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The Stability of a Oscillating Rotating Streaming Fluid Jet with an Self-Gravitating Force

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Abstract

The stability of a oscillating rotating streaming fluid jet with an self-gravitating force is discussed. The problem is formulated and the basic equations are solved. A general Eigen-value relation is derived studied analytically and results are confirmed numerically. The fluid jet is purely stable in the non-ax symmetric perturbation while in the ax symmetric mode it is unstable for small wave number. The streaming has the effect of reducing the stable states not only in the ax symmetric mode but also in those of non-ax symmetric. The self-gravitating force is destabilizing only in the symmetric mode (m=0) for a small range of wave numbers, but it is stabilizing for all other perturbation. This phenomenon is interest, academically and during the geological drilling in the crust of the earth as we have superposed gas-oil layer mixture fluids. The stability behavior of the model comes after destabilizing behavior of the model when it be reduced and suppressed.

Keywords: Capillary force • Self gravitatin • Hydro magnetic

Introduction

The object of the present work is to investigate The stability of a oscillating rotating streaming fluid jet with an self-gravitating force. The MHD stability of full fluid cylinder pervaded by uniform magnetic field has been documented by Chandrasekhar [1]. Kendall performed experiments to obtain and examine the stability of annular fluid jet. Moreover, he did attract and draw the attention for investigating the stability of this model in general for its crucial astrophysical applications. The classical of the capillary instability of a gas cylinder submerged into a liquid are given for first time by Chandrasekhar for axisymmetric perturbation. Hasan and Abdelkhalek, Elazab et al., and Drazin and Reid gave the dispersion relation valid for all axisymmetric and non-ax symmetric modes [2-4]. Cheng discussed the instability of a gas jetin an incompressible liquid for all modes of perturbation. However, we haveto mention here that the results given by Cheng [5]. Kendall [6] performed experiments with modern equipment to check the breaking up of that model. Barakat HM study the magneto hydrodynamic (MHD) Stability of Oscillating Fluid Cylinder with Magnetic Field [7]. Barakat [8] discuss the axisymmetric magneto-hydrodynamic (MHD) self-gravitating stability of fluid cylinder. Mehring and Sirignano [9] discuss the axisymmetric capillary waves on thin annular liquid sheets. Barakat [10] discuss the The Instability of a Un compressible Oscillating Fluid Cylinder with an Axial Magnetic Field. Hamdy M Barakat [11] discuss Self-Gravitating Stability of a Fluid Cylinder Embedded in a Bounded Liquid, Pervaded by Magnetic Field, for all Symmetric and Asymmetric Perturbation Modes. The aim of the present study the stability of a oscillating rotating streaming fluid jet with an self-gravitating force.

Formulation of the problem

We consider an infinite circular Cylinder of oscillating rotating streaming fluid jet with oscillating velocity $\underline{U} = (0, 0, U e^{\omega t})$ and rotating with angular velocity $\underline{\Omega} = (0, 0, \Omega)$ the fluid is assumed to be non-viscous, incompressible and perfectly conducting. We shall use a cylindrical polar coordinates (r, φ, z) system with the z-axis coinciding with the axis of the annular jet. The basic equationare the hydro magnetic equation of motion, continuity equation.

The equation of motion

$$\rho\left(\frac{\partial \underline{u}}{\partial t} + (\underline{u}\cdot\nabla)\underline{u}\right) = -\nabla P + \nabla V^{i} + \frac{1}{2}\nabla\left|\underline{\Omega}\wedge\underline{r}\right|^{2} + 2(\underline{u}\wedge\underline{\Omega}) \qquad (1)$$

Continuity equation

$$\nabla\cdot\underline{u} = 0 \qquad (2)$$

The Poisson's equation satisfying the gravitational field interior the fluid, and Laplace's equation satisfying the gravitational potential of the medium surrounding the fluid cylinder.

$$\nabla^{2}V^{i} = -4\pi G\rho \qquad (3)$$

$$\nabla^{2}V^{e} = 0 \qquad (4)$$

Where (U) and (ω) are the amplitude and oscillation frequency of the velocity, ρ, P are the mass density and kinetic pressure; G is the gravitational constant, V^{i} is the gravitational potential interior the fluid cylinder and V^{e} the gravitational potential exterior the fluid cylinder.

Equilibrium state

In the unperturbed state the system of the basic equation (1) – (4) take the form

$$\nabla\left(\frac{p_0}{\rho} - \frac{1}{2}\nabla\left|\underline{\Omega}\wedge\underline{r}\right|^{2} - V_0^{i}\right) \qquad (5)$$

$$\underline{u_0} = 0, \nabla\cdot\underline{u_0} = 0 \qquad (6)$$

$$\nabla^{2}V_0^{i} = -4\pi G\rho \qquad (7)$$

$$\nabla^{2}V_0^{e} = 0\cdot \qquad (8)$$

These equation are simplified with $\frac{\partial}{\partial z} = 0$ and $\frac{\partial}{\partial\varphi} = 0$ and the resulting system after simplification is solved. The solution obtained are matched at $r = R_0$ across the boundary surface of the fluid. The finite solution can be easily found.

Perturbation analysis

For small departure from the unperturbed state, every physical quantity $Q(r, \varphi, z, t)$ could be expressed as

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$$Q(r,\varphi,z;t) = Q_0(r) + \varepsilon_0(t)Q_1(r,\varphi,z) \quad (9)$$

Where Q_1 stands for P, $\underline{U}$, V^i, V^e, the amplitude of perturbation $\varepsilon(t)$ at time t is

$$\varepsilon(t) = \varepsilon_0 \exp(\sigma\, t) \quad (10)$$

Where (σ) is the growth rate of the instability or rather the oscillation frequency if ($\sigma = iw$ with $i = \sqrt{-1}$) is imaginary and ε_0 is the amplitude at $t = 0$. The perturbed radii distances f the gas cylinder is given by

$$\text{where}\ r = R_0 + \varepsilon_0\, exp(\sigma\, t + i(kz + m\phi)) \quad (11)$$

Where (k) is the longitudinal wave number and (m an integer) is the transverse wave number. The second term on the right-hand side of equation (11) represent the surface wave elevation normalized with respect to R_0 and measured from the equilibrium position. The linearized perturbation equation deduced from the fundamental equations (1)- (4) are given by

$$\frac{\partial U_1}{\partial t} + (\underline{U}\cdot\underline{\nabla})\underline{U_1} = -\underline{\nabla}\Gamma + 2(\underline{U_1} \wedge \underline{\Omega}) \quad (12)$$

$$\Gamma = \frac{P_1}{\rho} - V_1^i \quad (13)$$

$$\underline{\nabla}\cdot\underline{U_1} = 0 \quad (14)$$

$$\nabla^2 V_1^i = 0 \quad (15)$$

$$\nabla^2 V_1^e = 0 \quad (16)$$

This system of equation is simplified on using the time dependence as given above by (10). From the view point of the linear theory and based on the linear perturbation technique, every perturbed quantity can be expressed as $exp(\sigma\, t + i(kz + m\phi))$ times an amplitude function of r . Consequently, on solving (12) we obtain

$$u_{1r} = \frac{-(\sigma + ikUe^{\omega t})}{\rho[(\sigma + ikUe^{\omega t})^2 + 4\Omega^2]}\cdot\frac{\partial\Gamma}{\partial r} + \frac{4im\Omega^2}{\rho[(\sigma + ikUe^{\omega t})^2 + 4\Omega^2]}\cdot\frac{\Gamma}{r} \quad (17)$$

$$u_{1\phi} = \frac{2}{\rho[(\sigma + ikUe^{\omega t})^2 + 4\Omega^2]}\cdot\frac{\partial\Gamma}{\partial\phi} + \frac{im\Gamma}{r(\sigma + ikUe^{\omega t})}\cdot\left[\frac{4\Omega^2}{\rho[(\sigma + ikUe^{\omega t})^2 + 4\Omega^2]} - 1\right] \quad (18)$$

$$u_{1z} = \frac{-ik\Gamma}{(\sigma + ikUe^{\omega t})} \quad (19)$$

From equation (14) and (17,18,19) we get

$$\frac{d^2\Gamma}{dr^2} + \frac{1}{r}\frac{d\Gamma}{d\,r} + \left(q - \frac{m^2}{r^2}\right) = 0 \quad (20)$$

$$q = k^2\left[1 + \frac{4\Omega^2}{(\sigma + ikU e^{\omega t})^2}\right] \quad (21)$$

The solution of equation (20) is given by

$$\Gamma = A\, J_m(qr)\, exp(i(kz + m\phi + \sigma\, t)) \quad (22)$$

Where A is an arbitrary constant to be determined and J_m the ordinary Bessel function of first kind of order m.

Similarly equation (15) and (16), based on the linearzed theory, are solved and first order perturbation V_1^i and V_1^e are given by

$$V_1^i = B\, I_m(kr)\, exp\,(i(kz + m\phi + \sigma\, t)) \quad (23)$$

$$V_1^e = C\, K_m(kr)\, exp(i(kz + m\phi + \sigma\, t)) \quad (24)$$

Where I_m and K_m are modified Bessel function of order m and B, C are constantsof integrations to be determined.

Boundary condition

The Solution represented by equations(22) – (24) must satisfy certain boundary conditions. Under the present circumstances these conditions can be given as follows.

(i) Kinematics boundary condition states that "The normal component of the velocity U_{1r} vector must be compatible with the velocity of the particles of the boundary surface at $r = R_0$

$$U_{1r} = \frac{\partial r}{\partial t} = \frac{\partial\,\Gamma}{\partial\, r} \quad (25)$$

(ii) The gravitational potential and its derivatives must be continuous across the surface.

(iii) The normal component of the total stress tensor must be continuous across the boundary surface from which we have the following dispersion relation:

$$\frac{(\sigma + ikUe^{\omega t})[(\sigma + ikUe^{\omega t}) + 4\Omega^2]}{[yJ_m'(y)(\sigma + ikUe^{\omega t}) + 2im\, J_m(y)]}J_m(y) = 2\pi G\rho - \Omega^2 - 4\pi G\rho K_m(x) I_m(x) \quad (25)$$

Where x is, the dimensional wavenumber, given by $x = kR_0$ and y is defiend by $y = qR_0$.

Equation (25) is the dispersion relation of gravitational streaming oscillating rotating fluid cylinder surrounded by self-gravitating vacuum. It relates the growth rate σ with the streaming oscillating velocity $Ue^{\omega t}$, angular velocity Ω, the wave numbers x, y, m and other parameters ρ, G and R_0

Stability Discussion

Before we discuss the ordinary stability, marginal stability and instability of the system under consideration, it is desirable to study the behaviors of the Bessel functions and also those of the compound functions contained in the relation(25).In view of the recurrence relations [12]

$$2I_m'(x) = I_{m-1}(x) + I_{m+1}(x), \quad (26)$$

$$2\,K_m'(x) = -K_{m-1}(x) - K_{m+1}(x) \quad (27)$$

Because $I_m(x)$ is monotonic increasing and positive definite $I_m(x) > 0$ for all modes of perturbation $m \geq 0$ and nonzero values of $x \neq 0$, while $K_m(x)$ is monotonically decreasing but never negative, i.e., $K_m(x) > 0$ we may show that

$$I_m'(x) > 0, K_m'(x) < 0 \quad (28)$$

Also for $m \geq 1$ for all values of $x \neq 0, we\ have$

$$2I_m(x)K_m(x) < 1 \quad (29)$$

For non-rotating ($i.e\,\Omega = 0$) and non-streaming fluid ($i.e\ U = 0$) the dispersion relation (25) reduces to that of Chandraskhar [13] . Moreover if we put $m = 0$, $\Omega = 0$, $U = 0$ in (25) we recover the relation derived their relation by using the principle of Fermi .

In absence of the streaming ($i.e\ U = 0$), the dispersion relation (25) can be written in the dimensionless form

$$[yNJ_m'(y) + 2mMJ_m(y)]\left[K_m(x)I_m(x) - \tfrac{1}{2} + M^2\right] + N(N^2 + 4M^2)J_m(y) = 0 \quad (30)$$

Where the dimensionless quantity M, N we defined as follow

$$N = \frac{\sigma}{\sqrt{4\pi G\rho}},\ M = \frac{\Omega}{\sqrt{4\pi G\rho}}. \quad (31)$$

If $x = 0$ the dispersion relation (30) takes the simpler form

$$N^2 - 2MN + m\left(M^2 - \tfrac{1}{2}\right) + \tfrac{1}{2} = 0\,. \quad (32)$$

Hence we get $N = M \pm \sqrt{(m-1)(\frac{1}{2} - M^2)}$. (33)

A neutral mode of oscillation is obtained if

$$M = \pm\frac{\sqrt{(m-1)}}{\sqrt{2m}}, \quad (34)$$

It is clear that the angular velocity must satisfy the following$\sqrt{2}M \leq 1$ (35)

Neglecting the rotation effect the dispersion relation (25) reduces to

$$(\sigma + ikUe^{\omega t})^2 = \frac{4\pi G\rho x\, I_m'(x)}{I_m(x)}\left[I_m(x)K_m(x) - \tfrac{1}{2}\right]. \quad (36)$$

Conclusion

The streaming has the effect of reducing the stable states not only in the ax symmetric mode but also in those of non-ax symmetric.

The self-gravitating force is destabilizing only in the symmetric mode ($m = 0$) for a small range of wave numbers, but it is stabilizing for all other perturbation.

The stability behavior of the model comes after destabilizing behavior of the model when it be reduced and suppressed.

This phenomenon is interest, academically and during the geological drilling in the crust of the earth as we have superposed gas-oil layer mixture fluids.

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