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The Power Approximation of Time Series with Using Fractional Brownian Motion

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Abstract

We propose the approximating sequence and some of characteristics of this sequence to coincide with the increments of the fractional Brownian motion (fractional Browniannoise) for the observed time series. We study the Hurst parameter estimation algorithm and check the quality of the approximation.

Keywords: The parameters estimation of Fbm; Description of the algorithm; Approximation of real time series

Introduction

We consider a mathematical model for the time series $S_1..S_n$. The primary processing is smoothing, removing the trend - leads to improved time series $x_1..x_n$- ("initial"). Consider $x_1,x_2..x_n$ as the observed values of some quantity at some moments of time. Let us choose a random process X (t), where $x_k = X(\frac{k}{n})$. This problem has a controversial solution, because different processes (with different distributions) may have the same trajectories. The subjective criteria for selection of X(t) is as follows.

1. Process should possess known characteristics, particularly the Gaussian process should be chosen.
2. X(t) should not be Markov, as the Markov communication does not provide an adequate description of real phenomena.

Fractional Brownian motion is defined as a Gaussian random process with characteristics [1]:

$$B_H(t),\ \mathrm{E}\,B_H(t)=0,\ B_H(0)=0,\ \mathrm{E}\,B_H(t)\,B_H(s)=\frac{1}{2}(t^{2H}+s^{2H}-|t-s|^{2H})$$

Smoothness of the trajectories of the process $B_H(t)$ is defined by the parameter H: almost all the trajectories satisfy the Holder condition:

$$|X(t)-X(s)|\le c|t-s|^{\alpha},\alpha<H,$$

which generalizes the known Levy's result for the Wiener process. The statement of problem to build a forecast for the initial time series.

The Parameters Estimation of Fbm

Consider $X(t)=\sigma B^H(H)$, then examine the increment

$$y_K = X\left(\frac{k}{n}\right) = -X\left(\frac{k-1}{n}\right),\ K=1,2,...,n.\cdot$$

The vector

$Y=(y_1,y_2,...,y_n)\sim N(0;V)$, where the correlation matrix

$$v_{jk}=\frac{\sigma^2}{n^{2H}}\left(\frac{|k-j+1|^{2H}+|k-j-1|}{2}-|k-j|^{2H}\right)=\frac{\sigma^2}{n^{2H}}S_{jk};$$

$\{y_k\}$ constitutes a Gaussian stationary sequence. Henceforth, the increments are going to be the subject of consideration. Consider the algorithm proposed for simultaneous estimation of parameters [2].

We check the method for simultaneous estimation of two unknown parameters fBm (H,σ) and propose a method for approximation of the time series by the power function from the increments of fractional Brownian motion.

Description of the Algorithm

Consider the absolute random moments of increments of fractional Brownian motion.

$$R_{jn}=\frac{1}{n}\sum_{k=1}^{n}|y_k|^j,\ j \text{ - real.}$$

Then calculate the mean

$$E_n(j)=E\,R_{jn}=\frac{\sigma^j}{n^{JH}}\cdot\frac{2^{\frac{J}{2}}\Gamma\left(\frac{J+1}{2}\right)}{\sqrt{\pi}}$$

The result was first proved in eqn. (6)

Theorem

With probability 1

$$\frac{R_{jn}}{E_n(j)}\to 1$$

In particular, when $j=1$ $\sqrt{\frac{\pi}{2}}\cdot\frac{n^H}{\sigma}R_{1n}\to 1$, (1)

we obtain a consistent estimate for σ

$$\hat{\sigma}_{1n}=n^H N\sqrt{\frac{\pi}{2}}R_{1n} \quad (2)$$

with the known value estimate for H:

$$\hat{H}_n=\frac{\ln\left(\sqrt{\frac{2}{\pi}}\cdot\frac{\sigma}{R_{1n}}\right)}{\ln n}.$$

ε is the canonical Gaussian vector with the following characteristics:

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$E\varepsilon = 0, E(\varepsilon,u)(\varepsilon,v) = (u,v)\ \dim\varepsilon = n..$

Then $\mathbf{y} = v^{\frac{1}{2}}\boldsymbol{\varepsilon}$,

therefore

$n = \mathrm{E}(\mathbf{å},\mathbf{å}) = E(V^{-1}\mathbf{y},\mathbf{y}) = \frac{n^{2H}}{\sigma^2}\mathrm{E}(S^{-1}\mathbf{y},\mathbf{y})$

And consequently the statistic

$\hat{\sigma}^2_{2n} = (n)^{2H-1}(S^{-1}\mathbf{y},\mathbf{y})$;

and here statistics $(n)^{2H-1}(S^{-1}y,y)$ is an unbiased estimate of the parameter σ^2.

Now we prove consistency of this estimate. Let us introduce the notation:

$$\hat{\sigma}_{2n} = \sqrt{n^{2H-1}(S^{-1}y,y)} \quad (1)$$

We use the formula for integration by parts for Gaussian measures that leads to the relation (by calculating the dispersion $D\hat{\sigma}^2_{2n} = n^{4H-2}E(S^{-1}y,y)^2 - \sigma$)

$$D\hat{\sigma}^2_{2n} = \frac{2\sigma^4}{n}$$

where $\hat{\sigma}_{2n}$ is consistent estimate of the parameter σ.

Equations (1) and (2) form a system, which is proposed to solve iteratively [3]. The essence of the algorithm is as follows: for an arbitrary value $H\in(0;1)$ let us calculate the estimate $\hat{\sigma}_{1n}$, matrix S^{-1} and the estimate $\hat{\sigma}_{2n}$. Then we iterate the values H (matrix S for different H) with some step

$$\frac{\hat{\sigma}_{2n}}{\hat{\sigma}_{1n}} = \frac{0{,}8}{R_{1n}}\sqrt{\frac{(S^{-1}y,y)}{n}} \approx 1,\ \left|\frac{\hat{\sigma}_{2n}}{\hat{\sigma}_{1n}} - 1\right| \to \min. \quad (2)$$

The values $\hat{H}$ of parameter H, which satisfies eqn. (2), is an estimate and

$$\hat{\sigma} = \frac{\hat{\sigma}_{1n} + \hat{\sigma}_{2n}}{2}$$

We performed a numerical experiment, which implements the algorithm proposed.

We proved that the estimate of parameter σ

$\hat{\sigma}_{2n} = \sqrt{n^{2H-1}(S^{-1}\mathbf{y},\mathbf{y})}$ (3) is consistent.

Let us calculate the estimate $\hat{\sigma}_{1n}$, matrix S^{-1} and estimate $\hat{\sigma}_2$.

$$\frac{\hat{\sigma}_{2n}}{\hat{\sigma}_{1n}} = \frac{0{,}8}{R_{1n}}\sqrt{\frac{(S^{-1}\mathbf{y},\mathbf{y})}{n}} \approx 1,\ \left|\frac{\hat{\sigma}_{2n}}{\hat{\sigma}_{1n}} - 1\right| \to \min. \quad (4)$$

$$\hat{\sigma} = \frac{\hat{\sigma}_{1n} + \hat{\sigma}_{2n}}{2}.$$

$$F_n = \frac{1}{n-1}\sum_{k=1}^{n-1}\mathrm{sgn}(y_k y_{k+1});\ Q_n = \frac{1}{n-1}\sum_{k=1}^{n-1}u_k$$

The generated increments are defined by

$z_k = n^H\ y^k,\quad n = 256; n = 1024$

$r_1 = \frac{1}{n}\sum_{k=1}^{n}|z_k| = n^H R_{1n}$

The numerical results verified by effectiveness of the proposed estimation algorithm are summarized in Tables 1-3.

Here

$$c_{kj} = \frac{0{,}8}{r_1}\sqrt{\frac{(S_j^{-1}\mathbf{z}_k,\mathbf{z}_k)}{n}},$$

Z_k is generated vector of increments fBm; S_j is normalized correlation matrix corresponding the Hurst exponents H_k, H_j. For each H_k (in the fixed line) C_{kj} are calculated the selection of parameter H_j with step ΔH_j=0,1. The generation of Z_k is performed for the values

n=256; H_k=0,4; H_k=0,7;

n= 1024; H_k=0,3; H_k=0,6; H_k=0,7.

Analysis of data in the table shows that for each H_k

$|c_{kj}-1|\to\min,\ H_j = H_k,\ \hat{H}_k = H_j,$

which shows effectiveness of the algorithm.

Approximation of Real Time Series

Let's $S_1..S_n$ be observed time series data of an arbitrary nature. Consider the procedure for its approximation of fractional Brownian motion. The first step of the algorithm – initial data processing is to bring them to a time series with zero trend $X_k=S_k-M(k)$, $M(k)$ is an approximation of the trend [4].

Let us select an adequate model of the corresponding random process for the actually observed time series:

$x_0 = 0$, $x_1 \ldots x_n$ — the observed values, $y_k = x_k - x_{k-1}$, $k = 2\ldots n$, $\mathrm{E}x_k = 0$.

The criterion of the value of the Gaussian increments will be the "excess coefficient"

$$d_n = \frac{\left(\frac{1}{n-1}\sum_{k=2}^{n}|y_k|\right)^2}{\frac{1}{n-1}\sum_{k=2}^{n}y_k^2} \quad (5)$$

H_j / H_k	0,1	0,2	0,3	0,4	0,6	0,7	0,8	0,9
n=256								
0,4	1,09	1,10	1,05	1,00	1,12	1,17	1,23	1,28
0,7	1,10	0,92	0,91	0,89	0,95	1,01	1,10	1,17
n=1024								
0,3	0,85	0,89	1,04	1,20	1,4	1,37	1,42	1,48
0,6	1,18	1,12	1,20	1,23	1,04	1,15	1,23	1,25
0,7	1,20	1,18	1,15	1,14	1,13	1,02	0,93	0,89

Table 1: The values of efficiency of the estimation algorithm.

H	n	An	Bn	Cn	Dn
0,2	256	-1,19	0,25	0,78	
	1024	-2,1	0,17	0,6	
	4096	-1,6	0,08	0,62	
0,3	256	-1,07	0,13		
	1024	-1,85	0,099		
	4096	-1,48	0,072		
0,4	256	-1,14	0,11		
	1024	-1,76	0,082		
	4096	-1,54	0,063		
0,6	256				0,21
	1024				0,13
	4096				0,09
0,7	256				0,14
	1024				0,072
	4096				0,044
0,8	256				0,14
	1024				0,019
	4096				0,031

Table 2: The values of efficiency of the estimation algorithm.

H	0,1	0,2	0,3	0,4	0,6	0,7	0,8	0,9
$\frac{\hat{\sigma}_2}{\sigma_1}$	2,1	1,3	1,01	0,91	0,96	0,97	1,02	1,4

Table 3: The values of efficiency of the estimation algorithm.

The general idea of approximation is in dimensional functional transformation g for each increment y_k, g- is the increasing odd function.

$$z_k = g(y_k)$$

Consider two-dimensional distribution $(y_k; y_j)$ with Gaussian density:

$$f(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}\ \sigma^2} \exp\left\{-\frac{x^2+y^2}{2\sigma^2(1-\rho^2)}\right\} \exp\left\{-\frac{\rho\, xy}{\sigma^2(1-\rho^2)}\right\},$$

$$\rho = \rho_{k-j} = \frac{1}{\sigma^2}\ \mathrm{E}\ y_k y_j$$

If $\rho > 0$, than

$$f(x_1, y_1) \geq f(x_2, y_2)\ \ |x_1| = |x_2|, |y_1| = |y_2|, x_1 y_1 > 0,\ \ x_2 y_2 < 0$$

$$\operatorname{sgn}\ \mathrm{E}(g(y_k)\, g(y_j))\ = \operatorname{sgn}\rho$$

Assume that a two-dimensional density distribution f_{k-j} (x,y) of increments $(y_k; y_j)$ of converted time series satisfies the following conditions:

1) $\varphi(x) = \int_{-\infty}^{\infty} f_{k-j}\ (x,y)\ dy$, $\mathrm{E}y_k = 0$

2) $\operatorname{sgn} \int\int_{-\infty}^{\infty} g(x) g(y) f_{k-j}(x,y)\ dx\ dy = \operatorname{sgn} \int\int_{-\infty}^{\infty} xy\ f_{k-j}(x,y) dx\ dy$.

$$\lim_{n\to\infty} \frac{\left(\frac{1}{n-1}\sum_{k=1}^{n-1}\left|g^{-1}(y_k)\right|\right)^2}{\frac{1}{n-1}\sum_{k=1}^{n-1}\left(g^{-1}(y_k)\right)^2} = \frac{2}{\pi}$$

$z_k = g^{-1}(y_k)$ is assumed to be a Gaussian random variable. Let's demonstrate the algorithm by a power function g.

Let's assume

$$z_k = \operatorname{sgn} y_k |y_k|^{\frac{1}{\lambda}},\ y_k = \operatorname{sgn} y_k |z_k|^{\lambda}, \lambda > 0. \tag{6}$$

Then

$$d_n = \frac{\left(\frac{1}{n-1}\sum_{k=1}^{n-1}|z_k|^{\lambda}\right)^2}{\frac{1}{n-1}\sum_{k=1}^{n-1}|z_k|^{2\lambda}},$$

$$\mathrm{E}|\xi|^{\alpha} = \frac{2^{\frac{\alpha}{2}}}{\sqrt{\pi}}\sigma^{\alpha}\Gamma\left(\frac{\alpha+1}{2}\right), \text{ to } d = \frac{1}{\sqrt{\pi}} \frac{\Gamma^2\left(\frac{\lambda+1}{2}\right)}{\Gamma\left(\lambda+\frac{1}{2}\right)}$$

Thus, the proposed approximation leads to the following model of original time series:

$$x_k = \sum_{j=1}^{k} \operatorname{sgn} y_j \cdot |z_j|^{\lambda}.$$

From the assumption of stationarity the sequence $\{y_k\}$ follows the stationarity of $\{Z_k\}$

$\mathrm{E}z_j\ z_k = Dz_j \cdot f\left(\frac{k-j}{n}\right), f(0) = 1, f(s) -$ is a decreasing function.

Let's approximate f by the exponential function:

$$\frac{\left(s+\frac{1}{n}\right)^{2H} + \left(s-\frac{1}{n}\right)^{2H}}{2} - s^{2H},$$

and approximate $\{x_k\}$ by the fractional Brownian motion:

$$z_k = \sigma^H\left(B_H\left(\frac{k}{n}\right) - B_H\left(\frac{k-1}{n}\right)\right), x_k = \sigma^{\lambda}\sum_{j=1}^{k}\operatorname{sgn} y_j \left|B_H\left(\frac{j}{n}\right) - B_H\left(\frac{j-1}{n}\right)\right|^{\lambda},$$

The approximation procedure of real time series is based on the value for one parameter d_n.

To check the properties of increments Z_k allow limit theorems for fractional Brownian motion proved in I. Nourdin.

Let's $B_H(t), 0 \leq t \leq 1$ - is fractional Brownian motion with Hurst exponent H, f- is twice differentiable function

$$\mathrm{E}\left(\left|f^{(k)}(B(t)\right|^p\right) < \infty, k = 1,2;\ \ p\text{ - is positive here:}$$

$$\xi_k = n^H\left(B\left(\frac{k+1}{n}\right) - B\left(\frac{k}{n}\right)\right) \cong \mathrm{N}(0;1).$$

In the studies mentioned above there have been proved the following limit relations

$$1.\ \frac{n^H}{n}\sum_{k=1}^{n} f\left(B\left(\frac{k}{n}\right)\right)\xi_k^3 \to -\frac{3}{2}\int_0^1 f'(B(s))ds,\quad H \in \left(0;\frac{1}{2}\right);$$

$$2.\ \frac{n^{2H}}{n}\sum_{k=1}^{n} f\left(B\left(\frac{k}{n}\right)\right)\left(\xi_k^2 - 1\right) \to \frac{1}{4}\int_0^1 f''(B(s))\,ds, H \in \left(0;\frac{1}{4}\right);$$

$$3.\ \frac{1}{n^H}\sum f\left(B\left(\frac{k}{n}\right)\right)\xi_k^3 \to 3\int_0^{B(1)} f(x)\,dx, H \in \left(\frac{1}{2};1\right)$$

$$\eta_n \to \eta,\ \mathrm{E}(\eta_n - \eta)^2 \to 0.$$

$$\alpha_k = n^H B\left(\frac{k}{n}\right) = \sum_{j=1}^{k-1} \xi_j,$$

and assuming in the first formula $f(x)=x$, $f(x) = x^2$, let's write

$$\frac{1}{n}\sum_{k=1}^{n}\alpha_k\xi_k^3 \to -\frac{3}{2},\ H\in(0;\frac{1}{2})$$

$$\frac{1}{n^{1+H}}\sum\alpha_k^2\ \xi_k^3\ \to 3\eta,\ H\in(0;\frac{1}{2})$$

$$\eta\cong \mathrm{N}(0;\frac{1}{2H+2})$$

In the second formula $f(x)=x^2$,

$$\frac{1}{n}\sum_{k=1}^{n}\alpha_k^2(\xi_k^2-1)\to\frac{1}{2},\quad H\in(0;\frac{1}{4}) \tag{3.18}$$

The third formula for $f(x)=x$ and $f(x)=1$:

$$\frac{1}{n^{2H}}\sum_{k=1}^{n}\alpha_k\xi_k^3\to\frac{3}{2}B^2(1)\quad H\in(\frac{1}{2};1) \tag{3.19}$$

$$\frac{1}{n^H}\sum_{k=1}^{n}\ \xi_k^3\to 3B(1)\quad H\in(\frac{1}{2};1)$$

With the (3.18)-(3.19) we can check a hypothesis T: the statistics $z_1 . z_n$ are proportional to the increments of fractional Brownian motion. Let's consider a checking algorithm (with a known H) [5]:

$$c=\frac{1}{n}\sum_{k=1}^{n}z_k^2,$$

$$z_k=\sqrt{c}\ \xi_k=\sqrt{c}\ n^H(B(\frac{k+1}{n})-B(\frac{k}{n}),$$

$$v_k=\sum_{j=1}^{k-1}z_j.$$

Let's construct the statistics

$$H\in(0;\frac{1}{2})\ ;\ ,\ H\in(0;\frac{1}{2})\ ;$$

$$B_n=\frac{1}{n^{1+H}}\sum v_k^2 z_k^3,\ \ H\in(0;\frac{1}{2});$$

$$C_n=\frac{1}{n}\sum v_k^2(\frac{z_k^2}{c}-1),\ \ H\in(0;\frac{1}{4});$$

$$D_n=\frac{1}{n^{2H}}\sum v_k z_k^3,\ \ H\in(\frac{1}{2};1);$$

Let's consider the application of algorithm.

As a first example consider the data: the daily data of solar activity (366 data, ftp://ftp.ngdc.noaa.gov). We calculated the initial time series $x_1 . x_{365}$ with increments $y_k=x_{k+1}-x_k,\ k=1,2..365$.

$$R_{1n}=\frac{1}{365}\sum_{k=1}|y_k|=0{,}141;\ \ R_{2n}=\frac{1}{365}\sum y_k^2=0{,}04$$

$$d=0{,}5,\quad \lambda=1{,}4,$$

$$z_k=\mathrm{sgn}\,y_k\cdot|y_k|^{0,7},\qquad k=1.\,365.$$

The second example: 1678 values of interest rate. The interest rate is given by the following formula in each time window.

$$S(t)=a+b\exp\{X(t)\},\ X(t)=\mathrm{h}\ \frac{S(t)-a}{S(0)-a},\ a{=}0{,}085$$

$$x_k=X(\frac{k-1}{n})=\mathrm{h}\ \frac{S_k-a}{S_1-a},x_1=0$$

$$y_k=x_{k+1}-x_k$$

$$R_{1n}=0{,}425;\ R_{2n}=0{,}541;\ d_n=\frac{R_{1n}^2}{R_{2n}}=0{,}334$$

$$\frac{1}{\sqrt{\pi}}\frac{\Gamma^2(\frac{\lambda+1}{2})}{\Gamma(\lambda+\frac{1}{2})}=0{,}334;\ \lambda=2$$

$$y_k=\mathrm{sgn}\,y_k|z_k|^2,\ z_k=\mathrm{sgn}\,y_k\cdot\sqrt{|y_k|}$$

Applying the estimation algorithm (4), we obtain

$$\hat{H}=0{,}3$$

The third example - monthly data interest rate of Germany Bundesbank (www.bundesbank.de) for 2003-2012, 120 data.

$$Y_k=X_{k+1}-X_K$$

$$R_{1n}=\frac{1}{119}\sum_{k=1}^{119}|y_k|=1{,}39,\quad R_{2n}=\frac{1}{119}\sum_{k=1}^{119}y_k^2=3{,}3,\quad d_n=\frac{R_{1n}^2}{R_{2n}}=0{,}59$$

Applying the method of Hurst parameter estimation by the formula (4), let's calculate the equation

$$\frac{\hat{\sigma}_2}{\hat{\sigma}_1}=1{,}0,\ H=0{,}4\ ;\ \hat{H}=0{,}4\ .$$

The fourth $y_k=x_{k+1}-x_k,\quad u_k=100y_k$ example- $S_1..S_n$ exchange rate, 1218 data, 2005-2009 (http://www.banque-france.fr).

Let's calculate the statistics R_{1n}, R_{2n}, n=1217.

$$R_{1n}=\frac{1}{n}\sum_{k=1}^{n}|u_k|=0{,}215,\qquad R_{2n}=\frac{1}{n}\sum_{k=1}^{n}u_k^2=0{,}52,\ \ d_n=\frac{R_{1n}^2}{R_{2n}}=0{,}089$$

$$\frac{1}{\sqrt{\pi}}\frac{\Gamma^2(\frac{\lambda+1}{2})}{\Gamma(\lambda+\frac{1}{2})}=0{,}089,\qquad \lambda=4$$

For the converted sequence defined by the equation

$$z_k=\mathrm{sgn}u_k\sqrt[4]{|u_k|}$$

$$r_{1n}=\frac{1}{n}\sum_{k=1}^{n}|z_k|=0{,}76;\qquad r_{2n}=\frac{1}{n}\sum z_k^2=0{,}84;\quad d_n=0{,}68$$

Let's calculate the value of the quantity

$$\frac{\hat{\sigma}_2}{\hat{\sigma}_1}=\frac{0{,}8}{r_1}\sqrt{\frac{(S^{-1}{}_H z,z)}{n}}$$

In this example, the minimum $\left|\frac{\hat{\sigma}_2}{\hat{\sigma}_1}-1\right|$ achieved by H-0,3; 0,8.

Let's check the quality of the approximation for each example. For the first example

$$c=\frac{1}{n}\sum_{k=1}^{n}z_k^2=0{,}144$$

$$D_n=\frac{1}{n^{1,2}}\sum v_k z_k^3=0{,}002,\ \frac{D_n}{c^2}=0{,}1$$

$$F(x)=2\Phi(\sqrt{\frac{2}{3}}x)-1,x>0$$

Let's choose a level of significance 0,1

$$\frac{D_n}{c^2} < \beta_2 = 4,08$$

The approximation of the time series is satisfactory and hypothesis T is accepted.

For the second example- the values of the bank interest rates

$$n = 336,\ \ H = 0,3;\ \ c = \frac{1}{n}\sum_{k=1}^{n} z_k^2 = 0,425$$

$$A_n = \frac{1}{335}\sum_{k=1}^{335} v_k z_k^3 = -0,16;\ B_n = \frac{1}{335^{1,3}}\sum_{k=1}^{335} v_k^2 z_k^3 = -0,08$$

The theoretical limit value

$$\lim_{n\to\infty} A_n = -1,5\,c^2 = -0,27$$

$$B_n c^{-\frac{5}{2}} = -0,68$$

$$B_n c^{-\frac{5}{2}} < \beta_1 = 3,07$$

The hypothesis T is accepted.

For the third example- the market rates of Bundesbank Germany.

$$n = 119;\ \ H = 0,4;\qquad c = \frac{1}{n}\sum_{k=1}^{n} z_k^2 = 3,3$$

$$A_n = -19,1;\ \ B_n = -0,8;\ \ A_n c^{-2} = -1,75;\ \ B_n c^{-\frac{5}{2}} = -0,04$$

The hypothesis T is accepted.

In the fourth example, we have received the uncertain answer. We calculate the value of statistics

$$A_n = \frac{1}{n}\sum v_k z_k^3 = -0,69$$

For $H > \frac{1}{2}$ $\quad A_n \geq 0$, then we should take a decision of hypothesis H=0,3 (The theoretical value A_n is equal -1,5.(0,84)²=-1,0)

Conclusion

Thus, we proposed in this research a new method for estimate of parameters fBm, approximation of the time series and types of mathematical models.

We checked the adequacy of models and efficiency of the algorithms on real examples.

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