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The Examples of Design of Normal Cycle Shells and Analyses of Stress-Strain State by Variation-Difference Method

Vyacheslav N Ivanov* and Alisa A Shmeleva

Peoples Friendship University of Russia (RUDN University), Moscow, Russian Federation

Abstract

The variation-difference method is a convenient numerical method for shells of complex forms. It is enough when only cinematic boundary conditions are satisfied because the method is based on the principle of Lagrange. Another advantage of the variation-difference method is the better opportunity to create computer programs based on it. For shell analysis in orthogonal coordinate system as well as for shell analysis in principal curvatures the system of equations describing stress-strain state can be simplified. In this paper the difference between analysis in orthogonal coordinate system and analysis in principal curvatures of the surface is considered. The main distinction of the analysis of shells in orthogonal curvilinear coordinate system is the necessity of determination of components which include curvature of torsion of coordinate lines. The addition of these components in the equations of the theory of shells for the coordinate system in principal curvatures gives possibility to analyze shells in common orthogonal coordinate system. In this article shell analysis in orthogonal coordinate system is applied to shells based on normal cyclic surfaces.

Keywords: Normal cycle surfaces • Non-conjugated orthogonal coordinates • Thin-walled shells • Variation-difference method

Introduction

One of the tasks of the modern town building is a design and working out the methods of analyses of space constructions of new complex forms [1]. There are the examples of using new complex forms of constructions in buildings in the 20-th 21-st centuries [2-5]. But the architects and mechanic-engineers demand to develop and to investigate the new forms of the surfaces, which there are described by the analytical equation and their inculcation in different branches of science and technics. At the Encyclopaedia of analytical surfaces [6] there are described 35 classes of the surfaces. One of class of surfaces is the class of normal cycle surfaces, which allow constructing much new forms of thin shells [7-10]. Cycle shells formed by the system of circles are comfortable for their realization. At second half of the 20-th century there has been worked out the finite element method (FEM) for stress- strain analyses of constructions, which there are mainly used for design of thin shell. But in most program complexes of FEM there are not taken into account the geometrical characteristics of the middle surface of the shell, which may bring to not correct results of design of the shells of complex form. The alternative method of analyses of thin-walled shells of complex geometry is a variation-difference method. The program complex VRMSHELL which used the base of variation-difference method there is worked out at the chair of strength of materials of Engineering department RUDN (today, the Department Civil Engineering, Engineering Academy, RUDN) used the geometrical characteristics of the middle surface of the shell [11,12].

At the article there given the base of the geometry of the normal cycle surfaces and the variation-difference method, shone the examples of the normal cycle surfaces and the example of calculation of stress-strain state of the shell.

***Address for Correspondence:** Vyacheslav N Ivanov, Peoples Friendship University of Russia (RUDN University), Moscow, Russian Federation, Tel: + 79031933021; E-mail: i.v.ivn@mail.ru*

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Geometry of normal cycle surfaces

Normal cycle surfaces there are formed by moving of a circle of change or constant radius at the normal plane of the directrix curve of centers of the generating circles. Vector equation of normal cycle surface:

$$\boldsymbol{\rho}(u,\upsilon)=\boldsymbol{r}(u)+R(u)\boldsymbol{e}(u,\omega), \tag{1}$$

where $\boldsymbol{\rho}(u,\upsilon)$ is a radius-vector of the surface; $\boldsymbol{r}(u)$ - radius-vector of the directrix line of centers of the generating circles; $R(u)$ is a function of radius of the generating circles; $\boldsymbol{e}(u,\omega)=\boldsymbol{\nu}\cos\omega+\boldsymbol{\beta}\sin\omega$ is the equation of the circle of unit radius at the normal plane of the line of the centers of the generating circles; ν, β are vectors of normal and bi-normal of the line of the circles; $\omega=v+\theta(u)$; v - polar angle at the normal plane of the of the line of the centers of the circles; $\theta(u)$ there is determining the initial polar angel from the normal of the directrix line.

Results and Discussion

On the base formulas of the differential geometry [13] there are received the formulas of the geometrical characteristics of the normal cycle surfaces:

$$A=\sqrt{R'^2+(s'-Rk_s\cos\omega)^2}\,;\quad B=R;\quad F=R^2(\theta'+\chi s');$$

$$L=\frac{1}{A}\{R'[s''-(2k_s\cdot R'+k_s'R)\cos\omega]-R'k_s\chi_s R\sin-(s'-Rk_s\cos\omega)[(s'-Rk_s\cos\omega)k_s\cos\omega+R'']\};$$

$$M=\frac{RR'k_s\sin\omega}{A};\qquad N=\frac{R}{A}(s'-Rk_s\cos\omega), \tag{2}$$

$k_s=ks'$; $\chi_s=\chi s'$; k,χ - curvature and torsion of the directrix line; $s'=|\boldsymbol{r}'|$;

$\frac{\partial ..}{\partial u}=..'$.

$$\text{If } F=0 \;\rightarrow\; \theta(u)=-\int\chi s'\,du+\theta_0, \tag{3}$$

There is received the orthogonal coordinates of the normal cycle surfaces. For plane directrix line $\chi=0$; $\theta(u)=\theta_0=const$.

It's seen from the formulas (2, 3) that the coordinate system of the normal cycle surfaces is orthogonal (F=0) but not conjugated (M≠0), so the generating circles are not the curves of principle curvatures of the normal cycle surfaces in common. The coefficient M may be equal zero if: 1) k=0 then the directrix curve is the right line and the surface is a surface of revolution; 2) $R'=0$, R=const, then the normal cycle surfaces is the tube surface.

The examples of normal cycle surfaces with different directrix and functions of generating circles there are shown on Figure 1.

The geometry of normal cycle surfaces there has been investigated at the articles [7-10].

Variation-difference method

The variation-difference method there is based on the Lagrange's principle that is the principle of minimum of the total energy of deformations [13], so as in finite element method:

$$E=U-T; \quad \delta E=\delta U-\delta T=0, \tag{4}$$

E is a total energy of deformation; U is a potential energy of deformation; T is the work of external forces; δ is a symbol of variation of functional [14].

$$U=\frac{1}{2}\iint_{\Omega}(N_1\varepsilon_1+N_2\varepsilon_2+2S\varepsilon_{12})d\Omega+\frac{1}{2}\iint_{\Omega}(M_1\chi_1+M_2\chi_2+2H\chi_{12})d\Omega=$$

$$=\frac{C}{2}\iint_{\Omega}\left[(\varepsilon_1+\nu\varepsilon_2)\varepsilon_1+(\varepsilon_2+\nu\varepsilon_1)\varepsilon_2+\frac{1-\nu}{2}S\varepsilon_{12}\right]d\Omega+\frac{D}{2}\iint_{\Omega}[(\chi_1+\nu\chi_2)\chi_1+(\chi_2+\nu\chi_1)\chi_2+(1-\nu)\chi_{12}]d\Omega;$$

$$T=\iint_{\Omega}(q_1\cdot u_1+q_2\cdot u_2+q_3\cdot u_3)d\Omega, \quad C=\frac{Eh}{1-\nu^2}, \quad D=\frac{Eh^3}{12(1-\nu^2)}. \tag{5}$$

$$\varepsilon_1=\frac{1}{A_1}\frac{\partial u_1}{\alpha_1}+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}u_2+k_1u_3; \quad \varepsilon_2=\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}u_1+\frac{1}{A_2}\frac{\partial u_2}{\alpha_2}+k_2u_3;$$

$$\varepsilon_{12}=\frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\left(\frac{u_1}{A_1}\right)+\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}\left(\frac{u_2}{A_2}\right)-2k_{12}\cdot u_3;$$

$$\chi_1=\frac{1}{A_1}\frac{\partial\vartheta_1}{\partial\alpha_1}+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}\vartheta_2-k_{12}\omega_3; \quad \chi_2=\frac{1}{A_2}\frac{\partial\vartheta_2}{\partial\alpha_2}+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}\vartheta_1+k_{12}\omega_3;$$

$$\chi_{12}=\tau_2+k_2\omega_1+k_{12}\varepsilon_2=\tau_1+k_1\omega_2+k_{12}\varepsilon_1 \tag{6}$$

$$\omega_1=\frac{1}{A_1}\frac{\partial u_2}{\partial\alpha_1}-\frac{1}{A_1A_2}u_1-k_{12}u_3; \quad \omega_2=\frac{1}{A_2}\frac{\partial u_1}{\partial\alpha_2}-\frac{1}{A_1A_2}u_2-k_{12}u_3; \quad \omega_3=\frac{1}{2A_1A_2}\left(\frac{\partial A_2u_2}{\partial\alpha_1}-\frac{\partial A_1u_1}{\partial\alpha_2}\right);$$

$$\tau_1=\frac{1}{A_1}\frac{\partial\vartheta_2}{\partial\alpha_1}-\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_{21}}\vartheta_1; \quad \tau_1=\frac{1}{A_2}\frac{\partial\vartheta_1}{\partial\alpha_2}-\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}\vartheta_2;$$

$$\vartheta_1=k_1u_1-k_{12}u_2-\frac{1}{A_1}\frac{\partial u_3}{\partial\alpha_1}; \vartheta_2=-k_{12}u_1+k_2u_2-\frac{1}{A_2}\frac{\partial u_3}{\partial\alpha_2},;$$

ε_i, χ_i are tangential and bending deformations of middle surface of the shell; N_i, M_i are tangential and bending forces; q_i are the projections of loads at the surface coordinate system of the shell, u_i are the displacements of middle surface; Ω – parameter of the area of the middle surface.

Using the physical and geometrical equation of the theory of the shells (6) the functional of total energy (4) there will be written in function of displacements.

For stress-strain analyses of the shell by the variation-difference method the surface of the shell there is used the coordinate net with equal or difference paces. Changing the derivations of displacements in the functional of total energy by numerical difference ratio and minimizing the functional there is received the system of the algebraic equations. The decision of the equation gives web displacements of the shell at the nods of the net. Using again the

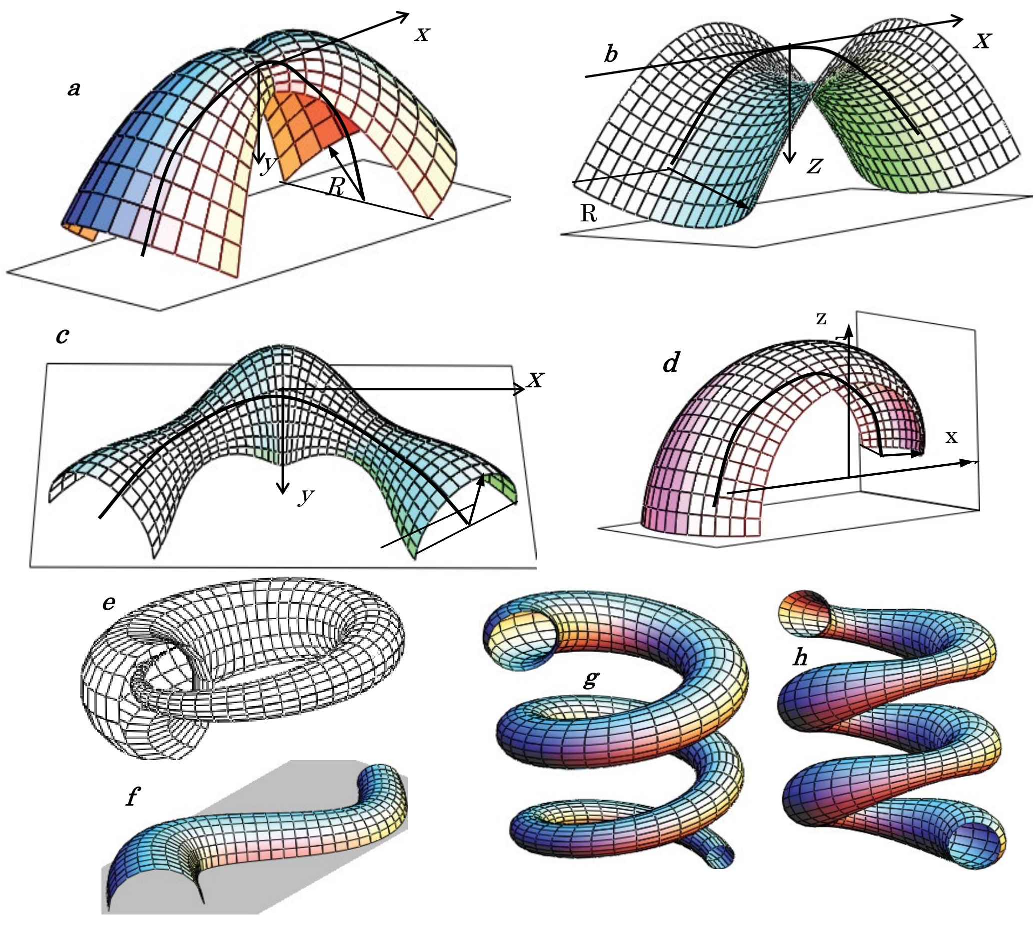

Figure 1. Normal cycle surfaces.
Directrix: a, b Parabola; c Hyperbola; d Evolving of the circle; e Ellipse; f Sinus; g, h - Helix;

numerical difference ratios there are received the strains and further the tangential and bending forces and stresses.

The program complex VRMSHELL includes the library of curves on base of which there are formed the sections of the surfaces of the classes including in complex and calculated the geometric characteristics and their necessary derivations. The program complex includes the classes of the surfaces: cylinders with different generating curves, surfaces of revolution, Joachimsthal's canal surfaces, Monge's surfaces. There may be analyzed the stress-strains of plates (bending and plane problem) and shallow shells in rectangular and polar coordinates. The Monge's class surfaces allow to analyze the plates on curved trapezium orthogonal plans [15]. There may be used different condition of support and elastic foundation and elastic supports as well. There may be analyzed the shells with holes and shells with any contours. The loads may be linear distributed on some part of the shell or along a coordinate lines or concentrated forces or moments. The loads may be orthogonal or tangent to the surface of the shell.

The beginning program there has been realized in coordinate of principle curvatures of the middle surfaces of the shells. The geometrical characteristics and geometrical equations of deformations of shells of common orthogonal (non-conjugated) coordinates have some differences of the characteristics and equations in the principle coordinates system of the surfaces. So there were made the necessary changing in the program complex, which make possible to analyze the normal cycle shells. The correction of the work of the block of the normal cycle shells there was checked up on the shells of rotation with some analytical decision and analyses of shells of rotation and tube shells using the blocks of the shells of rotation and Monge's shells included in the program complex.

The example of analyses of normal cycle shell

The stress-strain condition of the normal cycle shell with ellipse directrix and cosine function of radius of generating circles (Figure 2) there was analyzed by the program complex. The parameters of the shell: directrix - x=acosu, y=bsinu, a=3 m, b=2 m, u=0÷2π; radius of generating circles -

R= R_0-c·cos2u, R_0=0.75 m; c=-0, 2 m; thickness of the shell h=0.1 m; module of elasticity E=3.5·10^8 kPa.

The analyses there was made on the internal presser q=1 kPa.

As the shell has 3 axis of symmetry the analyses of strain stress state on internal presser there was made on 1/8 part of the shell (Figure 3). The epure of the tangential normal forces there are shown at Figure 4.

The epure of the bending moments are shown at Figure 5.

For comparison in Figure 6 in the brackets are the given results of analyses of the ellipsoidal tube shell with radius R_0=0.75 m. The parameters of the directing ellipse are the same as at the ellipsoid cosine shell a=3 m, b=2 m. The coordinates systems of the middle surfaces of the shells is different but they have at four sections u=±π/4. u=±π3/4 the generating circle with equal radius. The results of analyses of the stress-strain states of the regarded shells show the difference at the distributions of the tangential normal forces but sizes the largest normal forces are closer still at the differ sections.

The circular bending moments at the most sections larger at the ellipsoidal cosine shell.

From the results of analyses of the ellipsoidal normal cycle surface with cosine function of generating radius and with internal pressure load there are seen that the circle normal forces N_v about at two times more the longitudinal normal forces Nu. The large normal force is at the internal side of shell u=±π/2, v=0

$N_v^{\max} = 1.013 kN/m$, $\sigma_{Nv}^{\max} = 10.13 kPa$, (*u*=0, *v*=π/2).

The large longitudinal normal force $N_u^{\max} = 0.49 kN/m$, $\sigma_{Nu}^{\max} = 4.9 kPa$, (*u*=±π/2, *v*=0,75π).

The circular bending moments M_v there are bigger then longitudinal bending moments M_u as well: , $M_u^{\max} = 0.93\ Nm/m$, $\sigma_{Mu}^{\max} = 0.556 kPa$, , (*u*=π/4, *v*=0). ,

$M_v^{\max} = 2.46\ Nm/m$, $\sigma_{Mv}^{\max} = 1.476 kPa$, (*u*==π/2, *v*=0).

The larges bending stresses are about 15 percent of the largest tangential normal stresses. But those stresses are at different section. The tangential normal stresses at the section of largest moment stresses (*u*==π/2, *v*=0) is σ_{Nv}=5,52 kPa. The differences are about 30 percent.

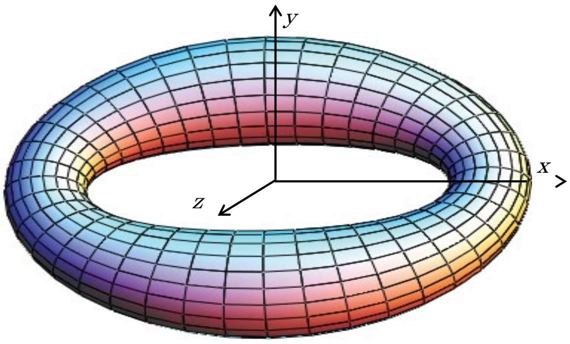

Figure 2. Ellipsoidal normal cycle shell.

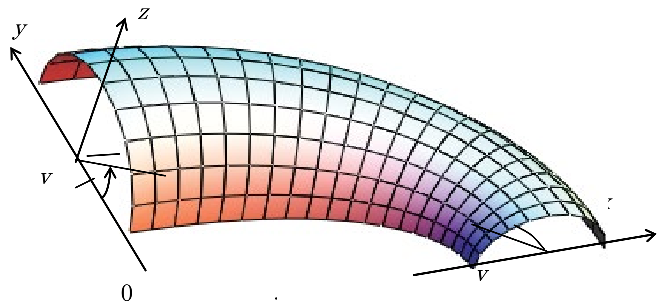

Figure 3. The analyzed section.

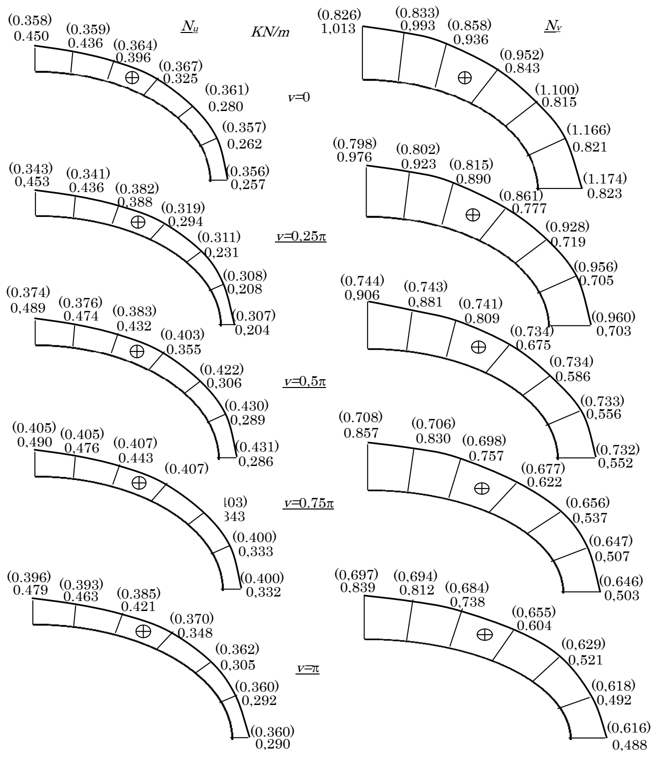

Figure 4. Tangential normal forces.

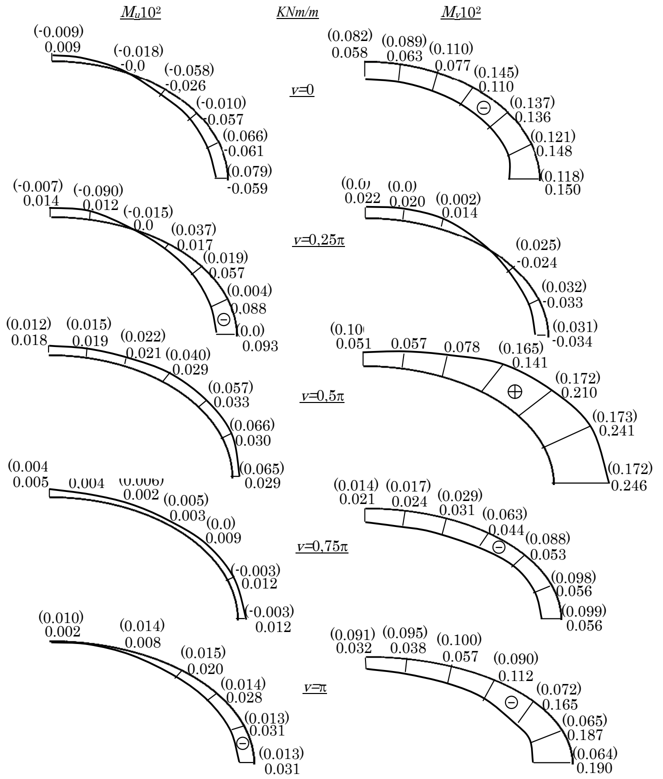

Figure 5. Bending moments.

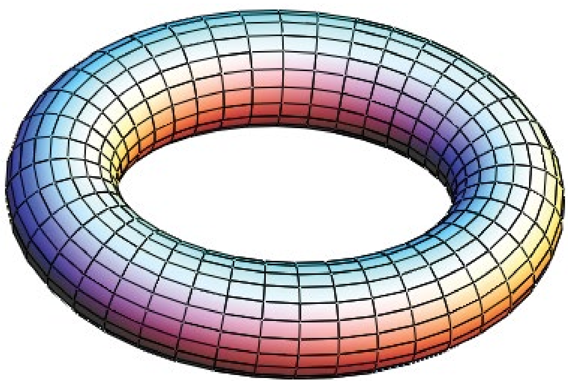

Figure 6. The ellipsoidal tube shell.

Conclusion

The block of the normal cycle shells allow to analyze any normal cycle shell with the plane directrix and the function of radius included at the library of curves of the program complex. If it is necessary to use some curve which is not at the library the new curve may be simply included at the library. There isn't any another changes that must be done. For using of space direcrix it will be necessary to do some changes at the program complex.

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