The New Generalized Difference Sequence Space \( \chi^2 \) over \( p \)-Metric Spaces Defined by Musielak Orlicz Function Associated with a Sequence of Multipliers

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Abstract

In the present paper, we introduce new sequence spaces by using Musielak-Orlicz function and a generalized \( B^*_\varphi \)-difference operator on \( p \)-metric space. Some topological properties and inclusion relations are also examined.

Keywords: Analytic sequence; Double sequences; \( \chi^2 \) space; Difference sequence space; Musielak-Orlicz function; \( p \)-metric space; Lacunary sequence; Ideal

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Introduction

Throughout \( w, x \) and \( L \) denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write \( w^d \) for the set of all complex sequences \( (x_{mn}) \) where \( m, n \in N \), the set of positive integers. Then \( w^d \) is a linear space under the coordinate wise addition and scalar multiplication. For some approximations results in Musielak-Orlicz-Sobolev spaces and some applications to nonlinear partial differential equations see equation 22. The growing interest in this field is strongly stimulated by the treatment of recent problems in elasticity, fluid dynamics, calculus of variations, and differential equations.

Some initial works on double sequence spaces is found in Bromwich [1]. Later on it was investigated by Hardy [2], Moricz [3], Moricz and Rhoades [4], Basarir and Solankan [5], Tripathy et al. [6-10], Turkmenoglu [11], Raj [11-14], and many others [15].

Let \( (x_{mn}) \) be a double sequence of real or complex numbers. Then the series \( \sum_{m,n=1}^{\infty} x_{mn} \) is called a double series. The double series \( \sum_{m,n=1}^{\infty} x_{mn} \) give one space is said to be convergent if and only if the double sequence \( (S_{nm}) \) is convergent, where:

\[
S_{nm} = \sum_{m,n=1}^{\infty} x_{mn}, \quad m = 1, 2, 3, \ldots
\]

A double sequence \( x = (x_{mn}) \) is said to be double analytic if,

\[
\lim_{m,n \to \infty} x_{mn} = 0
\]

The vector space of all double analytic sequences are usually denoted by \( L^2 \). A sequence \( x = (x_{mn}) \) is called double entire sequence if

\[
\sum_{m,n=1}^{\infty} x_{mn} \rightarrow 0 \quad \text{as} \quad m,n \to \infty.
\]

The vector space of all double entire sequences are usually denoted by \( G^2 \). Let the set of sequences with this property be denoted by \( L^2 \) and \( G^2 \) is a metric space with the metric

\[
d(x, y) = \sup_{m,n \geq 1} \left\| x_{mn} - y_{mn} \right\| = \sup_{m,n \geq 1} \left\| x_{mn} - y_{mn} \right\|, \quad m,n \geq 1, 2, 3, \ldots
\]

Consider a double sequence \( x = (x_{mn}) \). The \( (m,n) \)-th section \( x^{(m,n)} \) of the sequence is defined by \( x^{(m,n)} = \sum_{m,n=1}^{\infty} x_{mn} \delta_{m,n} \) for all \( m,n \in N \),

\[
\delta_{m,n} = \begin{pmatrix}
0 & 0 & 0 & \ldots & 0 & \ldots \\
0 & 0 & 0 & \ldots & 0 & \ldots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \ldots & 0 & 0 & 0 & \ldots \\
0 & \ldots & 0 & 0 & 0 & \ldots \\
\end{pmatrix}
\]

with 1 in the \( (m,n) \)th position and zero otherwise.

A double sequence \( x = (x_{mn}) \) is called double gai sequence if \( \left\| (m+n)^{1/n} x_{mn} \right\| \to 0 \) as \( m,n \to \infty \). The double gai sequences will be denoted by \( \chi^2 \).

Let \( M \) and \( \Phi \) be mutually complementary Orlicz functions. Then, we have:

(i) For all \( u, y \geq 0 \),

\[
y \leq M(u) + \Phi(y), \quad \text{Young’s inequality} \quad [16]
\]

(ii) For all \( u \geq 0 \),

\[
u(y) = M(u) + \Phi(y(u)).
\]

(iii) For all \( u \geq 0 \) and \( 0 < \lambda < 1 \),

\[
M(\lambda u) \leq \lambda M(u)
\]

Lindenstrauss and Tzafriri [16] used the idea of Orlicz function to construct Orlicz sequence space

\[\chi^2 \]

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The space \( \ell_{\infty} \) with the norm

\[
\|f\| = \inf \left\{ \rho > 0 : \sum_{i=1}^{\infty} M \left( \frac{|f_i|}{\rho} \right) \leq 1 \right\},
\]

becomes a Banach space which is called an Orlicz sequence space. For a given Musielak Orlicz function \( \mu \), the Orlicz sequence space \( M \) is defined by:

\[
M = \left\{ x = (x_n) : \sum_{i=1}^{\infty} M \left( \frac{|x_i|}{\mu_i} \right) \leq \infty \right\}.
\]

A sequence \( f = (f_m) \) of Orlicz function is called a Musielak-Orlicz function. A sequence \( g = (g_n) \) defined by \( g_n = \sup \{|u| - \mu_n(u) : u \geq 0\}, \) where \( m, n = 1, 2, \ldots \), is called the complementary function of a Musielak-Orlicz function \( f \).

Let \( X \) be a real vector space of dimension \( w \), where \( n \leq w \). A real valued function \( d_x(x_1, \ldots, x_k) = ||(d_1(x_1), \ldots, d_k(x_k))|| \) on \( X \) satisfying the following four conditions:

(i) \( ||(d_1(x_1), \ldots, d_k(x_k))|| = 0 \) if and only if \( d_1(x_1), \ldots, d_k(x_k) \) are linearly dependent,

(ii) \( ||(d_1(x_1), \ldots, d_k(x_k))|| \) is invariant under permutation.

(iii) \( ||(\alpha d_1(x_1), \ldots, d_k(x_k))|| = \alpha \| (d_1(x_1), \ldots, d_k(x_k))\|, \alpha \in \mathbb{R} \).

(iv) \( d_x((x_1, y_1), (x_2, y_2), \ldots, (x_k, y_k)) = \sum_{i=1}^{k} d_x(x_i, y_i) \) for \( 1 \leq p < \infty \).

A trivial example of \( p \) product metric of \( n \) metric spaces is the \( p \) norm of the \( n \)-vector of the norms of the \( n \) subspaces.

A Cauchy sequence in \( X \) converges to some \( L \in X \), then \( X \) is said to be complete with respect to the \( p \)-metric. Any complete \( p \)-metric space is said to be \( p \)-Banach metric space.

Let \( X \) be a linear metric space. A function \( w : X \times X \to \mathbb{R} \) is called paranoan, if:

1. \( w(x, y) \geq 0 \), for all \( x, y \in X \);
2. \( w(-x, y) = w(x, y) \), for all \( x, y \in X \);
3. \( w(x+y, y) \leq w(x, y) + w(y, x) \), for all \( x, y \in X \);
4. If \( (x_m) \) is a sequence of scalars with \( \sigma_n \to \sigma \) as \( m, n \to \infty \) and \( (x_m) \) is a sequence of vectors with \( w(x_m, x) \to 0 \) as \( m, n \to \infty \), then \( w(\sigma_n x_m - \sigma x) \to 0 \) as \( m, n \to \infty \).

A paranoan \( w \) for which \( w(x, 0) = 0 \) implies \( x = 0 \) is called total paranoan and the pair \((X, w)\) is called a total paranoan space. It is well known that the metric of any linear metric space is given by some total paranoan by (Willansky, 1984).

\( \eta = (\eta_n) \) a nondecreasing sequence of positive reals tending to infinity and \( \phi_n = 1 \) and \( \phi_{n+1} \leq \phi_n \).

The generalized de la Vallee-Poussin means is defined by:

\[
\lambda_n(x) = \frac{1}{\phi_n} \sum_{\ell=1}^{n} \sum_{\ell=1}^{\ell} \phi_{n-\ell} x_{m},
\]

Where \( I_m = \{m-\lambda, 1, \ldots, 1\} \). For the set of sequences that are strongly summable to zero, strongly summable and strongly bounded by the de la Vallee-Poussin method.
The notion of $\lambda$- double gai and double analytic sequences as follows: Let $\lambda = \left(\lambda_n\right)_{n=1}^{\infty}$ be a strictly increasing sequences of positive real numbers tending to infinity, that is:

$$0 < \lambda_1 < \lambda_2 < \ldots < \lambda_n \to \infty$$

and said that a sequence $x = (x_n) \in w$ is $\lambda$-convergent to 0, called the $\lambda$- limit of $x$, if $B_{\lambda}^p (x) \to 0$ as $m,n \to \infty$. Where:

$$B_{\lambda}^p (x) = \left\{ \lim_{m,n \to \infty} \frac{1}{\lambda_m} \sum_{q=1}^{m} \sum_{r=1}^{n} (\lambda^{-q}_{r} - \lambda^{-q}_{r+1} - \lambda^{-q}_{r+1} + \lambda^{-q}_{r+1}) |x_{mr}| p \right\}^{1/p}$$

The sequence $x = (x_n) \in w$ is $\lambda$–double analytic if $$\lim_{m,n \to \infty} \frac{1}{\lambda_m} \sum_{q=1}^{m} \sum_{r=1}^{n} (\lambda^{-q}_{r} - \lambda^{-q}_{r+1} - \lambda^{-q}_{r+1} + \lambda^{-q}_{r+1}) |x_{mr}| = 0.$$ This implies that:

$$\lim_{m,n \to \infty} \frac{1}{\lambda_m} \sum_{q=1}^{m} \sum_{r=1}^{n} (\lambda^{-q}_{r} - \lambda^{-q}_{r+1} - \lambda^{-q}_{r+1} + \lambda^{-q}_{r+1}) (m + n) |x_{mr}| = 0.$$ which yields that the limit $$\lim_{m,n \to \infty} \frac{1}{\lambda_m} \sum_{q=1}^{m} \sum_{r=1}^{n} (\lambda^{-q}_{r} - \lambda^{-q}_{r+1} - \lambda^{-q}_{r+1} + \lambda^{-q}_{r+1}) |x_{mr}| = 0.$$ is $\lambda$–convergent to 0.

Let $f = (f_{mn})$ be a Musielak-Orlicz function and $$X = \left\{ (d(x),d(x)) : x \in \mathbb{R} \right\},$$ be a $p$–metric space, $g = (g_{mn})$ be double analytic sequence of strictly positive real numbers. By $w^2 (p - X)$ we denote the space of all sequences defined over:

$$\{ X_n \} \in \left\{ (d(x),d(x)) : x \in \mathbb{R} \right\}.$$ The following inequality will be used throughout the paper. If $0 \leq a_n \leq s_n \leq 1$, then:

$$\left[ a_n \right]^{1/p} + \left[ b_n \right]^{1/p} \leq K \left[ a_n^{1/p} + b_n^{1/p} \right]$$

for all $n, r$ and $m, n, r \in \mathbb{R}$. Also $\phi^{1/p} \leq \max \{ 1, \phi \}$ for all $\phi > 0$.

Let $$X = \left\{ (d(x),d(x)) : x \in \mathbb{R} \right\}$$ be an $p$–metric space and let $s(w^2 - x)$ denote the space of $X$–valued sequences. Let $q = (q_{mn})$ be any bounded sequence of positive real numbers and $f = (f_{mn})$ be a Musielak-Orlicz function. We define the following sequence spaces in this paper:

$$X = \left\{ (d(x),d(x)) : x \in \mathbb{R} \right\},$$ where:

$$\lim_{m,n \to \infty} \left[ \lambda^{-q}_{r} - \lambda^{-q}_{r+1} - \lambda^{-q}_{r+1} + \lambda^{-q}_{r+1} \right] |x_{mr}| = 0.$$ If we take $f_{mn} (x) = x$, we get:

$$\left[ \lambda^{-q}_{r} - \lambda^{-q}_{r+1} - \lambda^{-q}_{r+1} + \lambda^{-q}_{r+1} \right] |x_{mr}| = 0.$$ Suppose that $B_{\lambda}^p (x) \neq 0$ for each $m,n \in \mathbb{N}$. Then

$$\left[ f_{mn} (x), d(x), d(x), \ldots, d(x), \ldots \right] \to \infty.$$ which is a contradiction. Therefore $B_{\lambda}^p (x) = 0$.

Let:

$$\left[ f_{mn} (x), d(x), d(x), \ldots, d(x), \ldots \right] \leq 1$$ and

$$\left[ f_{mn} (x), d(x), d(x), \ldots, d(x), \ldots \right] \leq 1.$$ Then by using Minkowski’s inequality, we have:
Since $\rho \subset \rho'$ by (Theorem 1).

Let the sequence $(f_{m})$ be any complex number. By definition, $g(x+y) = \inf \left\{ f_{m}(f_{m}(x+y),d(x,0),d(x,0),\ldots,d(x,0)) \right\}$.

Then:
\[
g(\lambda x) = \inf \left\{ f_{m}(f_{m}(\lambda x),d(x,0),d(x,0),\ldots,d(x,0)) \right\} = 1.
\]

Finally, to prove that the scalar multiplication is continuous. Let $\lambda$ be any complex number. By definition, $g(\lambda x) = \inf \left\{ f_{m}(f_{m}(\lambda x),d(x,0),d(x,0),\ldots,d(x,0)) \right\}$.

Thus:
\[
g(\lambda x) = \inf \left\{ f_{m}(f_{m}(\lambda x),d(x,0),d(x,0),\ldots,d(x,0)) \right\} = 1.
\]

Therefore, $g(x+y) \leq g(x) + g(y)$.

To prove the inclusion:
\[
\left[ f_{m}(B_{p_{n}}^{\rho}(x+y),d(x,0),d(x,0),\ldots,d(x,0),0) \right]^{\text{max}} \subset \left[ f_{m}(B_{p_{n}}^{\rho}(x),d(x,0),d(x,0),\ldots,d(x,0),0) \right]^{\text{max}} + \left[ f_{m}(B_{p_{n}}^{\rho}(y),d(x,0),d(x,0),\ldots,d(x,0),0) \right]^{\text{max}}.
\]

So we have:
\[
g(x+y) = \inf \left\{ f_{m}(f_{m}(x+y),d(x,0),d(x,0),\ldots,d(x,0)) \right\} \leq \inf \left\{ f_{m}(f_{m}(x),d(x,0),d(x,0),\ldots,d(x,0)) \right\} + \inf \left\{ f_{m}(f_{m}(y),d(x,0),d(x,0),\ldots,d(x,0)) \right\}.
\]

Therefore,
\[
g(x+y) \leq g(x) + g(y).
\]

Finally, to prove that the scalar multiplication is continuous. Let $\lambda$ be any complex number. By definition, $g(\lambda x) = \inf \left\{ f_{m}(f_{m}(\lambda x),d(x,0),d(x,0),\ldots,d(x,0)) \right\}$.

Thus:
\[
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Thus:
\[
g(\lambda x) = \inf \left\{ f_{m}(f_{m}(\lambda x),d(x,0),d(x,0),\ldots,d(x,0)) \right\} = 1.
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Finally, to prove that the scalar multiplication is continuous. Let $\lambda$ be any complex number. By definition, $g(\lambda x) = \inf \left\{ f_{m}(f_{m}(\lambda x),d(x,0),d(x,0),\ldots,d(x,0)) \right\}$.

Thus:
\[
g(\lambda x) = \inf \left\{ f_{m}(f_{m}(\lambda x),d(x,0),d(x,0),\ldots,d(x,0)) \right\} = 1.
\]
Proposition 2

(i) If $0 < \inf q_{m} \leq q_{m} < 1$ then
\[
\left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y} \subset \left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y}.
\]

(ii) If $1 \leq q_{m} \leq \sup q_{m} < \infty$, then
\[
\left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y} \subset \left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y}.
\]

Proof
The proof is standard, so we omit it.

Proposition 3

Let $f = (f_{m})$ and $f = (f_{m})$ are sequences of Musielak Orlicz functions, we have
\[
\left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y} \cap \left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y} \subset \left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y}.
\]

Proof
The proof is easy so we omit it.

Proposition 4

For any sequence of Musielak Orlicz functions $f = (f_{m})$ and $q = (q_{m})$ be double analytic sequence of strictly positive real numbers. Then
\[
\left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y} \subset \left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y}.
\]

Proof
The proof is easy so we omit it.

Proposition 5

The sequence space $\left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y}$ is solid.

Proof
Let $x (x_{m}) \in \left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y}$. (i.e)
\[
\sup_{m,n} \left[ \Lambda^{2}_{m,n} B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right]^{y} < \infty.
\]
Let $(a_{m,n})$ be double sequence of scalars such that $|a_{m,n}| \leq 1$ for all $m,n \in \mathbb{N} \times \mathbb{N}$. Then we get:
\[
\sup_{m,n} \left[ \Lambda^{2}_{m,n} B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right]^{y} \leq \sup_{m,n} \left[ \Lambda^{2}_{m,n} B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right]^{y}.
\]

Proposition 6

The sequence space $\left[ \Lambda^{2}_{m,n} \left\| B_{q}^{*} (x), (d(x,0), d(x,0), \ldots, d(x,0), 0) \right\| \right]^{y}$ is monotone.

Proof
The proof follows from Proposition 5.
We define $s_{\phi}$ as
\[ s_{\phi}(x) = \sup_{m,n} \frac{\sup_{x \in B(x_m, r, s)} |\phi(x)|} {r,s} \]
where $\phi(x)$ is not monotone.

Example 1
Consider $x = (x_m)$ where
\[ x_m = \begin{cases} 1 & \text{if } m \text{ is odd} \\ 0 & \text{if } m \text{ is even} \end{cases} \]
Let $\alpha_{x_m} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$, for all $m,n \in \mathbb{N}$.
Then $\alpha_{x_m} x_m \notin \left[ x_m^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right] \supseteq \left[ \left. x_m^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right| \right]$.
Hence
\[ \left[ x_m^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right] \supseteq \left[ \left. x_m^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right| \right] \]
is not solid.

Proposition 10
The sequence space $\left[ x_m^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right]$ is not monotone.

Proof
The proof follows from Proposition 9.

A sequence $x = (x_m)$ is said to be $\phi-$ statistically convergent or $s_{\phi}$-statistically convergent to 0 if for every $\epsilon > 0$,
\[ \lim_{n \to \infty} \left| \left| f_n \left( x_m^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right) \right| \right|_{\infty} \geq \epsilon = 0 \]
where the vertical bars indicate the number of elements in the enclosed set. In this case we write $s_{\phi} - \lim x = 0$ or $x_{\lim}(x_s)$ and $s_{\phi} = \{ x \in \mathbb{R} : s_{\phi} - \lim x = 0 \}$.

Proposition 11
For any sequence of Musielak-Orlicz functions $f = (f_m)$ and $q = (q_m)$ be double analytic sequence of strictly positive real numbers. Then
\[ \left[ x_m^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right] \supseteq \left[ x_{\lim}^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right] \]

Proof
Let $x \in \left[ x_m^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right]$ and $\epsilon > 0$. Then
\[ f_{\lim} \left( x_m^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right) \supseteq \epsilon \leq \epsilon \]
from which it follows that $x \in \left[ x_{\lim}^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right]$.

To show that $\left[ x_{\lim}^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right]$ strictly contain
\[ \left[ x_{\lim}^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right] \]
We define $x = (x_m)$ by
\[ x_m = mn \]
if $rs - \sqrt{\phi_m} < mn \leq r, s$ and $(x_m) = 0$ otherwise. Then:
\[ x \in \left[ \left. x_{\lim}^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right| \right] \]
and for every $\epsilon (0 < \epsilon \leq 1)$,
\[ \left| f_{\lim} \left( x_{\lim}^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right) \right| \supseteq \epsilon = \left| x \right|_{\infty} \begin{cases} \frac{1}{\phi_m} \rightarrow 0 & \text{as } rs \rightarrow \infty \\ \epsilon \rightarrow 0 & \text{as } rs \rightarrow \infty \end{cases} \]
\[ = \left| x \right|_{\infty} \begin{cases} \frac{1}{\phi_m} \rightarrow 0 & \text{as } rs \rightarrow \infty \\ \epsilon \rightarrow 0 & \text{as } rs \rightarrow \infty \end{cases} \]
\[ \text{where } \left| \right| \text{denotes the greatest integer function. On the other hand,} \]
\[ \left| f_{\lim} \left( x_{\lim}^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right) \right| \supseteq \infty \text{ as } rs \rightarrow \infty \]
\[ \text{i. e. } x_{\lim} \rightarrow 0 \left[ x_{\lim}^\sigma_n \beta^*(\epsilon), (d(x_0), d(x_0), \ldots, d(x_{-1}, 0), \ldots) \right] \]

Conclusion
Approximations results in Musielak-Orlicz spaces are applicable in nonlinear partial differential equations. We proposed a generalized triple sequence spaces and discuss general topological properties with respect to a sequence of Musielak-Orlicz function. Our result generalizes and unifies the results of several author’s in the case of classical Orlicz spaces. One can extend our results for more general spaces.

Competing Interests
The authors declare that there is not any conflict of interests regarding the publication of this manuscript.

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