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The Theory and Models of Keynesian Disequilibrium Macroeconomics

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Abstract

This paper critically reviews and examines the relationship between the origin of disequilibrium macroeconomic thinking by John Maynard Keynes, and the development of Keynesian disequilibrium macroeconomic models. Given that the two strands of literature are both plentiful, I will focus on discussing the essence of Keynesian disequilibrium thinking, and its implications of relevant models in the context of Keynes-Metzler-Goodwin and Weidlich-Haag-Lux approaches.

Introduction

This paper examines the interplay between the origin of disequilibrium macroeconomic thinking from John Maynard Keynes in his *General Theory*, and the subsequent development of Keynesian disequilibrium macroeconomic models. Given that these two strands of literature are both plentiful, I will focus on the essence of Keynes's disequilibrium thinking, and discuss how it influences the development of relevant disequilibrium macroeconomic models, mainly in the context of deterministic nonlinear dynamics of Keynes-Metzler-Goodwin and Weidlich-Haag-Lux approaches.

Disequilibrium macroeconomics was originally envisioned by Keynes [1]. It had once enjoyed a brief popularity in the 1970s since Leijonhufvud [2] published his famous book "On Keynesian Economics and the Economics of Keynes"[1]. Yet it had almost disappeared in the 1980s due to the popularization of theory of rational expectations and the emergence of Neo-classical synthesis.

Yet it becomes apparent in the aftermath of the 2007-2008 GFC and the subsequent world-wide recession that disequilibrium macroeconomics offers a better explanation about macroeconomic instability. Indeed, Keynes wrote the *General Theory* nearly 80 years ago in his effort to explain the Great Depression. Although his analytical framework was static in nature, he had in mind a dynamic theory, since disequilibrium is essential in understanding Keynes's *General Theory*: saving and investment are independently determined; wages and prices are rigid in the short run; and expectations are formed by the self-fulfilling sentiments and herd mentality. Later on, Minsky deepens Keynes's analysis by emphasizing the crucial role of finance in propagating macroeconomic instability.

On the other hand, the development of theories in non-linear dynamics provides a mathematical backbone for the study of disequilibrium macroeconomic models. Three areas are of particular importance: *(i)* the seminal work of Poincare [3], originally applied in the study of celestial mechanics, which had laid the foundation for the modern qualitative-geometric approach to the analysis of non-linear dynamical systems; *(ii)* the development of Bifurcation Theory, especially the Hopf Bifurcation theorem [4]; *(iii)* the discovery of chaos in deterministic dynamic system [5,6]. The use of Hopf bifurcation

theorem to demonstrate the existence of a limit cycle, as a parameter of interest passes through a critical bifurcation point, has been widely applied in numerous fields in natural science. To a lesser degree in economics, it is applied in identifying and modelling endogenous business cycles of various kind[2]; the discovery of deterministic chaos, characterized by the irregular fluctuations in the Lorenz system, had inspired the study of complexity economics, which looks into the emergence of complex phenomena as a result of local interactions of heterogeneous agents with simple, deterministic laws - mimicking many complex phenomena observed in reality such as herd behaviour.

An important use of disequilibrium macroeconomic models is to understand the dynamic and heterogeneous interactions amongst the core sectors of the aggregate economy, in order to see which parameters are stabilizing or destabilizing, and which parameters, particularly parameters that are associated with policy interventions such as Tobin-type taxes, have the most influence in switching the economy between the regions of stability and instability, thus offering us an insight over the effectiveness of policy interventions in qualitative terms. In the context of deterministic models, the term "stability" (or "instability") refers to the local properties of the steady state, and the existence of transition from stability to instability (in the form of persistent, self-sustaining fluctuation) can be mathematically formulated and proven with the use of *Hopf bifurcation theorem*. Given its aggregative nature, this type of models is sometimes coined "macro-founded". It offers a particular advantage over the traditional "micro-founded" models of stochastic intertemporal optimization type such as RBC or New-Keynesian DSGE models, which are difficult to be analyzed in terms of the dynamic linkages and feedbacks between various sectors of the macro-economy [7].

This paper expounds two fundamental approaches of modelling Keynesian disequilibrium macro-dynamics: the Keynes-Metzler-Goodwin (KMG) approach and the Weidlich-Haag-Lux (WHL) approach. These two approaches represent one of the most stylized and most recent development of Keynesian disequilibrium macro-dynamics. They are also probably the most relevant in addressing

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[2]See [29] for detailed discussion.

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[1]Leijonhufvud is perhaps the first author that interprets Keynes in terms of disequilibrium phenomena. He argues that the traditional IS/LM formulation of Keynes's theory fails to explain phenomena such as "involuntary unemployment", which is central for Keynes's explanation of unemployment and depression. Leijonhufvud advocates a "cybernetic" approach to macroeconomics where the dynamic adjustments of prices and quantities are explicitly considered without imposing the standard Walrasian equilibrium concept.

Leijonhufvud's critique of standard IS-LM interpretation of Keynes that lacked disequilibrium phenomena. The KMG model is formulated by Chiarella and Flaschel [8] and later on, Chiarella et al. [7]. "Keynes" refers to the disequilibrium between savings and investment, the sticky adjustment of prices and wages, and the causal nexus from financial to real markets; "Metzler" to inventory dynamics and "Goodwin" to the dynamics of distributive shares. It synthesizes and extends the disequilibrium macro-dynamic models in the 1960s and 1970s in a systematic and hierarchial manner. The interaction of three agents (households, firms, and government) across five markets (labour market, goods market, money market, bond market, and equities market) are considered. On the other hand, the WHL approach attempts to model the dynamics of "animal spirits". It is originally inspired by earlier work of Weidlich and Haag [9] that models the interacting population dynamics. Lux [10], in his seminal work, studies how herd behaviour and sentiment contagion contributes to asset price bubbles/crashes. The framework is further applied in Franke [11] and Charpe et al. [12], which study the dynamics of "animal spirits" and real-financial interaction in a macroeconomic setting.

The rest of the paper is organized as follows: section 4 traces back to the origin of Keynes's disequilibrium macroeconomic thinking, as well as Minsky's interpretation of Keynes. Section 5 introduces the modelling of Keynesian disequilibrium macroeconomic models in terms of KMG and WHL approaches, particularly the latter one. Section 6 concludes with a discussion regarding the interplay between these two strands of literature. It also paves a few path for future research.

The Origin of Disequilibrium Macroeconomic Thinking

The second half of the 19th century and the beginning of the 20th century had witnessed an increasing instability of capitalism, in the form of more severe economic crises and an increasing number of bankruptcies. The worsening macroeconomic instability culminates in the "Black Thursday" on the 24th of October, 1929 and the subsequent Great Depression of the 1930s, which is marked as the most prolonged and severe depression of the 20th century.

Classical economists at that time, however, believed that crises would not occur, and full employment is guaranteed due to the self-adjusting market. They based their analysis on the Say's law: goods produced will be be sold since "supply creates its own demand". Saving from household always equals business investment. Furthermore, wages and prices are assumed to be highly flexible. The increase of product prices will quickly be matched by a rise of costs, which eliminates the incentive to expand output. The belief of Say's law, coupled with the assumption of flexible price, insures that business will be able to sell their goods to either consumers or investment; full-employment is automatically maintained, as long as there is no involuntary unemployment - workers are always willing to work at any wages.

The Great Depression is clearly a significant event that had casted doubt over the myth of the self-adjusting markets. The Classical doctrines had outlived its ideological usefulness in explaining the severe depression. When it came to policy recommendations, the Classical economists could recommend nothing but a general cut in all wages. It was obvious that unconventional policies were needed to restore aggregate demand and business confidence on a scale that could only be achieved by a drastic government intervention [13].

The "disequilibrium" thinking of John Maynard Keynes

Keynes's *General Theory* was born out of the need to understand and explain the Great Depression. In a nutshell, Keynes made his departure from the Classical doctrine mainly in three aspects:

- The rejection of Say's law;
- The rejection of flexible wages and price;
- "Animal spirits".

The first departure of Keynes's analysis lies in his rejection from the Say's law. While Say's law says "supply creates its own demand", Keynes runs the causation other way around. Keynes believes that an active management of aggregate demand by means of monetary and fiscal policy is crucial in maintaining full employment. Otherwise involuntary employment would occur and the economy would reach a sub-optimal equilibrium, prevailing with prolonged unemployment. With regard to interest rate determination, Keynes refutes the concept of self-adjusting interest rates mechanism driven by saving-investment equilibrium. He argues that saving does not necessarily equal to investment, since household income is the major determinant of saving; firms' investment decisions, on the other hand, are mainly determined by profit expectations. Interest rate is primarily determined by liquidity preference in the money market.

The second departure of Keynes's analysis, which is more received and becomes the main ingredient of the New-Keynesian macroeconomics, is his rejection of flexible price and wage assumption. Price would be rigid, or in a disequilibrium dynamical adjustment process due to the existence of monopoly and the resistance of workers for cutting wages.

Perhaps the most fundamental departure of Keynes's thinking from the Classical school lies in his view over how non-rational, psychological factors play a role in influencing human behaviours and propagating aggregate business cycles, due to the presence of radical uncertainty. Keynes coined the term "animal spirits" to address this aspect in *Chapter 12 - The State of Long-term Expectation*, in which he writes:

*"Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than mathematical expectations, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of **animal spirits** - a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities."*

Akerlof and Shiller [14], in their book "Animal Spirits: How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism" goes further beyond and identifies five types of animal spirits: confidence, fairness, corruption, money illusion, and stories. This paper will, however, only focus on the confidence factor given its primary importance. Akerlof and Shiller used the term "Confidence Multiplier", which is borrowed from the concept of Keynesian consumption multiplier, in describing the self-fulfilling nature of confidence: on an individual level, an optimistic attitude leads to positive actions that reinforces the confidence; on an aggregate level, confidence is contagious in the form of herd-type behaviours. This is well depicted in Keynes's famous beauty contest metaphor:

"It is not a case of choosing those [faces] that, to the best of one's judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where

we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees."

"Animal spirit" alludes to a state in disequilibrium - a constant deviation of human expectations and behaviours from the ones being rationally determined. It is crucial in Keynes's explanation of economic expansion and recession: a state of over-optimism leads to inflated asset prices, over-investment and over-consumption, as well as over-expansion of credit in the financial sector, which eventually leads to its own defeat.

Although Keynes's theory is conservative in nature: his analytical framework does not deviate from the Walrasian general equilibrium theory, the three aspects of Keynes's disequilibrium thinking had clearly paved a path for the emergence of disequilibrium macroeconomics during the 1960s[1], and later on, inspired the work of Hyman Minsky.

Minsky's interpretation of Keynes

"Keynesian economics as the economics of disequilibrium is the economics of permanent disequilibrium."

Minsky HP [15].

It is the work of Hyman Minsky that brings a new light over a conventional, Hicksian IS/LM interpretation of Keynes's General Theory. Minsky views Keynes's theory as an endogenous theory of business cycle that explicitly considers the crucial role of the financial instability in propagating macroeconomic fluctuations, as he writes in his famous book "John Maynard Keynes":

"The missing step in the standard Keynesian theory is the explicit consideration of capitalist finance within a cyclical and speculative context...finance sets the pace for the economy. As recovery approaches full employment...soothsayers will proclaim that the business cycle has been banished and debts can be taken on...But in truth neither the boom, nor the debt deflation...and certainly not a recovery can go on forever. Each state nurtures forces that lead to its own destruction."

The instability and interconnection between the real and financial sector lies mainly in two aspects in Minsky's analysis: *(i)* the role of speculative and Ponzi borrowers[2]; *(ii)* the role of financial institutions. The first aspect is elaborated in Minsky's well-known "Financial Instability Hypothesis". Minsky argues that stability is inherently unstable: the economic boom in the preceding period nurtures a self-fulfilling sense of optimism that leads to an increasing portion of speculative and Ponzi borrowers in the financial market. However, the speculative boom cannot last forever. The "Minsky moment" occurs when investors are over-indebted. As they start to sell off their asset in order to meet debt repayments, asset price turns south. Consequently, the financial sector transits from stability to instability. Over the course of business cycle, it is the expansionary phase that ultimately leads to its own destruction. The financial and real cycles are inherently interconnected.

The second aspect, however, is more or less implicit in Minsky's

discussion[3]. Financial institutions, particularly banks, play a crucial role in propagating credit cycle, since banks create money by creating loans that simultaneously create deposits, as is discussed by Minsky's predecessor Joseph Schumpeter and Knut Wicksell. Specifically, Minsky stresses on the role of banker's confidence over the state of credit, as he wrote:

"The state of credit reflects banker's view toward borrowers...A revision by banker of their views about the appropriate leverage to use in financing positions in capital assets will not necessarily cause an immediate revision in the market value of these assets...But such a revision of bankers' views can have a strong impact upon equity prices."

This aspect is further developed by Minsky's followers, in the Post-Keynesian literature of endogenous money theory. It can be generally categorized into three groups: the horizontalist approach, the structuralist approach, and the state money approach [16]. Horizontalist approach is initially advocated by Basil Moore[4] in the 1970s and 1980s that stresses the endogenous nature of money and reserves, arguing that "loans make deposits, deposits make reserves". Structuralists, which had emerged in 1990s and advocated by Charles Goodhart[5], on the other hand, take a more active role of banks' lending behaviour, given the profit-seeking nature of banks. The state money approach, advocated by Wray [17] and Mosler [18] amongst several other so-called *"neo-chartalists"*, agrees upon most aspects of the horizontalist and structuralist proposition. Yet it stresses the mechanism of fiat money system of the public sector and studies the macroeconomic impacts of taxation and government deficit from an accounting perspective. In a historical context, Charles [19] vividly depicts the boom/bust of credit cycles and the subsequent economic expansions/recessions over a course of historical events, which further vindicates Minsky's insights over the role of financial institutions in propagating financial and macroeconomic instability.

To sum up, the disequilibrium thinking of Keynes involves four aspects:

1. Saving ≠ investment;

2. Price and wages rigidity;

3. "Animal spirits";

4. Financial instability and real-financial nexus.

Models of Keynesian disequilibrium macroeconomics

The Keynes-Metzler-Goodwin model

The three decades since 1960s had witnessed an emergence of literature in the mathematical modelling of disequilibrium monetary macro-dynamics, both in Neo-classical and Keynesian schools of thought. This strand of literature is further synthesized and extended by Chiarella and Flaschel [7] and Chiarella et al. [8] in the form of Neo-classical Tobinian model, the Keynes-Wicksell model, and the textbook Keynesian AS-AD model of Turnovsky [20] and Sargent [21]. It leads to the formulation of Keynes-Metzler-Goodwin (KMG) Model that overcomes several drawbacks of these previous model types.

The (neo)-classical discussion of monetary growth can be traced

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[3]See [28] for a detailed survey over the development of disequilibrium macroeconomics during the 1960s.

[4] Minsky defines three categories of borrowers, *i.e.* the hedging borrower, whose current cash flow can serve both interest and principle; the speculative borrower, who can only serve the interest payable; and the Ponzi borrower, who thrives on an inflating asset price and has to rely on borrowing in order to pay back both interest and principle.

[5] See chapter six - "Financial Institutions, Financial Instability, and the Pase of Investment", of [15], "John Maynard Keynes".

[6]See [33].

[7]See [31].

back to Tobin [22,23] and its subsequent extensions[1]. In Tobinian model, money is typically considered as an asset in addition to real capital, and the money market disequilibrium is the core driving force behind inflation and inflationary expectations. Goods market, on the other hand, is assumed to follow the Say's Law and always stays in equilibrium (Saving = Investment). These features are captured in the following equations of a prototype Tobinian model of monetary growth[2]:

$$W = M / p + K, \tag{1}$$

$$I = S = Y - \delta K - C - G = \dot{K}, \tag{2}$$

$$\hat{w} = \beta_w (V - \bar{V}) + \kappa_w \hat{p} + (1 - \kappa_w)\pi, \tag{3}$$

$$\hat{p} = \beta_p \left(\frac{M - M^d}{pK} \right) + \eta\pi + (1 - \eta)(\mu_0 - n), \tag{4}$$

$$\dot{\pi} = \beta_{\pi_1}(\hat{p} - \pi) + \beta_{\pi_2}(\mu_0 - n - \pi), \tag{5}$$

where W is the real wealth, M is the money supply (index d : demand, growth rate μ_o), p is price, K is capital stock, I (S) is investment(saving), W is nominal wage, π is expected rate of inflation, V is the rate of employment, n is the natural growth rate, and βx, Kx, and η are the adjustment parameters.

The assumption of Say's Law, coupled with the role of monetary market disequilibrium in determining inflation are surely questionable features of the Tobin-type model from a Keynesian disequilibrium perspective. The Keynes-Wicksell model, which synthesizes Goodwin [24] and Rose [25], dispenses the Say's Law by introducing independently determined saving and investment functions, and uses both the Labour and Goods market disequilibrium as the cause of inflation, as is represented by the following equations:

$$W = (M + B + p_e E) / P, \tag{6}$$

$$S = Y - \delta K - C - G, \tag{7}$$

$$I = i(\rho - r + \pi)K + nK, \tag{8}$$

$$\hat{w} = \beta_w (V - \bar{V}) + \kappa_w \hat{p} + (1 - \kappa_w)\pi, \tag{9}$$

$$\hat{p} = \beta_p \left(\frac{I - S}{K} \right) + \kappa_p \hat{w} + (1 - \kappa_p)\pi, \tag{10}$$

Where p_e is the price of equities, ρ is the rate of profit, and r is the nominal rate of interest.

The wealth function now has two additional assets (private equity and government bond) that replace the capital (K) in the prototype Tobin model. Most importantly, the investment function is independently determined by the rate of profit, rather than the saving function.

The Keynes-Wicksell model, however, still suffers from an important drawback: it is a supply-side model, since full-capacity growth is assumed. It is a problematic feature since (i) the price adjusts sluggishly while the quantity adjust instantly; (ii) for a true "Keynesian" model it requires a detailed treatment on the demand-driven mechanism. It leads to the formulation of the Keynes-Metzler-Goodwin (KMG) model, which is derived from a more traditional IS-LM approach, yet with IS disequilibrium. The Metzlerian inventory dynamics is

introduced to address the Saving-Investment disequilibrium. The demand-side dynamic multiplier, coupled with Metzlerian inventory dynamics, are captured by the following equations:

$$\dot{Y} = \gamma Y^e + \beta_{y^e}(Y^d - Y^e), \tag{11}$$

$$\dot{N} = Y - Y^d = S - I, \tag{12}$$

where Y^e is the expected aggregate demand and N is the stock of inventories.

In a stylized manner, the KMG model captures the Keynesian demand-side multiplier with saving-investment disequilibrium, the Metzlerian inventory dynamics, as well as the Goodwinian profit-squeeze mechanism. The formulation of KMG model is an important development toward a rigid mathematical formalization of Keyne's disequilibrium thinking: it provides a *"macro-foundation"* as a viable alternative to the more received *micro-founded* New-Keynesian model types.

Modelling the dynamics of "animal spirits"

An important drawback of the KMG model discussed in the previous section, is that it overlooks the "animal spirits" aspect of Keynesian disequilibrium thinking, which is essential in Keynes's explanation of business cycles. In part, this is due to the "macro-founded" nature of the KMG model that overlooks certain behavioural aspects at the micro level. Arguably, however, a true Keynesian model should take into account the dynamics of "animal spirits", *i.e.* how waves of optimism and pessimism of economic agents at micro level cause speculative and herd behaviours, which ultimately lead to fluctuations at macro level.

Modelling speculative behaviour: the "fundamentalist-chartist" approach

An emerging number of empirical studies in finance literature has found abnormality in asset prices in terms of the existence of serial correlations and excess volatilities, which casts doubts over the traditional asset pricing models of efficient market hypothesis and rational expectations school. It has inspired the development of Heterogeneous Agent Models (HAMs) that was initially aimed to study speculative behaviours in the financial market and later on, it has found applications to a broader scope of issues in Keynesian disequilibrium macroeconomics[3]. In this strand of literature, agents are typically categorized into two groups: the *fundamentalists* and the *chartists*. The fundamentalists represent the rational agents that provide a stabilizing force toward the fundamental price, whereas the chartists represent trend chasers that are self-fulfilling and destabilizing.

Beja and Goldman [26] is amongst the first work that takes this "fundamentalist-chartist" approach. The stabilizing fundamentalists' excess demand is driven by the deviation of actual price from the fundamental price, while the destabilizing chartists' excess demand is driven by the perceived historical price trends:

$$\dot{p} = D_t^f + D_t^c, \tag{13}$$

$$D_t^f = a(w - p), \tag{14}$$

$$D_t^c = b(\psi - g), \tag{15}$$

where D_t^f (D_t^c) is the excess demand of fundamentalists (chartists), W is the underlying equilibrium price, P is the current price, Ψ is the speculators' assessment of the price trend, and g is the opportunity

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[8]See [34] for a detailed survey.

[9]The equations presented in the following sections only describe partial structure. See [8] for full descriptions of the model.

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[10]See [32] for a detailed survey of HAMs.

growth rate of alternative investments in disequilibrium trading with comparable securities.

Modelling herd behaviour and "animal spirits": the Weidlich-Haag-Lux approach: In addition to the speculative behaviour captured by Beja and Goldman [26] type model at an individual level, herd behaviour is another important aspect of self-fulfilling animal spirits, at an aggregate level. The seminal work of Lux [10] is perhaps the first work that formalizes the dynamics of herd behaviour and mutual mimetic contagion in speculative financial market. This approach has its origin from social science in the earlier work of Weidlich and Haag, which attempts to model the interacting populations in a more general context. The basic idea of Weidlich-Haag-Lux approach is to model a population of agents that choose and switch between two attitudes in probabilistic terms. The agents interact with each other based on average opinion, as well as other non-herding factors. A Master equation that captures the average opinion as a mean-field variable is applied to simplify the analysis of the stochastic system. The Lux model also inspires numerous works in macroeconomics that incorporates "animal spirits" as the core driving factor of business fluctuations.

The basic set-up in Lux model is as follows: there are $2N$ speculative traders who hold either optimistic or pessimistic sentiment. There are n_+ number of optimists and n_- number of pessimists such that $n_+ + n_- = 2N$. Let $n \equiv 0.5(n_+ - n_-)$ and $x \equiv n/N$, we have an index $x \in [-1,1]$ that describes the average opinion of traders. Hence $x > 0 (< 0)$ corresponds to a situation of predominant optimism (pessimism).

The contagion process in Lux model is modelled in terms of transitory probability. Let p_{+-} be the probability that a pessimist would switch to an optimist, and likewise for p_{-+}. It is plausible that these two transitory probabilities will depend on the average opinion: a predominant optimism will lead to a higher likelihood that pessimists switch to optimists, and vice versa. Hence at any point in time we expect $n_- p_{+-}$ to switch from n_- n_+, while $n_+ p_{-+}$ to switch from n_+ to n_-. It follows that $dn_+/dt = n_- p_{+-} - n_+ p_{-+}$ and $dn_-/dt = n_+ p_{-+} - n_- p_{+-}$. Since $n = 0.5(n_+ - n_-)$ and $x = n/N$, we obtain[1]:

$$\dot{x} = (1-x)p_{+-}(x) - (1+x)p_{-+}(x). \tag{16}$$

Lux makes three assumptions over the transitory probabilities p_{+-} and p_{-+}: *(i)* they must be positive; *(ii)* the transition from pessimism to optimism is larger than the opposite direction if the predominant sentiment is already optimistic and vice versa; and *(iii)* the relative change in the probability to switch from pessimism to optimism increases linearly with changes in x, in a symmetric manner in both directions ($dp_{+-}/p_{+-} = adx$, and $dp_{-+}/p_{-+} = -adx$). Hence the most appropriate functional form would be:

$$p_{+-}(x) = vexp(ax), \tag{17}$$

$$p_{-+}(x) = vexp(-ax). \tag{18}$$

By substituting equation (17-18) into (16) we have

$$\dot{x} = v[(1-x)exp(ax) - (1+x)exp(-ax)]. \tag{19}$$

Equation (19) forms the core of the contagion mechanism. The parameter a plays a crucial role in determining the local stability condition. When $a \leq 1$, equation Beja and Goldman: has a unique equilibrium at $x = 0$; when $a \geq 1$, this equilibrium becomes unstable

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and two additional, stable equilibria emerges.

Lux further extends the model by adding a speculative element, which is in line with Beja and Goldman: it is assumed that price is driven by the fundamental and speculative demand ($\dot{p} = \beta(D_N + D_F)$) where D_N is the speculative demand and D_F s the fundamental demand. Lux further adds the price change to the opinion formation process, since a rising price will make pessimists more likely to switch to optimists, and vice versa. Hence,

$$p_{+-} = vexp(a_1 \dot{p}/v + a_2 x), \tag{20}$$

$$p_{-+} = vexp(-a_1 \dot{p}/v - a_2 x). \tag{21}$$

Franke adopts the Weidlich-Haag-Lux approach to model firm's sentiments in a disequilibrium macro-dynamical model. The model is characterized by a population of heterogeneous firms that constantly switch between optimistic and pessimistic attitudes. The sentiment, measured by the average opinion of firms, influences the investment decisions of firms and leads to variations of output gap. The model also incorporates a Phillips curve to capture the inflation climate and a Taylor rule equation to capture the interest rates. The two dimensional ODE system is given by Equation (22-23):

$$\dot{x} = v[(1-x)\exp(as) - (1+x)\exp(-as)], \tag{22}$$

$$\dot{\pi}^c = \alpha[\gamma\pi^* + (1-\gamma)(\pi^c + \kappa\eta x) - \pi^c], \tag{23}$$

Where π is the inflation climate, γ captures the credibility of the central bank, K is the slope of the Phillips curve, η is the proportionality factor linking the output gap to x.

The Franke paper is a highly-stylized framework that formalizes the modelling of "animal spirits" in this small-scale macroeconomic model. It can be viewed as providing an alternative micro-foundation for macro-dynamic model from a Keynesian perspective, which is fundamentally different from the dominating paradigm of representative, utility-maximizing agent approach. This approach is sufficiently simple and flexible to be applied to a broader scope of decision problem in economics.

Another stimulating work is Charpe et al. which further extended by Lux and Franke proposes a so-called *"Dynamical Stochastic General Disequilibrium (DSGD)"* model. It features the continuous dynamical adjustment process on interacting real and financial markets. The financial market is destabilizing in the presence of speculative behaviour of heterogeneous agents driven by "Animal Spirits". The financial market is populated with a changing portion of fundamentalists and chartists. The real market, on the other hand, follows a much simplified version of KMG model, in the sense that the real side is always stable in the absence of speculative activities on the financial side. The baseline 4D system is written as:

$$\dot{Y} = \beta_y[(a_y - 1)(Y - Y_o) + a_k(p_k - p_k^o)K + A], \tag{24}$$

$$\hat{p}_k = \beta_k \alpha_k [f(Y, p_k, \pi_k^e) - 1], \tag{25}$$

$$\dot{\pi}_k^e = \beta_{\pi_k^e}[\frac{1+x}{2}\hat{p}_k(Y, p_k, \pi_k^e) - \pi_k^e], \tag{26}$$

$$\dot{x} = \beta[(1-x)\exp(as) - (1+x)\exp(-as)], \tag{27}$$

where a_y is the propensity to spend, a_k asures the reaction of investment demand to deviations between the actual and the steady state, A is the autonomous consumption, α_k captures the partial demand for capital that actually enters the financial market, and π_k^e ected rate of return.

The DSGD approach is another viable step toward a more Keynesian macro-dynamic model that incorporates "animal spirits" in inducing financial instability and macroeconomic fluctuations. It captures the highly inter-connected real and financial markets: the former is treated as stable and the latter unstable due to a population of speculative, contagious agents with fundamentalist-chartist type interaction. This approach paves a path for numerous areas of ongoing research in Keynesian disequilibrium macro-dynamics. For example, it is subsequently adopted to an open-economy scenario in Flaschel et al. which investigates the exchange rate dynamics in a two-country framework[1] [27].

Conclusion

In his attempt to reorient economic theory and explain the Great Depression, Keynes envisioned the idea of disequilibrium macroeconomics in his *General Theory*, which consists of four crucial aspects: *(i)* the independently determined saving and investment; *(ii)* the price and wage rigidity; *(iii)* the self-fulfilling "animal spirits" and *(iv)*, the intrinsic connection and interaction between real sector and financial sector. It had inspired not only economic thinkers such as Axel Leijonhufvud (who is amongst the first economist that interprets Keynes in disequilibrium terms) and Hyman Minsky (who brings modern capitalist finance in the interpretation of Keynes), but also mathematicians and dynamic modellers who strive to formalize Keynes's disequilibrium thinking with the use of advanced tools in nonlinear dynamic systems [30-34].

This paper discusses two stylized approaches of modelling Keynesian disequilibrium macro-dynamics: the Keynes-Metzler-Goodwin (KMG) approach and the Weidlich-Haag-Lux (WHL) approach that are complementary to each other. The former emphasizes on the investment-saving disequilibrium, wage-price spiral, and sluggish inventory adjustments; whereas the latter emphasizes on the role of "animal spirits" in inducing macroeconomic fluctuations.

One particular area that could potentially benefit from this line of research is the modelling of instability from the irrational behaviours of financial institutions. It is evident that the irrational and predatory behaviours of commercial banks and investment banks, coupled with a loosening regulatory environments are the key contributors to the recent 2007-2008 GFC. It would be worthwhile to incorporate the modelling of "animal spirits" of financial institutions in the spirit of WHL approach discussed previously.

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[12]This model is in line with [30].