The Homotopy Analysis Approach and Homotopy Renormalization Method's Difference in Fluid Mechanics

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Introduction

An analytical approximation method for highly nonlinear problems, the homotopy analysis method (HAM), is contrasted with the homotopy renormalization method (HTR) in this paper. The Von Kármán swirling viscous flow, the Blasius viscous flow, the magnetohydrodynamic flow and the free-convective boundary-layer flow are four fluid mechanics problems that are compared in depth. At the same order, it is discovered that the HAM approximations are significantly more accurate than the HTR approximations. Additionally, while the HTR struggles in such situations, higher-order approximations with smaller errors can be fairly easily obtained by the HAM. All of these substantiate the HAM's superiority in fluid mechanics for highly nonlinear problems. In a variety of scientific and engineering fields, there are numerous nonlinear phenomena. Examination of such peculiarities frequently utilizes bother procedures, which depend on little (or enormous) actual boundaries called annoyance amounts in the overseeing conditions or limit conditions. However, the perturbation quantity is not present in every nonlinear problem. Based on the basic idea of homotopy in topology, the homotopy analysis method (HAM) is an analytical approach to nonlinear problems.

Description

We have a powerful and effective method for obtaining analytical solutions thanks to this approach. The following aspects represent the majority of the HAM's benefits. First, in contrast to perturbation methods, the HAM holds true even when the nonlinear problem does not have any small or large parameters. Second, it is simple to modify so that the series solution's convergence can be controlled. Thirdly, it grants us a great deal of discretion when selecting the initial approximation, auxiliary linear operators and base functions. As a result, it is able to efficiently provide analytical solutions that are highly precise. The HAM has been effectively applied to various nonlinear issues in the areas of science, designing and money specifically, a few huge discoveries have been accounted for in the field of liquid mechanics. For instance, Liao successfully solved the water-wave equations by finding multiple steady-state nearly resonant gravity waves in deep water using the HAM method [1]. Te-amplitude steady-state wave groups with multiple near-resonances exist in finite water depths as a result of Liao's work. In addition, Yang and Li used the HAM to discover analytical solutions for two primary waves traveling at any included angle. In addition, Zhong and Liao were able to successfully obtain the analytical series solution for the first time for the limiting Stokes wave at the extreme wave height at any water depth. The validity of applying the HAM to complex nonlinear problems is demonstrated by these works [2].

For asymptotic solutions to nonlinear differential equations, Liu proposed

Taylor renormalization (TR), a renormalization technique based on the Taylor expansion. Liu proposed the homotopy renormalization method (HTR), which combines homotopy deformation with the TR method, so that the TR method could be used to solve a wider range of nonlinear equations. The Blasius viscous flow, magnetohydrodynamic flow, free-convective boundary-layer flow and Von Kármán swirling viscous flow problems have all been solved analytically with approximate solutions using the HTR. The HTR provides approximate solutions to nonlinear equations, whereas HTR approximations typically have limited accuracy and are typically of the first order. Due to the intricate calculations required to deal with the renormalization equation, it is difficult to obtain higher-order HTR approximations. Additionally, the renormalization equation can be used to approximate the HTR's closed equations system. As a result, for nonlinear equations, the HTR typically offers straightforward low-order approximations. The HTR has trouble providing precise solutions, especially for problems with high nonlinearities [3].

In fluid mechanics, the HAM is a well-known analytical tool for solving nonlinear problems. Nonlinear fluid problems have also been solved using the newly proposed HTR method, which is based on the homotopy theory used in the HAM. In any case, these works didn't contrast the HTR and the HAM for the viability and accommodation to get logical arrangements. As a result, one might be curious about how the HTR and HAM frameworks differ and whether the new approach performs better than the HAM when dealing with nonlinear fluid issues. To demonstrate the advantages of the HAM over the HTR in terms of obtaining analytical approximate solutions for nonlinear fluid problems, we compare the HAM and HTR using theoretical frameworks and four well-known fluid mechanics nonlinear problems [4,5].

Conclusion

In this paper, we compare and contrast the advantages of the HAM and the HTR for obtaining analytical approximate solutions to problems involving nonlinear fluids. We show that the HAM is more convenient and efficient than the HTR by comparing the theoretical frameworks and using four well-known flow problems as examples. Analytic approximations can be obtained without having to construct and solve the HTR's renormalization equations system thanks to the HAM's straightforward procedure. In addition, the HAM's approximate results are significantly more accurate than the HTR's at the same order. Additionally, the HTR is difficult to use for higher-order approximations with smaller errors, whereas the HAM is easy to use for symbolic computation software like Mathematica. The straightforward, exact and logical arrangements acquired by the HAM concur well with the relating mathematical outcomes. The HAM also provides significant physical quantities with high accuracy, such as the skin friction, heat transfer and pressure difference. The HAM's superiority and potential for use in dealing with highly nonlinear fluid issues are demonstrated by all of these factors. This paper has looked at the hypothetical structures of the homotopy investigation strategy (HAM) and homotopy renormalization technique (HTR) and the rough insightful arrangements given by the HAM and HTR for four notable nonlinear liquid issues. We can more easily obtain highly accurate analytical approximations without having to construct and deal with the HTR's renormalization equations system by following the straightforward HAM procedure.

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Conflict of Interest

None.

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