

## The Illusive Alpha and Useless Beta Mathematical Elegance without Market Relevance

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### Abstract

The concepts of alpha ( $\alpha$ ) and beta ( $\beta$ ) have attained ubiquitous presence in the investment jargon of our times and are the bedrock of Modern Portfolio Theory (MPT) and the Nobel Prize-winning Capital Asset Pricing Model (CAPM). Portfolio managers routinely boast of their superior skills in generating alpha-their excess return relative to their benchmark. Copying the newest fad, hedge funds and proprietary trading desks tack on 'alpha' to the name of newfangled funds. Strategies are hawked as "portable alpha" and claimed to be the panacea for outperforming an index. On Main Street, the local postman-turned-investor logs on to Seeking *Alpha.com* to get the inside tip for the next big "two-bagger". Alas, few mangers-and even fewer retail investors-have been able to consistently generate this ever-elusive alpha.

**Keywords:** Capital asset pricing model; Compensation; Investors

### Introduction

The paper argues that this seeming impossibility to consistently generate alpha is not due to investor ability. It is due to the very definition of alpha and its mathematical formulation based on regression that renders its value to be zero. Instances of non-zero alpha are due to sampling variation and not human skill. The authors show that many widely followed benchmark indices with publicly traded components-in our case, the S&P 500-in the long run will not have any alpha left in their components, thus making it impossible to generate alpha consistently by taking long-only positions in the constituents of that index. In other words alpha is illusive and not elusive unless an investor deviates from provisions of CAPM [1-5]. An implication of this result, which is demonstrated with real-world cases, is that alpha can indeed be generated by taking both long and short positions in index's components, actively trading during the investment horizon, or trading assets that are not components of the index. CAPM also asserts that investors will only be compensated for taking systemic risk manifested in the market portfolio as all other company-specific or asset-specific risk can be diversified away. This statement runs counter to the experience of market practitioners with a case-in-point of mergers. For example, on November 2, 2009 Berkshire Hathaway (BRK) announced their plans to acquire the remaining 77.4% of Burlington Northern Santa Fe Corp (BNI) not already owned by BRK for \$100/share in cash and stock [6]. This deal price represented a 32% premium to BNI's November 2 closing price. The merger, which Buffett deemed "a huge bet...on the railroad industry", caused Burlington peers Union Pacific Corp (UNP), CSX Corp (CSX), and Norfolk Southern Corp (NSC) to gain an average of 6.8% the following day, November 3. How could these one-day moves not be related to the specific business of these companies and their railroad sector? The paper also argues that the concept of beta as a measure of the non-diversifiable risk of a portfolio is, like alpha, of dubious practical use and value [7]. It is common knowledge that beta is the slope of the line when regressing the returns of an asset on that of the market less the risk-free rate. In performing these regressions one crucial piece of information is often not mentioned in financial literature, namely the  $R^2$ -i.e. the predictive power or usefulness-of the regression. Said differently, what percentage of the return of the "asset risk-free rate" can be explained by the return of the "market return risk-free rate"? This important question is rarely asked and  $R^2$  is rarely examined

[8]. The authors show that  $R^2$  for most stocks in S&P 500 is not high enough to bequest on the regression for the resulting beta to represent a significant predictive power. The authors present the argument that it is reasonable to assume investors are indeed compensated for taking non-systemic risk and beta is useful in quantifying the level of this compensation. Accordingly, the authors propose a reformulation of the equation where alpha is zero by definition and beta is used to measure the reward for assuming company-specific or asset-specific risks. This reformulation avoids the use of "risk-free", a key concept of CAPM that is rarely discussed yet remains ambiguous and never fully defined. For example, does "risk-free" mean free of market risk (i.e. changing rates), credit risk (i.e. no defaults/downgrades), liquidity risk? Is it reasonable to assume that rates will not change during an investment horizon one year or longer? In short, illusive alpha and useless beta are a case of mathematical elegance with questionable market relevance and the proposed reformulation of these concepts brings them closer to the experience of practitioners and market realities.

### The Illusive Alpha

In this section the authors argue that in the long run the alpha of an asset that is a component of an index composed of tradable assets such as the S&P 500 will be zero. In summary, the concept of alpha and beta can be applied to any two tradable assets, with CAPM representing a special case with an asset and an index that includes the asset. The authors demonstrate that that alpha of any asset relative to another cannot be negative in the long run, otherwise it would be possible to construct a portfolio with a risk/reward exceeding that of the asset, forcing down the price of the asset, which would boost the return of the asset and eliminate negative alpha. This result is used to show that the very definition of CAPM renders the alpha of an asset that is part of

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an index zero [9,10]. Given any two tradable assets A and B and risk-free rate  $f$ , one can formally regress the return of asset A ex-risk-free rate on return of B ex-risk-free rate and determine the alpha and beta (intercept and slope) of this regression. In fact CAPM is the special case where asset B is the “market” namely

$$R_A - f = \alpha + \beta \times (R_M - f) \quad (1)$$

For the case of an index like the S&P 500, the theory asserts that  $\alpha$  is zero because all asset specific risks can be diversified and investors will only be compensated for that portion of risk of the asset that is systemic and related to the market. This risk is non-diversifiable and is determined by  $\beta$ . There are many instances that are inconsistent with this assertion. For example, consider the hypothetical case where an investor named Jack owned shares of General Motors (GM) with beta 1.75 and his wife Jill owned shares of Ford (F) with beta 1.5. The S&P 500 served as the market portfolio and the risk-free rate was 0%. Jack put his blind trust into CAPM and assumed any company-specific risk was diversified away and Jill and his returns were entirely related to the movement of the S&P 500 [11,12]. While Jack would brag of his financial acumen and mathematical prowess, Jill’s common sense knew CAPM did not reflect the reality of the market. It was no surprise to Jill when Jack suffered dearly from GM’s 2009 bankruptcy as Ford continued to operate.

Returning to the mathematics underlying CAPM [13], the regression of  $R_A$  on  $R_M$  is carried out mechanically without enough attention paid to the underlying assumptions associated with regression analysis. Specifically, in a simple linear regression

$$Y = \alpha + \beta \times X + e$$

where  $X$  is the predictor,  $Y$  is the response, and  $e$  is the error. Errors are assumed to be independent and identically distributed (iid) random variables with mean 0 and standard deviation  $\sigma$ . In a simple linear regression the response  $Y$  is not assumed to be part of the predictor  $X$ , however under CAPM asset A is a component of the market portfolio M, the predictor in CAPM equation. Of course there are econometric models consisting of a set of linear equations where a response variable in one equation can be a predictor in another. However, in a simple linear regression the relation between the predictor and the response is through the regression equation. Under CAPM the market portfolio does include the asset A and this very fact will be shown to result in a equal to 0.

The following simple, yet general, approach illustrates the salient points of the argument that alpha should be zero by definition. First it is shown that alpha (the intercept) of regressing the return of a tradable asset says A on another tradable asset say B cannot be consistently negative. If this were true the risk adjusted return of asset B will be strictly superior to that of asset A and the price of asset A will adjust to eliminate this anomaly. It should be pointed out that if asset B is the “market” the regression is the CAPM formulation.

First, refer to (1) and suppose  $\alpha < 0$ . Next, the risk (volatility) of asset A is calculated and is compared with that of B. For a simple linear regression

$$Y_i = \alpha + \beta \times X_i + e_i$$

Let  $E[X] = \sum X_i / n$ ,  $E[Y] = \sum Y_i / n$  where  $n$  is the number of observations and  $Y_i = \alpha + \beta \times X_i$ ,

$$\text{then } \sum (Y_i - E[Y])^2 = \sum (Y_i - \alpha - \beta \times X_i)^2 + \sum e_i^2 \quad (2)$$

In words, the total sum of squares equals the sum of squares due to regression plus the sum of squares about the regression because

$$E[Y] = \alpha + \beta \times E[X] \text{ or } \sum (Y_i - E[Y])^2 = \beta^2 \times \sum (X_i - E[X])^2$$

Replacing the right side of equation (2) with the formula above and dividing by  $n$  results in

$$\sum (Y_i - E[Y])^2 / n = \beta^2 \times \sum (X_i - E[X])^2 / n + \sum e_i^2 / n$$

In other words, (realized volatility of asset A)<sup>2</sup> =  $\beta^2 \times$  (realized volatility of market)<sup>2</sup> +  $\sum e_i^2 / n$ .

Since  $\sum e_i^2 \geq 0$ , hence

realized volatility of asset A >  $|\beta| \times$  realized volatility of asset B  
risk adjusted return of asset A = expected return of A / volatility of A

Substituting from the above we get

$$(R_A - f) / \text{Vol}(A) = [\alpha + \beta \times (R_B - f)] / [\sqrt{(\beta^2 \text{Vol}(B)^2 + \sum e_i^2 / n)}]$$

If  $\alpha < 0$  then

risk adjusted return of asset A =

$$(R_A - f) / \text{Vol}(A) < \beta \times (R_B - f) / [\sqrt{(\beta^2 \text{Vol}(B)^2 + \sum e_i^2 / n)}] < \beta \times (R_B - f) / \beta \text{Vol}(B)$$

= risk adjusted return of asset B

In particular if B is the market portfolio M then  $R_A - f = \alpha + \beta \times (R_M - f)$ , hence  $\alpha \geq 0$ .

With the above result it is an easy step to show that alpha is zero by definition if the asset is part of the index. This analysis will also show that one way to generate alpha is to trade an asset that is not part of the index. The approach is to reconstruct the index as the weighted sum of asset A and asset B where asset B is the index without asset A. Because the index and reconstructed index are the same they should have identical returns and this will be possible if either  $\alpha_A = \alpha_B = 0$  or the weight of A in the index is zero, in other words it is not part of the index. Consider asset A and the market portfolio M. For illustration, assume A to be Apple Inc. (AAPL) and M to be the S&P 500 of which AAPL is a component. We assume S to be the S&P 500 without AAPL as a component (i.e. the “S&P 499”). Let  $R_A$ ,  $R_S$ ,  $R_M$  be the returns of A, S and M respectively. If  $f$  is the risk free rate, according to CAPM

$$R_A - f = \alpha_A + \beta_A \times (R_M - f)$$

$$R_S - f = \alpha_S + \beta_S \times (R_M - f)$$

And based on the above result  $\alpha_A \geq 0$  and  $\alpha_S \geq 0$ .

Let  $w$  be the relative weight of A in the index (typically relative market capitalization). Then

$$R_M = w \times R_A + (1 - w) \times R_S$$

Substituting for  $R_A$  and  $R_S$  above

$$R_M = w \times (\alpha_A + \beta_A \times R_M) + (1 - w) \times (\alpha_S + \beta_S \times R_M), \text{ or}$$

$$R_M = [w \times \alpha_A + (1 - w) \times \alpha_S] + [w \times \beta_A + (1 - w) \times \beta_S] \times R_M$$

In order for this equality to hold for all values of  $R_M$  the component not related to  $R_M$  should be zero, namely

$$w \times \alpha_A + (1 - w) \times \alpha_S = 0$$

Since it was shown that  $\alpha_A \geq 0$  and  $\alpha_S \geq 0$ , the above equation will hold if at least one of the following conditions is satisfied:

1.  $\alpha_A = \alpha_S = 0$ , i.e. there is no alpha; or
2.  $w = 0$  which is tantamount to A having a zero weight in the index (i.e. not part of the market portfolio).

Consequently under the assumptions of CAPM, the alpha of an asset that is part of the index will be zero by definition. No wonder investors have been unable to consistently produce this illusive alpha!

### Alphantom, the Phantom Alpha

While it has been shown that CAPM inherently results in an alpha of zero, luckily for mutual funds and portfolio managers it is still possible to generate alpha (which is termed alphantom) because CAPM is based on passive investment over a time horizon. As a result, it is possible to “generate” alpha by deviating from CAPM as follows:

- Selecting assets that are not part of the benchmark or index.
- Using a long/short-not long only-strategy.
- Actively trading during the investment horizon.

The following are some examples of actual claims of alpha generation that really are due to deviations from CAPM criteria:

#### Quaker global tactical allocation fund

Described as a “multi-cap global equity portfolio investing in a diversified portfolio of U.S and American Depository Receipts of foreign companies”, the fund would appear to be appropriately benchmarked against the MSCI World Index. A closer look at the investment prospectus, however, reveals that the fund can “utilize options and futures contracts to hedge the portfolio”. Given the MSCI World Index tracks equity market performance, but not options or futures contracts, the fund holds assets that are not part of the benchmark. The fund is also fairly active with an annual turnover rate of more than 185%. But most interesting was a single line on the fund’s brochure (emphasis ours): “short portfolio up to 25% portfolio *in an effort to generate alpha.*” Is our fund manager in on the secret? (Table 1).

#### Vanguard market neutral fund

This moderately active fund with an annual turnover of 91% has a “goal to “neutralize,” or limit, the effect of stock market movement on returns...using long- and short-selling strategies.” It’s possible to overlook these two deviations (actively trading and long/short strategy) but the benchmark deviation had us scratching our heads. While outperforming Citigroup’s 3-Month US T-Bill Index is a worthy objective, benchmarking a portfolio of equities to an index tracking the performance of short-term U.S. government debt instruments would only generate alphantom (Table 2).

#### Hennessy Gas Utility Index Fund

Looking at these two funds side-by-side serves as a good example of the underlying causes of alphantom. The Gas Fund “seeks to track the total return of the American Gas Association Stock Index.” It invests at least 85% of its assets in long equity positions for each component in the index in a similar proportion to the index. With an appropriate benchmark and a low annual turnover rate of 20%, it is no surprise that the fund closely mirrors the performance of the benchmark. The alphantom is likely explained by some trading activity and a small percentage of cash/cash-equivalent holdings not found in the benchmark (Table 3A).

Average Annual Returns as of 9/30/2012	1 YEAR	3 YEAR	Since inception
Quaker Global Tactical Allocation Fund	14.08%	4.93%	-6.90%
Benchmark: MSCI World Index	21.59%	7.47%	-1.01%
Alphantom	-7.51%	-2.54%	-5.89%

Table 1: Quaker global tactical allocation fund.

Average Annual Returns (as of 12/31/2012)	1 YEAR	3YEAR	5 YEAR
Vanguard Market Neutral Fund	-1.50%	1.67%	-3.09%
Benchmark: citigroup 3-month US T-Bill Index	0.07%	0.08%	0.44%
Alphantom	-1.57%	1.59%	-3.53%

Table 2: Vanguard market neutral fund.

Average Annual Returns (as of 12/31/2012)	1 YEAR	3YEAR	5 YEAR
Hennessy Gas Utility Index Fund	7.45%	14.76%	6.08%
Benchmark: AGA Stock Index	6.77%	13.92%	5.20%
Alphantom	0.68%	0.84%	0.88%

Table 3 A: Hennessy Gas Utility Index.

### Hennessy Technology Fund

On the other hand, consider the Technology Fund where 80% of the net assets are in securities found in the NASDAQ Composite Index benchmark. While this percentage is similar to the Gas Fund, the Technology Fund invests in more than common stock-it also invests in preferred stocks, warrants and securities convertible into common stock like convertible bonds and convertible preferred stock. Additionally, the fund has a high annual turnover rate of 140%. It is no surprise then that the returns of the Technology Fund generate significant alphantom (negative alphantom in this case) (Table 3B).

### Berkshire Hathaway

Of all investment managers hailed for generating alpha, Warren Buffet is the most legendary. Having consistently outperformed the S&P 500, his title as the “Oracle of Omaha” is well deserved-or is it? In its 2011 annual report, Berkshire Hathaway compares its per-share book value against the return of the S&P 500 including dividends. Given Mr. Buffet’s mantra of long-term value investing we can assume his style is passive and long only. This leaves us with the source of his alphantom-the S&P 500 benchmark. Sure Berkshire Hathaway holds considerable shares of common stock in the American blue chips like American Express, Coca-Cola and Johnson & Johnson. However, we cannot ignore shareholdings in European and Asian companies not in the S&P 500 such as POSCO (South Korea), Munich Re (Germany), Tesco (United Kingdom) and Sanofi (France). This is only the tip of the iceberg. Surely the tens of private companies Berkshire owns are not in the S&P 500, nor the many insurance operations that lead the balance sheet. And we cannot forget the various holdings in preferred and convertible securities, warrants, bonds and the self-proclaimed “financial weapons of mass destruction” derivatives. While we admire Warren, we have to reclassify his alpha as phantom alpha (Figure 1).

### The Useless Beta

Like alpha, beta has become ubiquitous and is found in the summary details for equities alongside data points such as bid/ask, market capitalization and 52-week range. Beta is supposed to indicate the riskiness of an asset relative to a benchmark; for American equities this is typically the S&P 500. Unfortunately, when discussing beta no mention is made of the  $R^2$  of the regression that is used to determine beta. In regression analysis special attention is always paid to  $R^2$ , the

percent of the variation of Y that is explainable by the regression.  $R^2$  represents the predictive power of regression-a small  $R^2$  and the model has little predictive power. A simple example illustrates the need to consider  $R^2$  prior to accepting and using the results of regression. Assume  $X_1 \dots X_{n-1}$  are random numbers representing  $n$  assets and define an index as  $R_M = (\sum X_i) / n$ . Now suppose  $X_1$  is mechanically regressed on  $R_M$ . The result is a non-zero intercept and slope. Yet this regression as a predictor is useless because of the small  $R^2$  of the regression. The following are a number of results from a sample size of 200 where the number of assets  $n=50$ : (Table 4a). Besides the lack of strong predictive power due to a low  $R^2$ , the resulting  $\beta$  for various stocks in S&P 500's is typically close to 1 for major components of this index irrespective of their sector. For example, at the writing of this article the  $\beta$  of a number of major components of S&P was observed as: (Table 4b) The economics

Average Annual Retruns (as of 12/31/2012)	1 YEAR	3YEAR	5 YEAR
Hennessy Technology Fund	8.67%	3.01%	-0.67%
Benchmark: NASDAQ Composite Index	17.73%	11.32%	3.78%
Alphantom	-9.06%	-8.31%	-4.45%

Table 3 B: Hennessy technology fund.

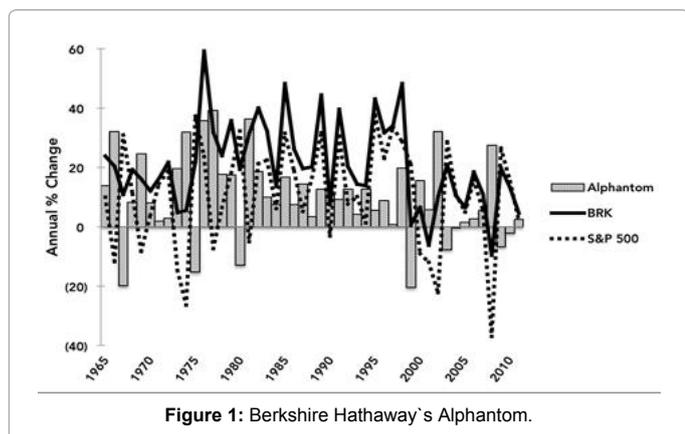


Figure 1: Berkshire Hathaway's Alphantom.

$\alpha$	$\beta$	$R^2$
0.13	0.70	0.01
-0.01	1.08	0.02
0.33	0.34	0.00

Table 4a: Use of beta in regression.

Company	Ticker	$\beta$	Industry
Microsoft	MSFT	0.77	Technology
EXXON-Mobil	XOM	0.85	Energy
Pfizer	PFE	0.81	Health care

Source: Yahoo! Finance

Table 4b: The  $\beta$  of a number of major components of S&P has been shown.

and sensitivity of these companies are unique to the business cycle and hence their risks are completely different which is not reflected in their  $\beta$ . This shortcoming is due to mechanically regressing one variable on another and as will be shown below the  $\beta$  of major components of S&P tend to be close to 1. Usually companies with relatively low market cap will have a  $\beta$  much greater than 1. For a diversified economy where no single company or sector is dominant, the correlation of the returns of a company with that of the economy rarely exceeds 80%. Let's consider  $R^2$  of the regression for the components of the S&P 500. As is shown in

Exhibit V, the predicative power of CAPM for stocks in the American economy that are components of the S&P 500 index is between 40% to 50% i.e., the  $R^2$  of the regression (Figure 2). The following relationships hold for the simple linear regression  $R^2 = [\text{Correlation}(R_A, R_M)]^2$  and  $\beta_A = \text{Correlation}(R_A, R_M) \times (\sigma_A / \sigma_M)$ , where  $\sigma_A$  and  $\sigma_M$  are the volatility of asset A and that of the market M respectively. In general, if an asset is a significant component of the market it moves in tandem with market. It follows that this asset's volatility cannot be significantly different from the volatility of the market. In other words  $(\sigma_A / \sigma_M)$  is close to one. Consequently, the  $\beta$  of the major components of S&P are generally close to one and often not exceeding it by much since  $|\text{Correlation}(R_A, R_M)| \leq 1$ ; i.e. they behave like the market irrespective of their sector. Further, stocks with high betas tend to have high volatility. According to the aforementioned arguments these stocks should typically have both a relatively low weights (i.e. market capitalization) relative to the other components of the index and a relatively small contribution to the market. Exhibit VI presents this high cap/low beta and low cap/high beta observation for components of the S&P 500 (Figure 3).

### An Alternative Formulation

A natural question arises at this time that I the alpha of a stock should be zero, then why not fit a regression with zero intercept to the data. One reason might be that such a forced model might lack some desirable properties of standard regression. Specifically according to CAPM:

$$R_A - f = \alpha_A + \beta_A \times (R_M - f) + e$$

It can be shown that

$$\beta_A = \text{cov}(R_A, R_M) / \text{var}(R_M) = \text{corr}(R_A, R_M) \times (\sigma_A / \sigma_M) \quad (3)$$

$$\alpha_A = (\sum a_i) / n - \beta_A \times ((\sum m_i) / n)$$

On the other hand let us follow the assertions of CAPM and assume  $\alpha_A = 0$ . The "right" model in this case should have no intercept, i.e.  $R_A = \beta_A \times R_M + e$ . If this model is chosen the following equation might not hold as the errors may have a bias:

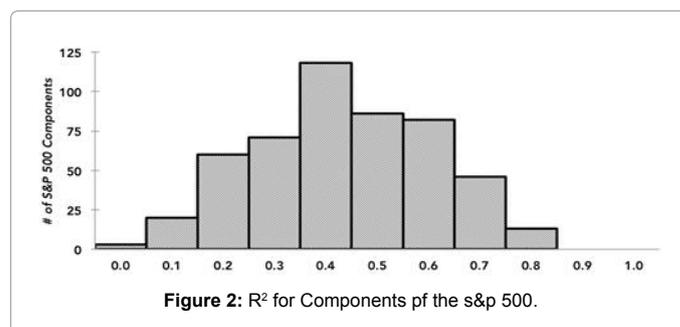
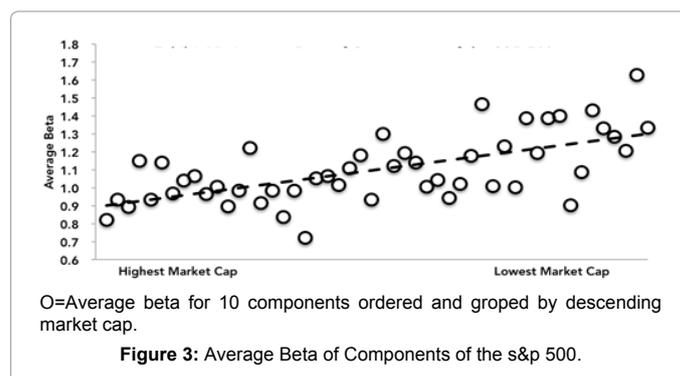


Figure 2:  $R^2$  for Components of the s&p 500.



O=Average beta for 10 components ordered and groped by descending market cap.

Figure 3: Average Beta of Components of the s&p 500.

total sum of squares=sum of squares due to regression+sum of square about regression.

Furthermore, the expression for  $\beta$  and  $\alpha$  will be slightly different from the above. Namely if  $a_i, m_i, i=1,2,\dots,n$ , are observations of the return of the asset  $R_A$  and the market  $R_M$ , when using a regression without intercept the  $\beta$  will be

$$\beta_A = (\sum a_i \times m_i) / (\sum m_i^2) = [\text{cov}(R_A, R_M) + \sum a_i \times \sum m_i] / [\text{var}(R_M) + ((\sum m_i) / n)^2]$$

Comparing the above formula with equation (3) shows a difference of inclusion of the expected values  $\sum a_i \times \sum m_i$  in the numerator and  $((\sum m_i) / n)^2$  in the denominator. Consequently, forcing a zero intercept can become problematical and practitioners using different time horizons could arrive at quite disparate  $\beta$ 's. Further by forcing the intercept to be zero, higher interest rates reduce the  $\beta$  of the stock, i.e. the stock should become less risky in a rising rate environment which is contrary to real life experience. It should also be pointed out that the term "risk free" is often used without much elaboration namely, does "risk free" mean free of market risk (i.e. changing rates)? This clearly does not hold if the investment horizon is one year or longer. Does it refer to the absence of credit risk (i.e. no defaults/downgrades)? Or does it refer to the absence of liquidity risk (i.e. unlimited borrow/lend capacity at that rate)? The following reformulation of the relationship between the return of an asset and the market results in a model that ensures  $\alpha$  will be zero and excludes the risk free rate. The reformulation is much more practical and in line with what is observed in the market, where practitioners believe that you are indeed compensated for assuming company and asset specific risk. Let  $Y_A = R_A - \sum R_A / n$  be the excess return of an asset compared to its average return. Similarly, for a benchmark the excess return relative to its average return is denoted by  $X_B = R_B - \sum R_B / n$ . This ensures that  $E[X] = E[Y] = 0$  and since in the linear regression  $E[Y] = \alpha + \beta \times E[X]$  the formula ensures that  $\alpha$  will be zero without having to resort to the market efficiency assumption and the claim that non-systemic risks can be hedged away. Regressing  $Y_A$  on  $X_B, Y_A = \beta_B \times X_B + e$ . Given that in this case  $\alpha$  is zero by construction we have

$$(R_A - \sum R_A / n) = \beta_B \times (R_B - \sum R_B / n) + e$$

The above can be rewritten as

$$\pi_A = (R_A - \beta_B \times R_B) = (\sum R_A / n - \beta_B \times \sum R_B / n) + e$$

Hence the return  $R_A$  consists of two components:  $\beta_B \times R_B$ , the part of the return due to the benchmark, and the difference  $(R_A - \beta_B \times R_B)$  is the return from assuming security specific risk.

The investor is actually compensated for taking security specific risk that is non-diversifiable. By taking this risk, the investor should expect an incremental return of  $(\sum R_A / n - \beta_B \times \sum R_B / n)$  with risk of  $\sigma_e$  (the standard deviation of the errors of the regression). By looking at the Sharpe Ratio of this excess return  $(\sum R_A / n - \beta_B \times \sum R_B / n) / \sigma_e$ , the investor can evaluate the risk/reward of assuming security specific risk.

### Benchmark Beta, Pure Return (Pi), Pure Security Risk (Kappa)

This formulation is mathematically coherent and closer to what is actually observed and practiced in the market. With a similar focus on market reality, the following concepts can further clarify the risk and return of an investment relative to a benchmark. This proposal has both mathematical coherence and market relevance. It is also a practical solution that can be consumed by ordinary investors. Synthesizing the formulas above, the following metrics generalize the current concept of

beta to handle different benchmarks and the cases where the underlying asset is not part of the benchmark:

- Benchmark Beta ( $\beta_B$ ) is the expected excess return of the security relative to its average return due to the excess return of the benchmark relative to the benchmark's average return.
- Pure Return ( $\pi$ ) is the security-specific expected excess return unrelated to the benchmark.
- Pure Security Risk ( $\kappa$ ) is the standard deviation of the Pure Return.

Consider the following application of Benchmark Beta, Pure Return and Pure Security Risk: (Table 5).

TICKER	NAME	BENCH mark Beta ( $\beta_B$ )	Pure return( $\eta$ ), %Daily	Pure return( $\eta$ ), % ANNUALLY	Pure sec. Risk( $\kappa$ ), %DAILY	Pure sec. Risk( $\kappa$ ), %Annual
MMM	3M Co	0.85	0.012	3.1	0.982	15.6
MO	Altria Group Inc	0.51	0.028	7	1.163	18.5
AMZN	Amazon.com Inc	1.07	0.103	26	2.3	36.5
AZO	Auto Zone Inc	0.62	0.096	24.2	1.437	22.8
BA	Boeing Co/The	1	0	0.1	1.455	23.1
CLX	Clorox Co/The	0.46	0.013	3.4	1.096	17.4
KO	Coca-Cola Co/The	0.56	0.016	4.1	1.064	16.9
CAG	ConAgraFoods INC	0.43	0.023	5.8	1.22	19.4
ED	Consolidated Edison Inc	0.48	0.012	3.1	0.909	14.4
DE	Deers & Co	1.26	0.017	4.4	1.788	28.4
DOW	Dow Chemical Co/The	1.33	0.012	3.1	2.02	32.1
FDX	FedEX corp	1.06	0.018	4.6	1.557	24.7
GOOG	Google Inc	0.91	0.014	3.6	1.622	25.7
HSY	Hersheyco/The	0.51	0.005	13.8	1.315	20.9
IR	Ingersoll-Rand PLC	1.3	0.027	6.8	1.797	28.5
ICE	International Exchange Inc	1.52	0.013	3.2	2.651	42.1
JPM	Jp Morgan Chase &Co	1.67	0.047	12	2.427	38.5
MSFT	Microsoft Corp	0.92	0.014	3.5	1.349	21.4
TAP	Molson Coors Brewing Co	0.59	0.005	1.4	1.464	23.2
NKE	NIKE Inc	0.9	0.048	12.1	1.52	24.1
NOC	Northrop Grumman Corp	0.75	0.006	1.5	1.309	20.8
DGX	Quest Diagnostics Inc	0.61	0.015	3.8	1.352	21.5
RTN	Raytheon Co	0.67	0.001	0.3	1.224	19.4
CRM	Salesforce.com Inc	1.25	0.116	29.3	2.474	39.3
STJ	St Jude Medical Inc	0.75	0.003	0.7	1.676	26.6
SBUX	Star bucks corp	1.07	0.1	25.2	1.891	30
STI	sun trust bank Inc	1.78	0.018	4.5	3.448	54.7
YUM	Yum!Brands Inc	0.87	0.053	13.4	1.423	22.6

Data: 01/01/2008-12/31/2012

Table 5: Bench mark beta, pure return and Kappa for select American equities.

## Conclusion

While mathematically elegant, CAPM's concepts of alpha and beta diverge from observed market realities. We have shown that the elusiveness of generating alpha is not due to market efficiency. Instead it is alpha's very definition renders that it illusive. Furthermore, we have shown that the concept of beta should not be used in isolation without attention to the  $R^2$  that measures the effectiveness of beta as a predictor. We have proposed a reformulation of the problem that aligns these concepts with what investors experience in the market and have introduced the concepts of Pure Return ( $\pi$ ) and Pure Security Risk ( $\kappa$ ).

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