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The Mathematics of the Brain

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There are many perspectives from which one can approach individual decision-making. Economics and other social sciences typically take an outward-oriented point of view, such that the main determinant of choices is not the set of cognitive capabilities of the individual but the object of the choice itself. Decisions are taken after carefully weighting benefits and costs, and these benefits and costs have to do with the object underlying the decision and not with the deliberative process on its own. A straightforward corollary of the outward-oriented decision-making perspective is that individuals are all alike: if what matters are the arguments of the choice and not who chooses, agents must decide exactly in the same way when faced with the same choice; in other words, full rationality must prevail. Furthermore, one does not need to know what happens in people's brains: what matters is the preference the individual reveals when consummating the choice. These arguments are the building block of most of the contemporary economic theory, arguments that we can trace back to Samuelson et al, or Becker et al. [1,2].

Advances on neuroscience, and respective applications to the field of neuroeconomics, have allowed for a fresh look on the foundations of decision-making. A detailed analysis of brain processes turns possible an inward-oriented perspective on human choices. There are psychological constraints that affect the way individuals perceive events, select and process information and choose across options, and these constraints are likely to vary from one individual to another. The literature on neuroeconomics has been able to identify and isolate some of the most powerful processes leading to human decision and to express them under the form of relatively simple mathematical models [3-10].[1]

A possible approach to the modeling of mental processes, the one we will focus in the remainder of this short note, is the recent paper by Alonso et al. [3]. These authors analyze some evidence provided by the neuroscience literature to justify a concrete organization of the human brain. Specifically, the brain is modeled as a collection of systems that are coordinated by a Central Executive System (CES). The role of the CES is to allocate limited cognitive resources to all the other systems, with each of these responsible for a specific and unique task. Brain systems compete for the resources and the CES manages the respective allocation, given the perceived cognitive demands of each task.

The mentioned study starts by identifying four fundamental features of brain activity. These are specialization, communication, centralization and scarcity of resources. When an individual has a task to perform, a system of neurons is formed; the neurons in a specific system are allocated exclusively to that particular task and remain attached to it as long as it occupies the mind of the individual; it is in this sense that we can talk about brain system specialization. Each brain system needs, then, to communicate its need of resources to a distinct entity, which will be the CES; the CES will be responsible for a fair and efficient allocation of the cognitive resources across systems, i.e., it functions as a central planner. Finally, one should note that mental resources are scarce, that is, the brain is unable to simultaneously attribute full attention to every task; cognitive resources must be distributed in such a way that it becomes possible to approach the highest possible number of tasks, each one with the highest possible efficiency.

Given the above properties, the functioning of the brain might be modeled as an agency problem where the CES is the principal and each system, allocated to the fulfillment of each task, represents an agent. Suppose the brain has, at a given moment in time, n tasks to perform; then, there will be n brain systems, each one responsible for one task. Tasks and systems are represented by $\ell \in L = \{1,2,\dots,n\}$. Let x_l be the resources allocated to system ℓ and Θ_l the amount of resources that allow to completely fulfill, without flaws, the same task. For each system there will be a performance function that measures how far the system is from holding the resources needed to complete the task with full success. The performance measure is $\Pi_\ell\left(x_\ell;\theta_\ell\right)$.

We interpret the performance measure as a loss function, i.e., we consider that $\Pi_\ell\left(x_\ell;\theta_\ell\right)=0$ whenever $x_\ell \geq_\ell$ and that $\Pi_\ell\left(x_\ell;\theta_\ell\right)<0$ for $x_\ell <_\ell$. The larger the distance between x_l and Θ_l the more negative is the performance outcome. A possible formalization is as follows,

$$\Pi_\ell\left(x_\ell;\theta_\ell\right)=\begin{cases}\alpha_\ell u_\ell\left(x_\ell-\theta_\ell\right) \text{if} x_\ell <_\ell \\ 0 \text{if} x_\ell \geq_\ell\end{cases} \tag{1}$$

In expression (1), α_ℓ is a positive parameter and $u_\ell\left(x_\ell-\theta_\ell\right)$ is a continuous and differentiable function such that $u_\ell^{'}>0$ and $u_\ell^{''}\leq 0$; also, $\lim_{x_\ell-\theta_\ell\to 0} u_\ell\left(x_\ell-\theta_\ell\right)=0$.

From the point of view of the CES, the brain solves the following optimality problem,

$$\max_{x_\ell} \max_{x_\ell} \sum_{l=1}^{n} \Pi_\ell\left(x_\ell;\theta_\ell\right)$$

$$\text{subject to} \sum_{l=1}^{n} x_\ell \leq k; x_\ell \geq 0, \forall \ell \in L \tag{2}$$

The control variable in the maximization problem is the amount of resources allocated to each system, x_l, and therefore the solution of the problem will be an array of quantities of resources that, optimally, are allocated to each system, given the full amount of available resources k. The problem has two constraints, a resource constraint and a feasibility constraint, which establish, respectively, a maximum and a minimum on the resources to be allocated to each system to the performance of each task.

[1]Fehr et al. [10] offer a detailed survey on the contribution of neuroscience to understand economic choice.

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Naturally, problem (2) is relevant only when the resource constraint is binding, i.e., when the amount of resources required to perform all the tasks flawlessly is a value larger than k $(\sum_{l=1}^{n} \theta_\ell > k)$. Under perfect knowledge or complete information of each system's needs by the CES, the problem is relatively trivial to approach and it delivers a straightforward solution with optimal allocation of cognitive resources to each task, given the available resources and the complexity of each task (translated on the value of Θ_l).

The problem becomes harder to address if information is not complete, i.e., if the CES does not have perfect knowledge about the resources required by each of the systems. In this case, Θ_l is unknown to the CES, at least for some of the systems or some of the tasks. The only information the CES has is that θ_ℓ is drawn from a distribution with cumulative distribution function $F^\ell(\theta_\ell)$, with $F^\ell(\theta_\ell)$ independent across systems.

The imperfect knowledge scenario implies that the CES will maximize the expected performance of the tasks. This requires considering that x_l is selected by looking at the needs of every system and not only to the needs of the system under consideration. Particularly, the CES considers now a resource variable $x_\ell(\theta_1, \theta_2, \ldots, \theta_\ell, \ldots, \theta_n)$ and solves a modified optimality problem,

$$\max_{E(x_\ell)} \max_{E(x_\ell)} \iint \cdots \int \sum_{l=1}^{n} \Pi_\ell \left[x_\ell(\theta_1, \theta_2, \ldots, \theta_\ell, \ldots, \theta_n); \theta_\ell \right] dF^1(\theta_1) dF^2(\theta_2) \ldots dF^n(\theta_n)$$

subject to:

$$\sum_{l=1}^{n} x_\ell(\theta_1, \theta_2, \ldots, \theta_\ell, \ldots, \theta_n) \le k;\ x_\ell(\theta_1, \theta_2, \ldots, \theta_\ell, \ldots, \theta_n) \ge 0, \forall \ell \in L \quad (2)$$

The specific form of $x_\ell(\theta_1, \theta_2, \ldots, \theta_\ell, \ldots, \theta_n)$ will depend on the criteria used to assess information asymmetries in the particular context one is assuming. The paper by Alonso et al. [3] makes a thorough analysis of the brain resource allocation problem under incomplete information and arrives to relevant conclusions, which allow for gaining important insights about how the brain approaches various tasks at the same time. The following results deserve to be highlighted:

- If the information is not complete and perfect, the individual performs flawlessly in simple cognitive tasks and under-performs by a large margin in difficult tasks; this contrasts with the full information outcome, where relatively moderate under-performance will prevail for every task, independently of its complexity;
- The efficient allocation of cognitive resources might be reached under an almost instantaneous dynamic biologically plausible process;
- Under incomplete information the allocation of resources will be path-dependent: previous allocations matter to the decision about the current allocation; this is in contrast with the full information outcome and it adds an important element to the analysis that allows for increased realism: individuals are not able to immediately switch their attention from one task to the other as their relative importance change; there is an inertia in reprogramming resource requirements, and this inertia is revealed when information is asymmetric or less than perfect;
- The brain specializes in specific tasks when they are considered more important and integrates various tasks into a single system if the brain considers them relatively unimportant. The human brain is an extremely complex unit, that responds to a multitude of stimulus and that prioritizes problems according to their relevance, salience, complementarities and the resources they require. Some evidence on how the brain works gives us the possibility of modeling the process through which decisions are effectively taken and choices are effectively made. This evidence points to the existence of a centralized unit capable of allocating resources to specific parts of the brain which are occupied with particular tasks. Such centralized unit faces two constraints: first, cognitive resources are not unlimited and, second, information about the true needs to complete each task is not necessarily perfect. Thus, the brain has to decide in an uncertainty context; it uses its best judgment to choose how many resources to allocate to each assignment in order to solve them the best it can.

Understanding the mechanisms of the brain is an essential first step to address economic problems. How the mind works determines not only individual choices in isolation but also choices in a social context or in a context of interaction. Trade relations, investment decisions, the organization of work processes all have behavioral roots, roots that ultimately must be searched on the organization of the human brain and how such organization triggers decisions.

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