrel{\text{def}}{=} \hbar\,\omega\,s$ and $\psi\,s\,(r,t) \stackrel{\text{def}}{=} us\,(r)e\text{-}i\omega t$ and $u(r) = us\,(r);\ \omega s=\omega\psi$

Theoretical and empirical analyses

Utilizing the results computed in Table 1, p. 10, and the observations in section 5 above, the following further calculations were made to draw conclusions and make propositions based on the results of the theoretical and empirical analyses:

Firstly, the gravitational constants, as calculated in section 4, are applied as per the usual analytic process in the frames of reference of the Sun, Earth and nuclei as central bodies [11].

Secondly, the hypotheses, in section 2, are applied and tested.

The results are as presented in Table 3 below:

#	m_i (Name)	R_i km x 10^6	T_i (earth-yr)	V_i (km/s)	R^3/T^2 (m^3/s^2)	$4\pi^2R^3/MT^2G_s$	m_i (kg)	Sun(acc.) $G_s.Ms/R_1^2$	$Force_G$ (N)
1	Mercury	58.2	0.241	47.9	3.4329E+18	6.8137E-11	3.20E+23	4.0010E-02	1.2803E+22
2	Venus	108	0.615	35.1	3.3686E+18	6.6861E-11	4.90E+24	1.1401E-02	5.5867E+22
3	Earth	150	1	29.8	3.4135E+18	6.7752E-11	5.98E+24	5.9893E-03	3.5816E+22
4	Mars	227	1.88	24.1	3.3472E+18	6.6437E-11	6.40E+23	2.5644E-03	1.6412E+21
5	Jupiter	778	11.86	13.1	3.3860E+18	6.7208E-11	1.90E+27	2.2085E-04	4.1961E+23
6	Saturn	1427	29.46	9.65	3.3863E+18	6.7213E-11	5.70E+26	6.5651E-05	3.7421E+22
7	Uranus	2871	84	6.8	3.3921E+18	6.7327E-11	8.70E+25	1.6246E-05	1.4134E+21
8	Neptune	4498	164.8	5.44	3.3890E+18	6.7266E-11	1.03E+26	6.6129E-06	6.8113E+20
9	Pluto	5910	248.4	4.75	3.3836E+18	6.7160E-11	5.40E+24	3.8245E-06	2.0652E+19
10				Average, μ	3.3888E+18	6.7262E-11			
B:	Atomic	(Hydrogen Atom)		M_a (kg)	3.35E-27			Gravitational	Law
#	m_i (Name)	R_i m × 10^{-12}	T_i(s)	V_i (km/s)	R^3/T^2 (m^3/s^2)	$4\pi^2Ra^3/M_aT_a^2\ G_a$	m_e-and (kg)	G_i (acc.) $G_i.M_a/R^2$	$Force_G$ (N)
11a	Hydrogen	52.8	1.52E-16	2187.279	6.3986	-	9.11E-31	8.08E-17	7.36E-47
11b	Hydrogen	52.8	1.52E-16	2187.279	6.3986	7.55E+28	9.11E-31	-	8.25E-08
B1:	Atomic	(Hydrogen Atom)		Q_a (C)	1.60E-19			Coulomb's	Law
#	Q_{e-} (Name)	R_i m × 10^{-12}	T_i (s)	V_i (km/s)	R^3/T^2	$4\pi^2Ra^3/M_aT_a^2\ G_a$	q_{e-} (c)	Coul. Constant $1/4\,\pi\varepsilon$	$Force_c$ (N)
11c	Hydrogen	52.8	1.52E-16	2187.279	6.3986	-	1.60E-19	8.99E+09	8.28E-08
C: Moon	(orbiting	the Earth)		ME (kg)	5.98E+24				
#	m_i (Name)	R_i km × 106	T_i (earth-yr)	V_i(km/s)	R /(m^3/s2)	$4\pi^2Ra^3/\ MT^2G_E$	m_i (kg)	Earth (acc) $Gs.M_E/R_i^2$	$Force_G$ (N)
12a	Moon	0.384	0.07479	1.02	1.02E+13		7.38E+22	0.002728	2.01E+20
12b	Moon	0.384	0.07479	1.02	1.02E+13	6.76E-11	7.38E+22	-	2.02E+20

Table 3. Calculation of force of attraction on planets, moon and hydrogen atom electron.

The results computed and listed in Table 3 above, show several important outcomes. The observations are listed as follows:

Coulomb's electrostatic law and sir Newton's gravitational law are fundamentally of the same natural phenomenon. Hence, their similarity in formulation;

It is therefore proposed that the model for the unifying electrostatic/gravitational field equation be:

Unifying potential: $V\,(R)=-Z/A\ \{1/4\pi\varepsilon_0\}\ \{q/a.m.u.\}\ \{q/m\}\ Mm/R$

For coulomb's law: $V\,(R)=-Z/A\ \{1/4\pi\varepsilon_0\}\ \{q/\ (1)\}\ \{q/1\}1/R$ (opposite charges)

Where I observe that: A=1, a.m.u=1, M=1 and m=1 is an implied special case of the general gravitational equation.

Planetary gravitational analyses:

Gravitational law: $V=-Z/A\ \{1/4\pi\varepsilon_0\}\{q/a.m.u.\}\{q/m\}\ Mm/R$

Where $G=6.674\ 28 \times 10^{-11}$ $N.m^2.kg^{-2}$ is a special case of the general gravitational formula:

$G=-Z/A\{1/4\pi\varepsilon_0\}\{q/a.m.u.\}\{q/m\}$

Such that solving for Z/A, we obtain:

$6.674\ 28 \times 10^{-11} = Z/A\ \{1/4\pi\varepsilon_0\}\{q/a.m.u.\}\{q/m\}$

$=Z/A\ \{1.5251892 \times 1029\}\ N.m^2.kg^{-2}$

Resulting in: $Z/A = 4.376\ 03413 \times 10^{-40} = 1/2.285174 \times 10^{39}$

Observations

From the above results, an important observation is made that the special dimensionless large number ($2.285\ 174 \times 10^{39}$) in the denominator of Z/A indicates the ratio of the number of nucleons for each proton in the planetary masses. This ratio may not necessarily be the same for all large masses, leading to different gravitational interactions. The above mentioned large number ($2.285\ 174 \times 10^{39}$) used to be the unknown factor whenever the strength of the Coulomb force was compared to the strength of the gravitational force. It follows from the above result that the large number is as a result of the ratio of protons (Z) to the number of nucleons (A) in the mass of the central body, M, in this case the sun. By applying Newton's gravitational constant for the sun at atomic for quantum mechanical interactions was an implicit assumption that the ratio of Z: A is the same for all reference frames, while the Z: A ratios at atomic level are distinctly different for each periodic table element. The aforesaid application of the Newtonian gravitational constant in quantum mechanics also led to the disparities in observations between the predictions of interactions at planetary level and interactions at atomic level. This resulted in the historic conclusion that gravity can be ignored in quantum mechanical analyses. This paper shows to the contrary, that gravity and electrostatic fields, force and potential are of the same origin and should produce the same or similar predictions and results for the respective orbital systems. It is evident from the result for G_f, in section 5 above, that the gravitational constant is relative to the centricity of orbital motion, based on the ratio of Z: A; The interpretation that gravity is the curvature of space-time is as a result of the influence and effect of the electrostatic field in space-time between masses. The orbital characteristics of planets around the sun and the earth show significantly comparable results for a similar gravitational constant, as listed in Table 2, lines 1^{-10} and 12a and 12b. Hence, we may apply the Newtonian constant $G=6.6742 \times 10^{-11}$ in calculations for interactions of bodies mi, in these two frames of reference, with minimal deviation; The gravitational constant for atomic nuclei, G_f (also referred to as Gatomic), based on the calculations for a Hydrogen (^2H) atom, is distinctly different from that of the sun and earth, using empirical data:

$G=G_{sun}$ ($=6.6742 \times 10^{-11}$) $\neq G_{Earth}$ (6.7583×10^{-11}) $\neq G_{Deuterium}$ (7.55×1028)

The general gravitational constant is $G_f=Z/A\ \{1.5251892 \times 10^{29}\}$ $N.m^2.kg^2$.

When the gravitational constant is calculated (using empirical data) based on the central body, M_i and the related orbital characteristics of the bodies, m_i revolving around the central body, M_i, then a computed relative gravitational constant G_f constant is applicable in that particular frame of reference for all motion, as per Table 2, lines 1 to 10, 12a and 12b.

It is observed from the results in Table 2, lines 11a and 11b that when the Newtonian gravitational constant G, based on planetary motion is imposed in the atomic frame of reference, then the results for the calculation of the gravitational force of attraction on the orbiting electron are distinguishably and significantly different from the results of the Coulomb force of attraction. When the relative gravitational constant, G_f, is applied in any frame of reference and compared to the Coulomb force of attraction, then the results of the Newtonian and Coulomb forces correspond significantly, as shown in Table 2, lines 11b and 11c, to within margin of error of ± 0.2622% (8.254×10^{-8} N ± 0.2622%), using the readily available atomic data. By applying the general gravitational formula to the naturally abundant isotopes of the periodic table, a list of relative gravitational constants is provided for each element, as per appendix A, according to the distinct sequence of atomic numbers. In appendix B the elements are listed from the largest relative gravitational constant to the least, with the accompanying periodic Table of constants. It is observed that this list shows a trend of atomic nuclei stability, from the most stable nuclei to the least stable nuclei. There are a set of 6 elements from Z=78 to 83, namely: Pt (Z=78), Au (Z=79), Hg (Z=80), Tl (Z=81), Pb (Z=82) and Bi (Z=83) which are classified as naturally stable, but are among elements with unstable nuclei.

With the exception of Hydrogen (^1H) with the highest relative gravitational constant of $1.513\ 13 \times 10^{+29}$ $N.m^2.kg^{-2}$ as depicted in Figure 1, there is a similar range for the relative gravitational constants of the remaining elements from O (Z=8) at $7.626\ 23 \times 10^{+28}$ $N.m^2.kg^{-2}$ to the lowest for U (Z=92) at $5.894\ 97\ 10^{+28}$ $N.m^2.kg^{-2}$.

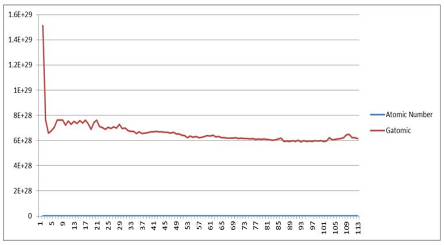

Figure 1. Trend of relative gravitational constant *vs.* atomic number.

Figure 2 depicts the inverse reduction trend of the relative gravitational constant per nucleon plotted versus the atomic number of the periodic table elements.

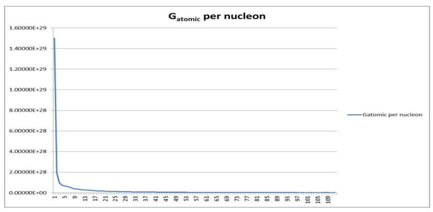

Figure 2. Trend of relative gravitational constant per nucleon *vs.* atomic number.

A further important result of the general gravitational derivation is that:

All orbital motion obeys Kepler's 3rd law and the universal relation is: $R^3/T^2=Z\ (q^2/16m\pi^3\varepsilon_0)=Z\ (6.414095)$

The above result may be applied to new astrophysics observations of galaxies. The results for Z from the data for the planets orbiting the sun are listed in Table 4 below:

A: Planetary		6.414095	1.53E+29		
#	m_i	Fixed	Fixed		
	(Name)	then Z_{sun}	then $A_{empirical}$	"Z/A"	Inverse of "Z/A"
1	Mercury	5.35E+17	1.20E+57	4.47E-40	2.24E+39
2	Venus	5.25E+17	1.20E+57	4.38E-40	2.28E+39
3	Earth	5.32E+17	1.20E+57	4.44E-40	2.25E+39
4	Mars	5.22E+17	1.20E+57	4.36E-40	2.30E+39
5	Jupiter	5.28E+17	1.20E+57	4.41E-40	2.27E+39
6	Saturn	5.28E+17	1.20E+57	4.41E-40	2.27E+39
7	Uranus	5.29E+17	1.20E+57	4.41E-40	2.27E+39
8	Neptune	5.28E+17	1.20E+57	4.41E-40	2.27E+39
9	Pluto	5.28E+17	1.20E+57	4.40E-40	2.27E+39
Average, μ		5.28E+17	1.20E+57	4.41E-40	2.27E+39

Table 4. Calculations of Z and A for the sun from empirical planetary data.

All gravitational interactions may be derived from the general gravitational potential energy equation:

$V=-Z/A\ \{1/4\pi\varepsilon_0\}\{q/a.m.u.\}\{q/m\}M_m/R$

$=-Z/A\ \{1.5251892 \times 1029\}\ M_m/R$

From the above general gravitational potential energy equation, I observe that in the absence of a proton in any of the interactions between masses or particles, the gravitational interaction will be zero. The following set of equations is provided in support of this observation, depending on the radial distance R between the particles:

$F_{e\text{-}e}= -0/A\ \{1.5251892 \times 10^{29}\}\ m_e m_e/R^2=0$

$F_{n\text{-}n}=-0/2.01733\ \{1.525189 \times 10^{29}\}\ m_n m_n/R^2=0$

$F_{n\text{-}e}=-0/1.008665\{1.525\ 1892 \times 10^{29}\}\ m_n m_e/R^2=0$

$F_{p\text{-}e}=-1/1.007825\{1.525\ 1892 \times 10^{29}\}\ m_p m_e/R^2 \neq 0$

$F_{p\text{-}n}=-1/2.014102\{1.5251892 \times 10^{29}\}\ m_p m_n/R^2 \neq 0$

$F_{p\text{-}p}= -2/2.014552\{1.525\ 1892 \times 10^{29}\}\ m_p m_p/R^2 \neq 0$

Conclusion

This study into the relative nature of the gravitational constant has provided material insight into the specific use of the computed gravitational constant, for a particular frame of reference.

The research was limited in the sense that only the readily available planetary data and that of the Hydrogen atom were utilized. More analyses may be carried out on other elements to evaluate and assess predictions as part of the work towards a Grand Unification Theory (GUT) and to compare with empirical results. However, based on the results computed and tabulated in Tables 1-3, as well as the observations made, it is my proposition that the gravitational force of attraction is electrostatic in nature, and by applying the relative gravitational constant principle we should be able to reciprocally determine the corresponding or equivalent and net covalent charge as well as corresponding magnetic fields of planetary bodies and other large bodies, based on their orbital characteristics and mechanics around a central body, M_i, similar to the interactions at atomic levels. Lastly, with these findings and other research work, we should be able to strive towards our quest for the unification of the four fundamental forces of nature, being: the strong and weak nuclear forces, electromagnetism and gravity and a better understanding of the universe as well as for further technological advancement.

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