

The Generalized Triple Difference Lacunary Statistical on Γ^3 Over P-Metric Spaces Defined by Musielak Orlicz Function

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Abstract

We introduce the generalized triple sequence spaces of entire difference lacunary statistical convergence and discuss general topological properties also inclusion theorems are with respect to a sequence of Musielak-Orlicz function.

Keywords: Analytic sequence; Triple sequences; Difference sequence; Γ^3 space; Musielak-Orlicz function; Lacunary sequence; Statistical convergence

Introduction

A triple sequence (real or complex) can be defined as a function $x: \mathbb{N} \times \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{R}$ (\mathbb{C}) where \mathbb{N}, \mathbb{R} and \mathbb{C} denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by Sahiner [1,2], Esi [3-5], Datta [6], Subramanian [7], Debnath [8] and many others.

A triple sequence $x=(x_{mnk})$ is said to be triple analytic if

$$\sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The space of all triple analytic sequences are usually denoted by Λ^3 . A triple sequence $x=(x_{mnk})$ is called triple entire sequence if

$$|x_{mnk}|^{\frac{1}{m+n+k}} \rightarrow 0 \text{ as } m,n,k \rightarrow \infty.$$

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [9] as follows

$$Z(\Delta) = \{x = (x_k) \in w : (\Delta x_k) \in Z\}$$

For $Z = c, c_0$ and ℓ_∞ where $\Delta x_k = x_k - x_{k+1}$ for all $k \in \mathbb{N}$

The difference triple sequence space was introduced by Debnath et al. (see [8]) and is defined as

$$\Delta x_{mnk} = x_{mnk} - x_{m,n+1,k} - x_{m,n,k+1} + x_{m,n+1,k+1} - x_{m+1,n,k} + x_{m+1,n+1,k} + x_{m+1,n,k+1} - x_{m+1,n+1,k+1}$$

and $\Delta^0 x_{mnk} = \langle x_{mnk} \rangle$.

Definitions and Preliminaries

Throughout the article $w^3, \Gamma^3(\Delta), \Lambda^3(\Delta)$ denote the spaces of all, triple entire difference sequence spaces and triple analytic difference sequence spaces respectively.

Subramanian introduced by a triple entire sequence spaces, triple analytic sequences spaces and triple gai sequence spaces [7]. The triple sequence spaces of $\Gamma^3(\Delta), \Lambda^3(\Delta)$ are defined as follows:

$$\Gamma^3(\Delta) = \{x \in w^3 : |\Delta x_{mnk}|^{\frac{1}{m+n+k}} \rightarrow 0 \text{ as } m,n,k \rightarrow \infty\},$$

$$\Lambda^3(\Delta) = \{x \in w^3 : \sup_{m,n,k} |\Delta x_{mnk}|^{\frac{1}{m+n+k}} < \infty\}.$$

1. Definition

An Orlicz function is a function $M: [0, \infty) \rightarrow [0, \infty)$ which is continuous, non-decreasing and convex with $M(0) = M(x) > 0$ for $M(x) > 0$ $x > 0$ and $M(x) \rightarrow \infty$ as $x \rightarrow \infty$ [10]. If convexity of Orlicz function M is replaced by $M(x+y) \leq M(x) + M(y)$ then this function is called modulus function. $M: [0, \infty) \rightarrow [0, \infty)$

Lindenstrauss and Tzafriri [11] used the idea of Orlicz function to construct Orlicz sequence space. A sequence $g=(g_{mnk})$ defined by

$$g_{mnk}(v) = \sup \{|v|u - (f_{mnk})(u) : u \geq 0\}, m,n,k = 1, 2, \dots$$

is called the complementary function of a Musielak-Orlicz function f . For a given Musielak-Orlicz function f the Musielak-Orlicz sequence space t_f is defined as follows [12]

$$t_f = \{x \in w^3 : I_f(|x_{mnk}|)^{\frac{1}{m+n+k}} \rightarrow 0 \text{ as } m,n,k \rightarrow \infty\},$$

Where I_f is a convex modular defined by

$$I_f(x) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} f_{mnk}(|x_{mnk}|)^{\frac{1}{m+n+k}}, x = (x_{mnk}) \in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} f_{mnk} \left(\frac{|x_{mnk}|^{\frac{1}{m+n+k}}}{mnk} \right)$$

is an extended real number.

2. Definition

Let $n \in \mathbb{N}$ and X be a real vector space of dimension w where nm . A real valued function $d_p(x_1, \dots, x_n) = \|(d_1(x_1, 0), \dots, d_n(x_n, 0))\|_p$ on X satisfying the following four conditions:

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(i) $\| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p = 0$ if and only if $d_1(x_1, 0), \dots, d_n(x_n, 0)$ are linearly dependent,

(ii) $\| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p$ is invariant under permutation,

(iii) $\| (\alpha d_1(x_1, 0), \dots, \alpha d_n(x_n, 0)) \|_p = |\alpha| \| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p, \alpha \in \mathbb{R}$

(iv) $d_p((x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)) = (d_x(x_1, x_2, \dots, x_n)^p + d_y(y_1, y_2, \dots, y_n)^p)^{1/p}$ for $1 \leq p < \infty$; (or)

(v) $d((x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)) := \sup \{ d_x(x_1, x_2, \dots, x_n), d_y(y_1, y_2, \dots, y_n) \}$,

For $x_1, x_2, \dots, x_n \in X, y_1, y_2, \dots, y_n \in Y$ is called the p product metric of the Cartesian product of n metric spaces [13].

3. Definition

Let X be a linear metric space. A function $\rho: X \rightarrow \mathbb{R}$ is called paranorm, if

(1) $\rho(x) \geq 0$ for all $x \in X$;

(2) $\rho(-x) = \rho(x)$ for all $x \in X$;

(3) $\rho(x + y + z) \leq \rho(x) + \rho(y) + \rho(z)$, for all $x, y, z \in X$,

(4) If (σ_{mnk}) is a sequence of scalars with $\sigma_{mnk} \rightarrow \sigma$ as $m, n, k \rightarrow \infty$ and $x = (x_{mnk})$ is a sequence of vectors with

$\rho(x_{mnk} - x) \rightarrow 0$ as $m, n, k \rightarrow \infty$ then $\rho(\sigma_{mnk} x_{mnk} - \sigma x) \rightarrow 0$ as $m, n, k \rightarrow \infty$

4. Definition

The triple sequence $\theta_{i, \ell, j} = \{(m, n, k)\}$ is called triple lacunary if there exist three increasing sequences of integers such that

$m_0 = 0, h_i = m_i - m_{i-1} \rightarrow \infty$ as $i \rightarrow \infty$ and

$n_0 = 0, \bar{h}_\ell = n_\ell - n_{\ell-1} \rightarrow \infty$ as $\ell \rightarrow \infty$

$k_0 = 0, \bar{h}_j = k_j - k_{j-1} \rightarrow \infty$ as $j \rightarrow \infty$

Let $m_{i, \ell, j} = m_i n_\ell k_j, h_{i, \ell, j} = \overline{h_i \bar{h}_\ell \bar{h}_j}$, and $\theta_{i, \ell, j}$ is determine by

$I_{i, \ell, j} = \{ (m, n, k) : m_{i-1} < m < m_i \text{ and } n_{\ell-1} < n \leq n_\ell \text{ and } k_{j-1} < k \leq k_j \}$,

$q_k = \frac{m_k}{m_{k-1}}, \bar{q}_\ell = \frac{n_\ell}{n_{\ell-1}}, \bar{q}_j = \frac{k_j}{k_{j-1}}$.

Main Results

The notion of λ -triple entire and triple analytic sequences as follows: Let $\lambda = (\lambda_{mnk})_{m, n, k=0}^\infty$ be a strictly increasing sequences of positive real numbers tending to infinity, that is

$0 < \lambda_{000} < \lambda_{111} < \dots$ and $\lambda_{mnk} \rightarrow \infty$ as $m, n, k \rightarrow \infty$

and said that a sequence $x = (x_{mnk}) \in w^3$

is λ -convergent to 0, called a the λ -limit of x if $\mu_{mnk}(x) \rightarrow 0$ as $m, n, k \rightarrow \infty$ where

$$\begin{aligned} \mu_{mnk}(x) &= \frac{1}{\varphi_{rst}} \sum_{m \in I_{rst}} \sum_{n \in I_{rst}} \sum_{k \in I_{rst}} (\Delta^{m-1} \lambda_{m, n, k}) - (\Delta^{m-1} \lambda_{m, n+1, k}) \\ &- (\Delta^{m-1} \lambda_{m, n, k+1}) + (\Delta^{m-1} \lambda_{m, n+1, k+1}) - (\Delta^{m-1} \lambda_{m+1, n, k}) + (\Delta^{m-1} \lambda_{m+1, n+1, k}) \\ &+ (\Delta^{m-1} \lambda_{m+1, n, k+1}) - (\Delta^{m-1} \lambda_{m+1, n+1, k+1}) \Big| \Delta^{m+1} x_{mnk} \Big|^{1/m+n+k} \end{aligned}$$

The sequence $x = (x_{mnk}) \in w^3$ is λ -triple difference analytic if $\sup_{uvst} |\mu_{mnk}(x)| < \infty$. If $\lim_{uvst} x_{mnk}(x) = 0$ in the ordinary sense of convergence, then

$$\begin{aligned} \lim_{mnk} \frac{1}{\varphi_{rst}} \sum_{m \in I_{rst}} \sum_{n \in I_{rst}} \sum_{k \in I_{rst}} (\Delta^{m-1} \lambda_{m, n, k}) - (\Delta^{m-1} \lambda_{m, n+1, k}) \\ - (\Delta^{m-1} \lambda_{m, n, k+1}) + (\Delta^{m-1} \lambda_{m, n+1, k+1}) - (\Delta^{m-1} \lambda_{m+1, n, k}) + (\Delta^{m-1} \lambda_{m+1, n+1, k}) \\ + (\Delta^{m-1} \lambda_{m+1, n, k+1}) - (\Delta^{m-1} \lambda_{m+1, n+1, k+1}) \Big| \Delta^{m+1} x_{mnk} \Big|^{1/m+n+k} = 0 \end{aligned}$$

This implies that

$$\begin{aligned} \lim_{mnk} |\mu_{mnk}(x) - 0| &= \lim_{mnk} \frac{1}{\varphi_{rst}} \sum_{m \in I_{rst}} \sum_{n \in I_{rst}} \sum_{k \in I_{rst}} (\Delta^{m-1} \lambda_{m, n, k}) \\ &- (\Delta^{m-1} \lambda_{m, n+1, k}) - (\Delta^{m-1} \lambda_{m, n, k+1}) + (\Delta^{m-1} \lambda_{m, n+1, k+1}) - (\Delta^{m-1} \lambda_{m+1, n, k}) \\ &+ (\Delta^{m-1} \lambda_{m+1, n+1, k}) + (\Delta^{m-1} \lambda_{m+1, n, k+1}) - (\Delta^{m-1} \lambda_{m+1, n+1, k+1}) \Big| \Delta^{m+1} x_{mnk} - 0 \Big|^{1/m+n+k} = 0 \end{aligned}$$

which yields that $\lim_{uvst} \mu_{mnk}(x) = 0$ and hence $x = (x_{mnk}) \in w^3$ is λ -convergent to 0. Let I^3 be an admissible ideal of $3^{\mathbb{N} \times \mathbb{N} \times \mathbb{N}}$, θ_{rst} be a triple difference lacunary sequence, $f = f_{mnk}$ be a Musielak-Orlicz function and $(X, \| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p)$ be a p -metric space, $q = (q_{mnk})$ be triple difference analytic sequence of strictly positive real numbers. By $w^3(p-X)$ we denote the space of all sequences defined over

$$\left(X, \| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right)$$

In the present paper we define the following sequence spaces:

$$\begin{aligned} \left[\Gamma_{f\mu}^{3\Delta^m q}, \| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right]_{\theta_{rst}}^{\Gamma^3} &= \\ \left\{ r, s, t \in I_{rst} : \left[f_{mnk} \left(\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right) \right]^{q_{mnk}} \geq \varepsilon \right\} \in I^3 & \\ \left[\Lambda_{f\mu}^{3\Delta^m q}, \| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right]_{\theta_{rst}}^{\Gamma^3} &= \\ \left\{ r, s, t \in I_{rst} : \left[f_{mnk} \left(\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right) \right]^{q_{mnk}} \geq K \right\} \in I^3 & \end{aligned}$$

If we take $f_{mnk}(x) = x$ we get

$$\begin{aligned} \left[\Gamma_{f\mu}^{3\Delta^m q}, \| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right]_{\theta_{rst}}^{\Gamma^3} &= \\ \left\{ r, s, t \in I_{rst} : \left[\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right]^{q_{mnk}} \geq \varepsilon \right\} \in I^3 & \\ \left[\Lambda_{f\mu}^{3\Delta^m q}, \| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right]_{\theta_{rst}}^{\Gamma^3} &= \\ \left\{ r, s, t \in I_{rst} : \left[\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right]^{q_{mnk}} \geq K \right\} \in I^3 & \end{aligned}$$

If we take $q = (q_{mnk}) = 1$ we get

$$\begin{aligned} \left[\Gamma_{f\mu}^{3\Delta^m}, \| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right]_{\theta_{rst}}^{\Gamma^3} &= \\ \left\{ r, s, t \in I_{rst} : \left[f_{mnk} \left(\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right) \right] \geq \varepsilon \right\} \in I^3 & \\ \left[\Lambda_{f\mu}^{3\Delta^m}, \| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right]_{\theta_{rst}}^{\Gamma^3} &= \\ \left\{ r, s, t \in I_{rst} : \left[f_{mnk} \left(\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right) \right] \geq K \right\} \in I^3 & \end{aligned}$$

In the present paper we plan to study some topological properties and inclusion relation between the above defined sequence spaces. $\left[\Gamma_{f\mu}^{3\Delta^m q}, \| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right]_{\theta_{rst}}^{\Gamma^2}$ and $\left[\Lambda_{f\mu}^{3\Delta^m q}, \| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \|_p \right]_{\theta_{rst}}^{\Gamma^3}$ which we shall discuss in this paper.

1. Theorem

Let $f=f_{mnk}$ be a Musielak-Orlicz function, $q=(q_{mnk})$ be a triple analytic difference sequence of strictly positive real numbers, the sequence spaces

$$\left[\Gamma_{f\mu}^{3\Delta^m q}, \left\| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^3} \text{ and}$$

$$\left[\Lambda_{f\mu}^{3\Delta^m q}, \left\| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^3} \text{ are linear spaces.}$$

Proof: It is routine verification. Therefore the proof is omitted.

2. Theorem

Let $f=f_{mnk}$ be a Musielak-Orlicz function, $q=(q_{mnk})$ be a triple analytic difference sequence of strictly positive real numbers, the sequence space

$\left[\Gamma_{f\mu}^{3\Delta^m q}, \left\| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^3}$ is a paranormed space with respect to the paranorm defined by

$$g(x) = \inf \left\{ \left[f_{mnk} \left(\left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \leq 1 \right\}.$$

Proof: Clearly $g(x) \geq 0$ for

$$x = (x_{mnk}) \in \left[\Gamma_{f\mu}^{3\Delta^m q}, \left\| (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^3}$$

Since $f_{mnk}(0)=0$ we get $g(0)=0$

Conversely, suppose that $g(x)$ then

$$\inf \left\{ \left[f_{mnk} \left(\left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \leq 1 \right\} = 0.$$

Suppose that $\mu_{mnk}(x) \neq 0$, for each $m, n, k \in \mathbb{N}$ Then

$$\left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \rightarrow \infty. \text{ It follows that}$$

$$\left(\left[f_{mnk} \left(\left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \right)^{1/H} \rightarrow \infty$$

which is a contradiction. Therefore $\mu_{mnk}(x)=0$. Let

$$\left(\left[f_{mnk} \left(\left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \right)^{1/H} \leq 1$$

and

$$\left(\left[f_{mnk} \left(\left\| \mu_{mnk}(y), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \right)^{1/H} \leq 1$$

Then by using Minkowski's inequality, we have

$$\begin{aligned} & \left(\left[f_{mnk} \left(\left\| \mu_{mnk}(x+y), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \right)^{1/H} \\ & \leq \left(\left[f_{mnk} \left(\left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \right)^{1/H} + \left(\left[f_{mnk} \left(\left\| \mu_{mnk}(y), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \right)^{1/H}. \end{aligned}$$

So we have

$$\begin{aligned} g(x+y) &= \inf \left\{ \left[f_{mnk} \left(\left\| \mu_{mnk}(x+y), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \leq 1 \right\} \\ &\leq \inf \left\{ \left[f_{mnk} \left(\left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \leq 1 \right\} + \inf \left\{ \left[f_{mnk} \left(\left\| \mu_{mnk}(y), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \leq 1 \right\} \end{aligned}$$

Therefore,

$$g(x+y) \leq g(x) + g(y).$$

Finally, to prove that the scalar multiplication is continuous. Let λ be any complex number. By definition,

$$g(\lambda x) = \inf \left\{ \left[f_{mnk} \left(\left\| \mu_{mnk}(\lambda x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \leq 1 \right\}.$$

Then

$$g(\lambda x) = \inf \left\{ (|\lambda|t)^{q_{mnk}/H} : \left[f_{mnk} \left(\left\| \mu_{mnk}(\lambda x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \leq 1 \right\}$$

where $t = \frac{1}{|\lambda|}$. Since $|\lambda|^{q_{mnk}} \leq \max(1, |\lambda|^{sup q_{mnk}})$, we have

$$g(\lambda x) \leq \max(1, |\lambda|^{sup q_{mnk}}) \inf$$

$$\left\{ t^{q_{mnk}/H} : \left[f_{mnk} \left(\left\| \mu_{mnk}(\lambda x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right) \right]^{q_{mnk}} \leq 1 \right\}$$

This completes the proof.

3. Theorem

(i) If the Musielak Orlicz function (f_{mnk}) satisfies Δ_2 - condition, then

$$\left[\Gamma_{f\mu}^{3\Delta^m q}, \left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^{3\alpha}} =$$

$$\left[\Gamma_g^{3\Delta^m q\mu}, \left\| \mu_{uvst}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^3}.$$

(ii) If the Musielak Orlicz function (g_{mnk}) satisfies Δ_2 - condition, then

$$\left[\Gamma_g^{3\Delta^m q\mu}, \left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^{3\alpha}} =$$

$$\left[\Gamma_{f\mu}^{3\Delta^m q}, \left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^3}$$

Proof: Let the Musielak Orlicz function (f_{mnk}) satisfies Δ_2 -condition, we get

$$\begin{aligned} & \left[\Gamma_g^{3\Delta^m q\mu}, \left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^{3\alpha}} \\ & \subset \left[\Gamma_{f\mu}^{3\Delta^m q}, \left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^3} \dots \dots \quad (1) \end{aligned}$$

To prove the inclusion

$$\begin{aligned} & \left[\Gamma_{f\mu}^{3\Delta^m q}, \left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^{3\alpha}} \\ & \subset \left[\Gamma_g^{3\Delta^m q\mu}, \left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^3}, \end{aligned}$$

$$\text{let } a \in \left[\Gamma_{f\mu}^{3\Delta^m q}, \left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^{3\alpha}}.$$

Then for all $\{x_{mnk}\}$ with

$$(x_{mnk}) \in \left[\Gamma_{f\mu}^{3\Delta^m q}, \left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^3}$$

we have

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} |\Delta^m x_{mnk} a_{mnk}| < \infty. \quad (1)$$

Since the Musielak Orlicz function (f_{mnk}) satisfies condition, then

$$(y_{mnk}) \in \left[\Gamma_{f\mu}^{3\Delta^m q}, \left\| \mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0)) \right\|_p \right]_{\theta_{rst}}^{j^3}, \text{ we get}$$

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \left| \frac{\varphi_{rst} \gamma_{mnk} \alpha_{mnk}}{\Delta^m \lambda_{mnk}} \right| < \infty. \text{ by (1). Thus}$$

$$(\varphi_{rst} a_{mnk}) \in \left[\Gamma_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3} =$$

$$\left[\Gamma_g^{3\Delta^m q\mu}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3} \text{ and hence}$$

$$(a_{mnk}) \in \left[\Gamma_g^{3\Delta^m q\mu}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3}. \text{ This gives that}$$

$$\left[\Gamma_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3\alpha}$$

$$\subset \left[\Gamma_g^{3\Delta^m q\mu}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3} \dots \dots \dots (2)$$

we are granted with (1) and (2)

$$\left[\Gamma_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3\alpha} =$$

$$\left[\Gamma_g^{3\Delta^m q\mu}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3}$$

(ii) Similarly, one can prove that

$$\left[\Gamma_g^{3\Delta^m q\mu}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3\alpha}$$

$$\subset \left[\Gamma_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3}$$

if the Musielak Orlicz function (g_{mnk}) satisfies Δ_2 -condition.

1. Proposition

The sequence space

$$\left[\Gamma_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3} \text{ is not solid}$$

Proof: The result follows from the following example.

Example: Consider

$$\Delta^m x = (\Delta^m x_{mnk}) = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \in \left[\Gamma_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3}. \text{ Let}$$

$$\Delta^m \alpha_{mnk} = \begin{bmatrix} -1^{m+n+k} & -1^{m+n+k} & \dots & -1^{m+n+k} \\ -1^{m+n+k} & -1^{m+n+k} & \dots & -1^{m+n+k} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}, \text{ for all } m, n, k \in \mathbb{N}$$

Then $\Delta^m \alpha_{mnk} x_{mnk} \notin \left[\Gamma_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3}$.

Hence

$$\left[\Gamma_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3} \text{ is not solid.}$$

2. Proposition

The sequence space

$$\left[\Gamma_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3}$$

is not monotone.

Proof: The proof follows from Proposition 3.4.

3. Proposition

The sequence space

$$\left[\Lambda_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3} \text{ is not solid.}$$

4. Proposition

The sequence space

$$\left[\Lambda_{f\mu}^{3\Delta^m q}, \|\mu_{mnk}(x), (d(x_1, 0), d(x_2, 0), \dots, d(x_{n-1}, 0))\|_p \right]_{\theta_{rst}}^{\Gamma^3}$$

is not monotone.

Conclusion

Through this paper we studied some topological properties and inclusion relation with respect to a sequence of Musielak-Orlicz function.

References

- Sahiner A, Gurdal M, Duden FK (2007) Triple sequences and their statistical convergence. Selcuk J Appl Math 8: 49-55.
- Sahiner A, Tripathy BC (2008) Some I related properties of triple sequences. Selcuk J Appl Math 9: 9-18.
- Esi A (2014) On some triple almost lacunary sequence spaces defined by Orlicz functions. Research and Reviews: Discrete Mathematical Structures 1: 16-25.
- Esi A, Catalbas MN (2014) Almost convergence of triple sequences. Global Journal of Mathematical Analysis 2: 6-10.
- Esi A, Savas E (2015) On lacunary statistically convergent triple sequences in probabilistic normed space. Appl Math and Inf Sci 9: 2529-2534
- Datta AJ, Esi A, Tripathy BC (2013) Statistically convergent triple sequence spaces defined by Orlicz function. Journal of Mathematical Analysis 4: 16-22.
- Subramanian N, Esi A (2015) The generalized triple difference of χ^3 sequence spaces. Global Journal of Mathematical Analysis 3: 54-60.
- Debnath S, Sarma Das BC (2015) Some generalized triple sequence spaces of real numbers. Journal of nonlinear analysis and optimization. 6: 71-79.
- Kizmaz H (1981) On certain sequence spaces. Canadian Mathematical Bulletin 24: 169-176.
- Kamthan PK, Gupta M (1981) Sequence spaces and series. Pure and Applied Mathematics, USA.
- Lindenstrauss J, Tzafriri L (1971) On Orlicz sequence spaces. Israel J Math 10: 379-390.
- Musielak J (1983) Orlicz Spaces and Modular Spaces. Springer
- Subramanian N, Murugesan C (2016) The entire sequence over Musielak p -metric space. Journal of the Egyptian Mathematical Society 24: 233-238.