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Generalized Lie Theory and Applications

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The Generalized Super Kaup–Newell Equation

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Description

It introduces a generalised Kaup-Newell spectral issue. There are an endless number of hierarchies of soltons equations that correspond to this spectral problem. Two new hierarchies of related soliton equations are constructed, and two more reductions of the generalised Kaup-Newell spectral issue are explored. Two new finite-dimensional integrable systems with non-confocal involutive integrals of motion are created by non-linearizing two reductive spectral issues. Integrable generalisation of the super Kaup-Newell(KN) isospectral issue and create a generalised super KN soliton hierarchy based on a Lie super-algebra B(0,1) and a super-trace identity. And a super bi-Hamiltonian form of the consequent super soliton hierarchy can be created. The presentation also includes a generalised super KN soliton hierarchy with self-consistent sources. As we are all aware, as soltons theory has advanced, super integrable systems connected to Lie super algebra have drawn increasing interest. As a result, many classical integrable equations have been expanded to become super fully integrable equations [1].

Hu and Ma in particular, have contributed greatly. The super-trace identity, which Hu introduced, is a useful tool for creating super Hamiltonian structures from super integrable equations. Ma provided the proof of the super-trace identity in 2008, establishing the super Hamiltonian structure of several super integrable equations. There are exact solutions to a large number of nonlinear partial differential equations, particularly soltons. Solving soliton equations can be done in a variety of ways, including the Hirota bilinear method, Wronskian methodology, and Darboux transformation. One of the most effective tools is the inverse scattering transformation (IST), which is closely related to the approaches outlined previously [2]. Since the method it uses to resolve nonlinear equations are similar to that of the linear Fourier transform, it is also known as the nonlinear Fourier transform. The IST has the benefit of being applicable across the entire soliton hierarchy. Recent studies demonstrate that the IST can solve soliton equations with self-consistent sources, non-isospectral soliton hierarchies, as well as standard soliton hierarchies [3].

Another efficient technique for resolving soliton equations is the Riemann-Hilbert (RH) strategy. In actuality, it closely relates to the IST. They both begin with identical matrix spectral problems that have constrained eigenfunctions that may be extended analytically to either the upper or lower half-plane. We must take into account the asymptotic conditions at infinity on the real axis by the IST to solve soliton equations in order to obtain scattering data. In actuality, the conditions under consideration serve as the answers to the associated RH puzzles. The RH issue is similar to the IST with reflectionless potentials when the jump matrix is an identity matrix, and -soliton solutions can be produced. There are numerous physical domains where the integrable equations with self-consistent sources are useful, including plasma physics, hydrodynamics, and interactions between solitary waves. For instance, the NLS equation with self-consistent sources may represent both the propagation of solitary waves in resonance- and non-resonant-mediums as well as the interaction between high-frequency static waves and ion-acoustic waves in plasma [4].

The Alfven waves in plasma physics and the sub-picosecond or femtosecond pulses in nonlinear optics are described by the derivative nonlinear Schrödinger equation, which also describes the Peregrine rogue waves. Analysis has been done on the relationships between the interaction and degeneration of two soliton-like solutions and the breather solution. The two types of formation procedures for Peregrine rogue waves have been taken into consideration: they can be produced by limiting the breathing solutions' indefinitely long periods, or they can be understood as soliton-like solutions with various polarity. As a special illustration, the trivial seed produces a breather solution and phase solution that yield a special Peregrine rogue wave (zero solution). We develop a super extension of the Kaup—Newell hierarchy connected to a 33 matrix spectrum issue and establish its super bi-Hamiltonian structures using the zero-curvature equation and the super trace identity. Furthermore, spectral parameter expansions are used to derive infinite conservation laws for the super Kaup—Newell equation [5]. The two-soliton solution of the DNLS equation under vanishing boundary conditions and nonvanishing boundary conditions is introduced as the so-called "paired soliton", which is now regarded as one kind of breather solution. By introducing an affine parameter, Chen and Lam revised the inverse scattering transform for the DNLS equation under nonvanishing boundary conditions, and then got the single breather solution, which can be reduced to the dark soliton and the bright soliton.

In addition, the interest in the two-soliton solution is due to its application, such as the development of the structure of soliton turbulence in integrable systems, as well as the soliton molecules in contemporary nonlinear optics. Nonlinear fluid waves and optical systems both exhibit similar soliton breathing behaviour. Studying the interaction of two solitons and associated structures is crucial, according to earlier studies. In this work, we show how two soliton-

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like solutions interact and degenerate along with their relationships to the breather solutions. Some classifications are also made in accordance with the distinct causes of the creation of Peregrine rogue waves. When studying Virasoro symmetric algebras, integrable couplings were first introduced as coupled systems of integrable equations. Finding integrable couplings in soliton theory is a crucial and fascinating topic [6]. Numerous techniques for finding integrable couplings have been developed in recent years. The endeavour to extend the classical integrable system's integrable couplings theory to the fractional integrable system is significant. Fractional integrable couplings of the AKNS hierarchy were proposed by Zhang and You. Then, using generalised fractional trace variational identity, we build the fractional Hamiltonian structures of the fractional integrable couplings of the Kaup-Newell hierarchy.

Conflict of Interest

None.

References

1. Rogers, C, W. K. Schief and P. Winternitz. "Lie-theoretical generalization and discretization of the Pinney equation." *J Math Anal Appl* 216 (1997): 246-264.
2. Dirr, G and U. Helmke. "Lie theory for quantum control." *GAMM-Mitteilungen* 31 (2008): 59-93.
3. Leach, P. G. L. "Applications of the Lie theory of extended groups in Hamiltonian mechanics: the oscillator and the Kepler problem." *ANZIAM J* 23 (1981): 173-186.
4. Mittal, Sushil, Saket Anand and Peter Meer. "Generalized projection based m-estimator: Theory and applications." *CVPR IEEE*, (2011) 2689-2696.
5. Miller Jr and Willard. "Lie theory and the Lauricella functions FD." *J Math Phys* 13 (1972): 1393-1399.
6. Fu, Zhongtao, Emmanouil Spyrakos-Papastavridis, Yen-hua Lin and Jian S. Dai. "Analytical expressions of serial manipulator jacobians and their high-order derivatives based on lie theory." *Int Conf Robot Autom* (*ICRA*) *IEEE* (2020) 7095-7100.

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