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The New Generalized Difference Sequence Space $\chi 2$ over p – Metric Spaces Defined by Musielak Orlicz Function Associated with a Sequence of Multipliers

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Abstract

In the present paper, we introduce new sequence spaces by using Musielak-Orlicz function and a generalized B_η^μ -difference operator on *p*–metric space. Some topological properties and inclusion relations are also examined.

Keywords: Analytic sequence; Double sequences; χ^2 space; Difference sequence space; Musielak-Orlicz function; *p*–metric space; Lacunary sequence; Ideal

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Introduction

Throughout *w*, χ and L^2 denote the classes *x* of all, gai and analytic scalar valued single sequences, respectively. We write w^2 for the set of all complex sequences (x_{mn}) where $m,n \in \mathbb{N}$, the set of positive integers. Then w^2 is a linear space under the coordinate wise addition and scalar multiplication. For some approximations results in Musielak-Orlicz-Sobolev spaces and some applications to nonlinear partial differential equations see equation 22. The growing interest in this field is strongly stimulated by the treatment of recent problems in elasticity, fluid dynamics, calculus of variations, and differential equations.

Some initial works on double sequence spaces is found in Bromwich [1]. Later on it was investigated by Hardy [2], Moricz [3], Moricz and Rhoades [4], Basarir and Solankan [5], Tripathy et al. [6-10], Turkmenoglu [11], Raj [11-14], and many others [15].

Let (x_{mn}) be a double sequence of real or complex numbers. Then the series $\sum_{m,n=1}^{\infty} x_{mn}$ is called a double series. The double series $\sum_{m,n=1}^{\infty} x_{mn}$ give one space is said to be convergent if and only if the double sequence (S_{mn}) is convergent, where:

$$S_{mn} = \sum_{i,j=1}^{m,n} x_{ij} \, (m,n = 1,2,3,\ldots)$$

A double sequence $x = (x_{mn})$ is said to be double analytic if,

$$\left|x_{mn}\right|^{\overline{m+n}} \to 0$$

The vector space of all double analytic sequences are usually denoted by L^2. A sequence $x = (x_{mn})$ is called double entire sequence if

$$\left|x_{mn}\right|^{\frac{1}{m+n}} \to 0 \text{ as } m,n \to \infty.$$

The vector space of all double entire sequences are usually denoted by G^2. Let the set of sequences with this property be denoted by L^2 and G^2 is a metric space with the metric

$$d(x,y) = sup_{m,n}\left\{\left|x_{mn} - y_{mn}\right|^{\frac{1}{m+n}} : m,n : 1,2,3,\ldots\right\}, \quad (1)$$

for all $x = \{x_{mn}\}$ and $y = \{y_{mn}\}$ in G^2. Let $\varphi = [\textit{finite sequences}]$.

Consider a double sequence $x = (x_{mn})$. The $(m,n)^{th}$ section $x^{[m,n]}$ of the sequence is defined by $x^{[m,n]} = \sum_{i,j=0}^{m,n} x_{ij}\delta_{ij}$ for all $m,n \in \mathbb{N}$,

$$\delta_{mn} = \begin{pmatrix} 0 & 0 & \ldots 0 & 0 & \ldots \\ 0 & 0 & \ldots 0 & 0 & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & \ldots 1 & 0 & \ldots \\ 0 & 0 & \ldots 0 & 0 & \ldots \end{pmatrix}$$

with 1 in the $(m,n)^{th}$ position and zero otherwise.

A double sequence $x = (x_{mn})$ is called double gai sequence if $\left((m+n)!\left|x_{mn}\right|\right)^{\frac{1}{m+n}} \to 0$ as $m,n \to \infty$. The double gai sequences will be denoted by χ^2.

Let *M* and Φ be mutually complementary Orlicz functions. Then, we have:

(i) For all $u,y \geq 0$,

$$uy \leq M(u) + \Phi(y), \text{ (Young's inequality) [16]} \quad (2)$$

(ii) For all $u \geq 0$,

$$u\eta(u) = M(u) + \Phi(\eta(u)). \quad (3)$$

(iii) For all $u \geq 0$, and $0 < \lambda < 1$,

$$M(\lambda u) \leq \lambda M(u) \quad (4)$$

Lindenstrauss and Tzafriri [16] used the idea of Orlicz function to construct Orlicz sequence space

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$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \quad for \quad some \quad \rho > 0 \right\},$$

The space ℓ_M with the norm

$$\|x\| = inf\left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \le 1 \right\},$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p (1 \le p < \infty)$, the spaces ℓ_M coincide with the classical sequence space ℓ_p.

A sequence $f = (f_{mn})$ of Orlicz function is called a Musielak-Orlicz function. A sequence $g = (g_{mn})$ defined by $gmn(v) = sup\{|v|u - fmn(u): u \ge 0\}$, $m,n = 1,2,\ldots$ is called the complementary function of a Musielak-Orlicz function f. For a given Musielak Orlicz function f, the Musielak-Orlicz sequence space t_f is defined by:

$$t_f = \left\{ x \in w^2 : I_f\left(|x_{mn}|\right)^{1/m+n} \to 0 \quad as \quad m,n \to \infty \right\},$$

where I_f is a convex modular defined by

$$I_f(x) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn}\left(|x_{mn}|\right)^{1/m+n}, x = (x_{mn}) \in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x,y) = sup_{mn}\left\{ inf\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn}\left(\frac{|x_{mn}|^{1/m+n}}{mn}\right)\right) \le 1 \right\}.$$

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [17] as follows

$Z(\Delta) = \{x = (x_k) \in w: (\Delta x_k) \in Z\}$,

for $Z = c, c_0$ and ℓ_∞, where $\Delta x_k = x_k - x_{k+1}$ for all $k \in \mathbb{N}$.

Here c, c_0 and ℓ_∞ denote the classes of convergent, null and bounded sclar valued single sequences respectively. The spaces $c(\Delta), c_0(\Delta), l_\infty(\Delta)$ and bv_p are Banach spaces normed by:

$$\|x\| = |x_1| + sup_{k\ge1}|\Delta x_k| \text{ and } \|x\|_{bv_p} = \left(\sum_{k=1}^{\infty}|x_k|^p\right)^{1/p}, (1 \le p < \infty).$$

Later on the notion was further investigated by many others. We now introduce the following difference double sequence spaces defined by:

$$Z(\Delta) = \left\{ x = (x_{mn}) \in w^2 : (\Delta x_{mn}) \in Z \right\},$$

Where $Z = L^2$, χ^2 and $\Delta x_{mn} = (x_{mn} - x_{mn+1}) - (x_{m+1n} - x_{m+1n+1}) = x_{mn} - x_{mn+1} - x_{m+1n} + x_{m+1n+1}$ for all $m,n \in \mathbb{N}$. The generalized difference double notion has the following representation: $\Delta^m x_{mn} = \Delta^{m-1}x_{mn} - \Delta^{m-1}x_{mn+1} - \Delta^{m-1}x_{m+1n} + \Delta^{m-1}x_{m+1n+1}$, and also this generalized difference double notion has the following binomial representation: $\Delta^m x_{mn} = \sum_{i=0}^{m}\sum_{j=0}^{m}(-1)^{i+j}\binom{m}{i}\binom{m}{j}x_{m+i,n+j}$.

Let $\eta = (\eta_{mn})$ be a sequence of nonzero scalars. Then, for a sequence space E, the multiplier sequence space E_η, associated with the multiplier sequence η, is defined as:

$$E_\eta = \left\{ x = (x_{mn}) \in w^2 : (\eta_{mn}x_{mn}) \in E \right\}.$$

The notion of sequence spaces associated with multiplier sequences was introduced by. Later on this notion was studied from different aspects by Tripathy and Sen [18], Tripathy and Hazarika [19] and many others [20].

Let $\eta = (\eta_{mn})$ be a sequence of nonzero scalars. Then, for a sequence space E, the multiplier sequence space E_η, associated with the multiplier sequence η, is defined as:

$$E_\eta = \left\{ x = (x_{mn}) \in w^2 : (\eta_{mn}x_{mn}) \in E \right\}.$$

Definition and Preliminaries

Let $n \in \mathbb{N}$ and X be a real vector space of dimension w, where $n \le w$. A real valued function $d_p(x_1,\ldots,x_n) = \|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p$ on X satisfying the following four conditions:

(i) $\|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p = 0$ if and and only if $d_1(x_1,0),\ldots,d_n(x_n,0)$ are linearly dependent,

(ii) $\|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p$ is invariant under permutation,

(iii) $\|(\alpha d_1(x_1,0),\ldots,d_n(x_n,0))\|_p = |\alpha| \|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p, \alpha \in \mathbb{R}$

(iv) $d_p((x_1,y_1),(x_2,y_2)\cdots(x_n,y_n)) = \left(d_X(x_1,x_2,\cdots x_n)^p + d_Y(y_1,y_2,\cdots y_n)^p\right)^{1/p} for 1 \le p < \infty;$

(or)

(v) $d((x_1,y_1),(x_2,y_2),\cdots(x_n,y_n)) := \sup\{d_X(x_1,x_2,\cdots x_n), d_Y(y_1,y_2,\cdots y_n)\}$,

for $x_1,x_2,\cdots x_n \in X, y_1,y_2,\cdots y_n \in Y$ is called the p product metric of the Cartesian product of n metric spaces is the p norm of the n-vector of the norms of the n subspaces.

A trivial example of p product metric of n metric space is the p norm space is $X = \mathbb{R}$ equipped with the following Euclidean metric in the product space is the p norm:

$$\|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_E = sup\left(|det(d_{mn}(x_{mn}))|\right) = sup\left(\left|\begin{matrix} d_{11}(x_{11},0) & d_{12}(x_{12},0) & \ldots & d_{1n}(x_{1n},0) \\ d_{21}(x_{21},0) & d_{22}(x_{22},0) & \ldots & d_{2n}(x_{1n},0) \\ . & & & \\ . & & & \\ . & & & \\ d_{n1}(x_{n1},0) & d_{n2}(x_{n2},0) & \ldots & d_{nn}(x_{nn},0) \end{matrix}\right|\right)$$

Where $x_i = (x_{i1}, \ldots x_{in}) \in \mathbb{R}^n$ for each $i = 1,2,\ldots n$.

If every Cauchy sequence in X converges to some $L \in X$, then X is said to be complete with respect to the p–metric. Any complete p–metric space is said to be p–Banach metric space.

Let X be a linear metric space. A function w: $X\mathbb{R}$ is called paranorm, if:

(1) $w(x) \ge 0$, for all $x \in X$;

(2) $w(-x) = w(x)$, for all $x \in X$;

(3) $w(x+y) \le w(x) + w(y)$, for all $x,y \in X$;

(4) If (σ_{mn}) is a sequence of scalars with $\sigma_{mn} \to \sigma$ as $m,n \to \infty$ and (x_{mn}) is a sequence of vectors with $w(x_{mn} - x) \to 0$ as $m,n \to \infty$, then $w(\sigma_{mn}x_{mn} - \sigma x) \to 0$ as $m,n \to \infty$.

A paranorm w for which $w(x) = 0$ implies $x = 0$ is called total paranorm and the pair (X,w) is called a total paranormed space. It is well known that the metric of any linear metric space is given by some total paranorm by (Willansky, 1984).

$\eta = (\phi_{rs})$ a nondecreasing sequence of positive reals tending to infinity and $\phi_{11} = 1$ and $\phi_{r+1,s+1} \le \phi_{rs} + 1$.

The generalized de la Vallee-Pussin means is defined by:

$$t_{rs}(x) = \frac{1}{\varphi_{rs}}\sum_{m\in I_{rs}}\sum_{n\in I_{rs}} x_{mn},$$

Where $I_{rs} = [rs - \lambda_{rs} + 1, rs]$. For the set of sequences that are strongly summable to zero, strongly summable and strongly bounded by the de la Vallee-Poussin method.

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The notion of λ– double gai and double analytic sequences as follows: Let $\lambda = (\lambda_{mn})_{m,n=0}^{\infty}$ be a strictly increasing sequences of positive real numbers tending to infinity, that is:

$$0 < \lambda_{00} < \lambda_{11} < \ldots \text{ and } \lambda_{mn} \to \infty \text{ as } m,n \to \infty$$

and said that a sequence $x = (x_{mn}) \in w^2$ is λ–convergent to 0, called a the λ– limit of x, if $B_\eta^\mu(x) \to 0 \quad as \quad m,n \to \infty$, Where:

$$B_\eta^\mu(x) = \frac{1}{\varphi_{rs}} \sum_{m\in I_{rs}} \sum_{n\in I_{rs}} \left(\Delta^{m-1}\lambda_{m,n} - \Delta^{m-1}\lambda_{m,n+1} - \Delta^{m-1}\lambda_{m+1,n} + \Delta^{m-1}\lambda_{m+1,n+1}\right)|x_{mn}|^{1/m+n}.$$

The sequence $x = (x_{mn}) \in w^2$ is λ–double analytic if $sup\left|B_\eta^\mu(x)\right| < \infty$. If $lim_{mn} x_{mn} = 0$ in the ordinary sense of convergence, then:

$$lim_{rs} \frac{1}{\varphi_{rs}} \sum_{m\in I_{rs}} \sum_{n\in I_{rs}} \left(\Delta^{m-1}\lambda_{m,n} - \Delta^{m-1}\lambda_{m,n+1} - \Delta^{m-1}\lambda_{m+1,n} + \Delta^{m-1}\lambda_{m+1,n+1}\right)\left((m+n)!|x_{mn} - 0|\right)^{1/m+n} = 0.$$

This implies that:

$$lim_{rs}\left|B_\eta^\mu(x) - 0\right| = \lim_{rs} \frac{1}{\varphi_{rs}} \sum_{m\in I_{rs}} \sum_{n\in I_{rs}}$$

$$\left(\Delta^{m-1}\lambda_{m,n} - \Delta^{m-1}\lambda_{m,n+1} - \Delta^{m-1}\lambda_{m+1,n} + \Delta^{m-1}\lambda_{m+1,n+1}\right)\left((m+n)!|x_{mn} - 0|\right)^{1/m+n} = 0.$$

which yields that limuvμμμμμ $lim_{uv}\mu_{mn}(x) = 0$ and hence $x = (xmn) \in w^2$ is λ–convergent to 0.

Let $f = (f_{mn})$ be a Mu-Orlicz function and $\left(X, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)$, be a p–metric space, $q = (q_{mn})$ be double analytic sequence of strictly positive real numbers. By $w^2(p - X)$ we denote the space of all sequences defined over:

$$\left(X, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right).$$

The following inequality will be used throughout the paper. If $0 \le q_{mn} \le sup q_{mn} = H, K = max\left(1, 2^{H-1}\right)$ then:

$$|a_{mn} + b_{mn}|^{q_{mn}} \le K\left\{|a_{mn}|^{q_{mn}} + |b_{mn}|^{q_{mn}}\right\} \tag{5}$$

for all m,n and amn, bmn $\in \mathbb{C}$. Also $|a|^{q_{mn}} \le max\left(1, |a|^H\right)$ for all a $\in \mathbb{C}$.

Let $\left(X, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)$ be an p–metric space and let $s(w^2 - x)$ denote the space of X–valued sequences. Let $q = (q_{mn})$ be any bounded sequence of positive real numbers and $f = (f_{mn})$ be a Musielak-Orlicz function. We define the following sequence spaces in this paper:

$$\left[\chi^{2q}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V =$$

$$\left\{x = (x_{mn}) \in s\left(w^2 - x\right) : lim_{rs}\left[f_{mn}\left(\left\|B_\eta^\mu(x), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right)\right\|_p\right)\right]^{q_{mn}} = 0\right\},$$

$$\left[\Lambda^{2q}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V =$$

$$\left\{x = (x_{mn}) \in s\left(w^2 - x\right) : sup_{rs}\left[f_{mn}\left(\left\|B_\eta^\mu(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mn}} < \infty\right\},$$

If we take $f_{mn}(x) = x$, we get:

$$\left[\chi^{2q}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V$$

$$= \left\{x = (x_{mn}) \in s\left(w^2 - x\right) : lim_{rs}\left[f_{mn}\left(\left\|B_\eta^\mu(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mn}} = 0\right\},$$

$$\left[\Lambda^{2q}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V =$$

$$\left\{x = (x_{mn}) \in s\left(w^2 - x\right) : sup_{rs}\left[f_{mn}\left(\left\|B_\eta^\mu(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mn}} < \infty\right\},$$

If we take $q = (q_{mn}) = 1$, we get:

$$\left[\chi^{2}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V =$$

$$\left\{x = (x_{mn}) \in s\left(w^2 - x\right) : \left[f_{mn}\left(\left\|B_\eta^\mu(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right] = 0\right\},$$

$$\left[\Lambda^{2}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V =$$

$$\left\{x = (x_{mn}) \in s\left(w^2 - x\right) : \left[f_{mn}\left(\left\|B_\eta^\mu(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right] < \infty\right\}.$$

In the present paper we plan to study some topological properties and inclusion relation between the above defined sequence spaces. $\left[\chi^{2q}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V$ and $\left[\Lambda^{2q}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V$ which we shall discuss in this paper.

Main Results

Theorem 1

Let $f = (f_{mn})$ be a Musielak-Orlicz function, $q = (q_{mn})$ be a double analytic sequence of strictly positive real numbers, the sequence spaces

$$\left[\chi^{2q}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V \text{ and,}$$

$$\left[\Lambda^{2q}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V \text{ are linear spaces.}$$

Proof

It is routine verification. Therefore the proof is omitted.

Theorem 2

Let $f = (f_{mn})$ be a Musielak-Orlicz function, $q = (q_{mn})$ be a double analytic sequence of strictly positive real numbers, the sequence space.

$\left[\chi^{2q}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V$ is a paranormed space with respect to the paranorm defined by:

$$g(x) = inf\left\{\left[f_{mn}\left(\left\|B_\eta^\mu(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mn}} \le 1\right\} = 0.$$

Proof

Clearly $g(x) \ge 0$ for $x = (x_{mn}) \in \left[\chi^{2q}_{fB_\eta^\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi\right]^V$ Since $f_{mn}(0) = 0$, we get $g(0) = 0$.

Conversely, suppose that $g(x) = 0$, then:

$$inf\left\{\left[f_{mn}\left(\left\|B_\eta^\mu(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mn}} \le 1\right\} = 0.$$

Suppose that $B_\eta^\mu(x) \ne 0$ for each $m,n \in \mathbb{N}$. Then $\left\|B_\eta^\mu(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p^\varphi \to \infty$. It follows that $\left(\left[f_{mn}\left(\left\|B_\eta^\mu(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mn}}\right)^{1/H} \to \infty$ which is a contradiction.

Therefore $B_\eta^\mu(x) = 0$.

Let:

$$\left(\left[f_{mn}\left(\left\|B_\eta^\mu(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mn}}\right)^{1/H} \le 1$$

and

$$\left(\left[f_{mn}\left(\left\|B_\eta^\mu(y), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mn}}\right)^{1/H} \le 1.$$

Then by using Minkowski's inequality, we have:

$$\left(\left[f_{mn}\left(\left\|B_\eta^\mu(x+y),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\right)^{1/H}$$

$$\le\left(\left[f_{mn}\left(\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\right)^{1/H}+$$

$$\left(\left[f_{mn}\left(\left\|B_\eta^\mu(y),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\right)^{1/H}.$$

So we have:

$$g(x+y)=inf\left\{\left[f_{mn}\left(\left\|B_\eta^\mu(x+y),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\le 1\right\}$$

$$\le inf\left\{\left[f_{mn}\left(\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\le 1\right\}+inf$$

$$\left\{\left[f_{mn}\left(\left\|B_\eta^\mu(y),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\le 1\right\}$$

Therefore,

$$g(x+y)\le g(x)+g(y).$$

Finally, to prove that the scalar multiplication is continuous. Let λ be any complex number. By definition,

$$g(\lambda x)=inf\left\{\left[f_{mn}\left(\left\|B_\eta^\mu(\lambda x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\le 1\right\}.$$

Then:

$$g(\lambda\quad x)=inf$$

$$\left\{((|\lambda|t)^{q_{mn}/H}:\left[f_{mn}\left(\left\|B_\eta^\mu(\lambda x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\le 1\right\}$$

Where $t=\frac{1}{|\lambda|}\cdot$ Since $|\lambda|^{q_{mn}}\le max\left(1,|\lambda|^{supq_{mn}}\right)$, we have:

$$g(\lambda\quad x)\le max\left(1,|\lambda|^{supq_{mn}}\right)inf$$

$$\left\{t^{q_{mn}/H}:\left[f_{mn}\left(\left\|B_\eta^\mu(\lambda x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\le 1\right\}$$

Theorem 3

(i) If the sequence (f_{mn}) satisfies uniform Δ_2– condition, then:

$$\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V\alpha}=$$

$$\left[\chi_g^{2qB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}.$$

(ii) If the sequence (g_{mn}) satisfies uniform Δ_2–condition, then:

$$\left[\chi_g^{2qB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V\alpha}=$$

$$\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}.$$

Proof

Let the sequence (f_{mn}) *satisfies uniform* Δ_2– condition, we get:

$$\left[\chi_g^{2qB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}\subset$$

$$\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V\alpha}$$

To prove the inclusion:

$$\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V\alpha}\subset$$

$$\left[\chi_g^{2qB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V},$$

let $a\in\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}$.

Then for all $\{x_{mn}\}$ with $(x_{mn})\in\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}$ we have:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}|x_{mn}a_{mn}|<\infty. \quad (6)$$

Since the sequence (f_{mn}) *satisfies uniform* Δ_2– condition, then:

$(y_{mn})\in\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}$, we get

$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left|\frac{\varphi_{rs}y_{mn}a_{mn}}{\Delta^m\lambda_{mn}(m+n)!}\right|<\infty$, by (Theorem 1).

Thus:

$(\varphi_{rs}a_{mn})\in\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}=$ and Hence

$$\left[\chi_g^{2qB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}$$

$$(a_{mn})\in\left[\chi_g^{2qB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}.$$

This gives that:

$$\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V\alpha}\subset$$

$$\left[\chi_g^{2qB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}$$

From this, we get:

$$\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V\alpha}=$$

$$\left[\chi_g^{2qB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}.$$

(ii) Similarly, one can prove that:

$$\left[\chi_g^{2qB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V\alpha}\subset$$

$$\left[\chi^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}$$

if the sequence (g_{mn}) satisfies uniform Δ_2– condition.

Proposition 1

If $0<q_{mn}<p_{mn}<\infty$ *for each m and n, then:*

$$\left[\Lambda^{2q}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}\subseteq$$

$$\left[\Lambda^{2p}_{fB_\eta^\mu},\left\|B_\eta^\mu(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]^{V}.$$

Proof

The proof is standard, so we omit it.

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Proposition 2

(i) If $0 < infq_{mn} \le q_{mn} < 1$ *then*

$$\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V} \subset$$

$$\left[\Lambda^{2}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V}.$$

(ii) If $1 \le q_{mn} \le supq_{mn} < \infty$, *then*

$$\left[\Lambda^{2}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V} \subset$$

$$\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V}.$$

Proof

The proof is standard, so we omit it.

Proposition 3

Let $f=(f_{mn})$ *and* $f'=(f'_{mn})$ *are sequences of Musielak Orlicz functions, we have*

$$\left[\Lambda^{2q}_{f'B^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V} \bigcap$$

$$\left[\Lambda^{2q}_{f''\mu}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V} \subseteq$$

$$\left[\Lambda^{2q}_{f'+f''B^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V}.$$

Proof

The proof is easy so we omit it.

Proposition 4

For any sequence of Musielak Orlicz functions $f=(f_{mn})$ *and* $q=(q_{mn})$ *be double analytic sequence of strictly positive real numbers. Then*

$$\left[\chi^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V} \subset$$

$$\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V}.$$

Proof

The proof is easy so we omit it.

Proposition 5

The sequence space $\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V}$ *c is solid.*

Proof

Let $x=(x_{mn}) \in \left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V}$, (i. e)

$$sup_{mn}\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V} < \infty.$$

Let (α_{mn}) be double sequence of scalars such that $|\alpha_{mn}| \le 1$ for all $m,n \in \mathbb{N}\times\mathbb{N}$. Then we get:

$$sup_{mn}\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(\alpha x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V} \le$$

$$\sup_{mn}\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V}.$$

Proposition 6

The sequence space $\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_{p}\right]^{V}$ *is monotone.*

Proof

The proof follows from Proposition 5.

Proposition 7

If $f=(f_{mn})$ *be any Musielak Orlicz function. Then*

$$\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{*}}_{p}\right]^{V} \subset$$

$$\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{**}}_{p}\right]^{V}$$ if and only

if $sup_{r,s\ge 1}\frac{\varphi^{*}_{rs}}{\varphi^{**}_{rs}} < \infty$.

Proof

Let $x \in \left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{*}}_{p}\right]^{V}$ and $N = sup_{r,s\ge 1}\frac{\varphi^{*}_{rs}}{\varphi^{**}_{rs}} < \infty$. Then we get:

$$\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{**}_{rs}}_{p}\right]^{V} =$$

$$N\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{*}_{rs}}_{p}\right]^{V} = 0.$$

Thus $x \in \left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{**}}_{p}\right]^{V}$. Conversely, suppose that

$$\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{*}}_{p}\right]^{V} \subset$$ and

$$\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{**}}_{p}\right]^{V}$$

$$x \in \left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{*}}_{p}\right]^{V}.$$

Then $\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{*}}_{p}\right]^{V} < \varepsilon$, for every $\varepsilon > 0$.

Suppose that $sup_{r,s\ge 1}\frac{\varphi^{*}_{rs}}{\varphi^{**}_{rs}} = \infty$, then there exists a sequence of members (rs_{jk}) such that $lim_{j,k\to\infty}\frac{\varphi^{*}_{jk}}{\varphi^{**}_{jk}} = \infty$.

Hence, we have:

$$\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{*}_{rs}}_{p}\right]^{V} = \infty.$$

Therefore $x \notin \left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{**}}_{p}\right]^{V}$, which is a contradiction.

Proposition 8

If $f=(f_{mn})$ be any Musielak Orlicz function. Then

$$\left[\Lambda^{2q}_{fB^{\eta}_{\mu}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^*}_p\right]^V = \left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi^{**}}_p\right]^V$$ if and only

if $sup_{r,s\geq 1}\frac{\varphi^*_{rs}}{\varphi^{**}_{rs}}<\infty, sup_{r,s\geq 1}\frac{\varphi^{**}_{rs}}{\varphi^*_{rs}}>\infty.$

Proof

It is easy to prove so we omit.

Proposition 9

The sequence space $\left[\chi^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V$ *is not solid.*

Proof

The result follows from the following example.

Example 1

Consider

$$x=(x_{mn})=\begin{pmatrix}1 & 1 & \ldots & 1\\ 1 & 1 & \ldots & 1\\ . & & & \\ . & & & \\ . & & & \\ 1 & 1 & \ldots & 1\end{pmatrix}\in\left[\chi^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V.$$

Let

$$\alpha_{mn}=\begin{pmatrix}-1^{m+n} & -1^{m+n} & \ldots & -1^{m+n}\\ -1^{m+n} & -1^{m+n} & \ldots & -1^{m+n}\\ . & & & \\ . & & & \\ . & & & \\ -1^{m+n} & -1^{m+n} & \ldots & -1^{m+n}\end{pmatrix}, \text{ for all } m,n\in\mathbb{N}.$$

Then $\alpha_{mn}x_{mn}\notin\left[\chi^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V.$

Hence

$\left[\chi^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V$ is not solid.

Proposition 10

The sequence space $\left[\chi^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V$ *is not monotone.*

Proof

The proof follows from Proposition 9.

A sequence $x=(x_{mn})$ is said to be ϕ– statistically convergent or s_ϕ –statistically convergent to 0 if for every $\varepsilon>0$,

$$lim_{rs}\,|\left\{\left[f_{mn}\left(\left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}|\geq\varepsilon\right\}=0$$

where the vertical bars indicates the number of elements in the enclosed set. In this case we write $s_\phi-limx=0$ or $x_{mn}0(s_\phi)$ and $s_\phi=\{x{:}\exists 0\in\mathbb{R}{:}\ s_\phi-limx=0\}$.

Proposition 11

For any sequence of Musielak Orlicz functions $f=(f_{mn})$ and $q=(q_{mn})$ be double analytic sequence of strictly positive real numbers. Then

$$\left[\chi^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V\subset\left[s^{2q}_{\varphi fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V.$$

Proof

Let $x\in\left[\chi^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V$ and $\varepsilon>0$. Then

$$\left[f_{mn}\left(\left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\geq|\left\{\left[f_{mn}\left(\left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}|\geq\varepsilon\right\}$$

from which it follows that $x\in\left[s^{2q}_{\varphi fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V$.

To show that $\left[s^{2q}_{\varphi fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V$ strictly contain $\left[\chi^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V$. We define $x=(x_{mn})$ by $(x_{mn})=mn$

if $rs-\left[\sqrt{\varphi_{rs}}\right]+\leq mn\leq rs$ and $(x_{mn})=0$ otherwise. Then:

$x\notin\left[\Lambda^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V$ and for every ε $(0<\varepsilon\leq 1)$,

$$|\left\{\left[f_{mn}\left(\left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}|\geq\varepsilon\right\}=\frac{\left[\sqrt{\varphi_{rs}}\right]}{\varphi_{rs}}\to 0 \text{ as } r,s\to\infty$$

i. e $x\to 0\left(\left[s^{2q}_{\varphi fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V\right)$, where []denotes the greatest integer function. On the other hand,

$$\left[f_{mn}\left(\left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p\right)\right]^{q_{mn}}\to\infty \text{ as } r,s\to\infty$$

i. e $x_{mn}\not\to 0\left[\chi^{2q}_{fB^{\mu}_{\eta}}, \left\|B^{\mu}_{\eta}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|^{\varphi}_p\right]^V$.

Conclusion

Approximations results in Musielak Orlicz spaces are applicable in nonlinear partial differential equations. We proposed a generalized triple sequence spaces and discuss general topological properties with respect to a sequence of Musielak-Orlicz function. Our result generalizes and unifies the results of several author's in the case of classical Orlicz spaces. One can extend our results for more general spaces.

Competing Interests

The authors declare that there is not any conflict of interests regarding the publication of this manuscript.

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