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Temperature Dependent Variable Properties on Mixed Convective Unsteady MHD laminar Incompressible fluid Flow with Heat Transfer and Viscous Dissipation

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Abstract

In the present paper we investigated that the numerical solution of mixed convective laminar boundary layer flow with uniform transverse magnetic field for a vertical porous plate. The fluid properties are considered the nonlinear functions of temperature as $\mu = \mu_\infty (T/T_\infty)^A$ and $\kappa = \kappa_\infty (T/T_\infty)^C$. For flue gases the values of the exponents A and C are taken as *A=0.7,* and *C=0.83*. Also time dependent suction velocity is considered. The governing equations are reduced into ordinary differential equations by introducing the suitable similarity variables and then solved numerically by using sixth ordered Runge-Kutta method and Nachtsheim Swigert iteration technique. The numerical results of our physical interest are shown graphically and tabular form.

Keywords: Mixedconvection; Viscous dissipation; MHD Laminar Fluid flow; Heattransfer; Temperature Dependent Variables Properties

Introduction

This is well known that the physical properties of the flow are functions of temperature. In light of this concept, Hodnett [1] studied the low Reynolds number flow of a variable property gas past an infinite heated circular cylinder. Bhat and Bose [2] examined the fluid flow with variable properties in a two dimensional channel. Zakerullah and Ackroyd [3] investigated the free convection flow above a horizontal circular disc for variable fluid properties. Correlations have been developed by Gokoglu and Rosner [4] for deposition rates in force convection systems with variable properties. The influence of variable properties on laminar fully developed pipe flow with constant heat flux across the wall studied by Herwing [5] and Herwig and Wickern [6], Herwig and Klemp [7]. Herwing [8] examined an asymptotic analysis of laminar film boiling on a vertical plate including variable property effects. Steady and unsteady MHD laminar convective flow due to rotating disk with the effects of variable properties and Hall current investigated by Maleque and Sattar [9-11]. Recently Asterios Pantokratoras [12] studied the effect of variable fluid properties on the classical Blasius and Sakiadis flow taking into account temperature dependent physical properties. More recently Maleque [13] considered combined temperature- and depth-dependent viscosity and Hall current on MHD convective flow due to a rotating disk.

Considering the importance of MHD combined convective flow, in present study we are to investigate the variable fluid properties (viscosity (μ) and thermal conductivity (k) transverse magnetic field and time dependent suction/injection velocity on an unsteady MHD laminar incompressible fluid flow for a vertical porous plate. Using suitable dimensionless similarity variables the governing partial differential equations of the MHD combined convective boundary layer flow are reduced to nonlinear ordinary differential equations and then by using Range-Kutta six order integration scheme and Nachtsheim-Swigert iteration technique the nonlinear ordinary differential equations are solved numerically. The obtained numerical results of physical properties are shown graphically and tabular form for the different values of the dimensionless parameters. Finally the effects of the relative temperature difference parameter *(γ)* on the skin friction and heat transfer coefficients are also examined.

Governing Equations

Let us consider an unsteady combined forced and free convective incompressible laminar boundary layer MHD electrically conducting viscous fluid flow past an infinite vertical porous plate *y=0*. Considering the flow is moving in upward direction with a constant velocity U_0. We take the plate in upward direction along the *x*-axis and *y*-axis is normal to plate. A uniform non conducting magnetic field $\vec{B}$ is perpendicular to the plate and parallel to the y-axis. The magnetic field is assumed as the form $\vec{B} = (0, B_0, 0)$. The equation of conservation of electric charge is $\nabla \cdot \vec{J} = 0$, where $\vec{J} = (J_x, J_y, J_z)$, the propagation direction is considered only along the y-axis and does not have any variation along the y-axis. Considering $\frac{\partial J_y}{\partial y} = 0$, which gives result J_y = constant. For an electrically non-conducting plate, this constant is zero and hence $J_y = 0$ everywhere in the flow. It is assumed that an infinite plate in extent along x-axis and hence all the physical quantities velocity and temperature are functions of *y* and *t*.

We assume that the dependency of the fluid properties, viscosity coefficient (*μ*) and thermal conductivity coefficient (*k*) are function of temperature alone and obey the following laws [10].

$$\frac{\mu}{\mu_\infty} = \left(\frac{T}{T_\infty}\right)^A, \qquad \frac{\kappa}{\kappa_\infty} = \left(\frac{T}{T_\infty}\right)^C \tag{1}$$

Where A and C are arbitrary exponents, μ_∞ is the uniform viscosity of the fluid and k_∞ is a uniform thermal conductivity of heat. The fluid is assumed to be flue gas for our present analysis. The exponents values are considered as *A=0.7,* and *C=0.83* for flue gases [14].

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Thus according to the above assumptions and Boussinesq's approximation, the continuity equation, the momentum equation and the heat equation of the problem are:

$$\frac{\partial v}{\partial y}=0, \tag{2}$$

$$\rho\left(\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial \mu}{\partial y}+\rho g\beta(T-T_\infty)$$

$$+\sigma' B_0^2(U_0-u) \tag{3}$$

$$\rho\, c_p\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial y^2}+\frac{\partial T}{\partial y}\frac{\partial \kappa}{\partial y}$$

$$+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma' B_0^2(U_0-u)^2 \tag{4}$$

The boundary conditions are:

$$\left.\begin{array}{l} u=0,\quad v=v(t),\quad \text{and}\ T=T_w\ \text{ at }\ y=0 \\ u=U_0,\qquad T=T_\infty\ \text{ for }\ y\to\infty. \end{array}\right\} \tag{5}$$

where u and v are components of velocity along x-axis and y-axis respectively; ρ is the fluid density, B_0 is the constant applied transverse magnetic field, g is the gravitational acceleration, β is the volume expansion coefficient for temperature, T and T_∞ are respectively the thermal boundary layer fluid temperature and the fluid temperature in the free stream, T_w is the plate temperature; σ' is the electrical conductivity; and $v_w(t)$, the normal velocity at the plate having a positive value for suction and negative value for injection.

Mathematical Formulations

The governing equations (2)-(4) under the boundary conditions (5) can be solve by considering the well-defined similarity technique to obtain the similarity solutions. Now we introduce the following non-dimensional suitable similarity variables:

$$\eta=\frac{y}{\delta(t)},\ \frac{u}{U_0}=f(\eta),\ T-T_0=\Delta T\,\theta \tag{6}$$

From the continuity equation (2) we have

$$v=-\frac{v_0\,\nu_\infty}{\delta\,(t)} \tag{7}$$

v_0 is the dimensionless suction/injection velocity. $v_0<0$ represents suction and $v_0>0$ represents injection.

The dimensionless quantities from equation (6) and v from equation (7) are introduced in equations (3)—(4) finally the nonlinear ordinary differential equations are obtained as:

$$f''+A\,\gamma\,(1+\gamma\,\theta)^{-1}f'\,\theta'$$

$$+\left[\eta\,\frac{\delta\,\delta'}{\nu_\infty}f'+G_r\,\theta\right](1+\gamma\,\theta)^{-A}$$

$$+\left[M(1-f)+v_0\,f'\right](1+\gamma\,\theta)^{-A}=0 \tag{8}$$

$$\theta''+C\,\gamma\left(1+\gamma\,\theta\right)^{-1}\theta'^2+\Pr\,E_c\left(1+\gamma\,\theta\right)^{A-C}f'^2$$

$$+M\Pr\,E_c\left(1+\gamma\,\theta\right)^{-C}\left(1-f\right)^2$$

$$+\Pr\left(1+\gamma\,\theta\right)^{-C}\left(\eta\frac{\delta\,\delta'}{\nu_\infty}\right)\theta'$$

$$+v_0\Pr\left(1+\gamma\,\theta\right)^{-C}\theta'=0 \tag{9}$$

Where,

$$G_r=\frac{\delta^2\,g\,\beta\,\Delta T}{U_0\,\nu_\infty}\ \text{(Grashof Number)}$$

$$M=\frac{\sigma'\ \delta^2 B_0^2}{\mu_\infty}\ \text{(Magnetic parameter)}$$

$$\Pr=\frac{\rho_\infty\,\nu_\infty\,c_p}{\kappa_\infty}\ \text{(Prandtl Number)}$$

$$E_c=\frac{U_0^2}{\Delta T\,c_p}\ \text{(Eckert Number)}$$

Where, $\gamma=\dfrac{\Delta T}{T_\infty}$ is the relative temperature difference parameter which is positive for a heated plate, negative for a cooled plate and zero for uniform properties.

The equations (8)-(9) are similar except for the term $\dfrac{\delta\,\delta'}{\nu_\infty}$, where time t appears explicitly. Thus $\dfrac{\delta\,\delta'}{\nu_\infty}$ must be a constant quantity for the similar condition. Hence following Maleque [15] can try a one class of solutions of the equations (8)-(9) by taking that

$$\frac{\delta\,\delta'}{\nu_\infty}=K\ \ \text{(Constant)} \tag{10}$$

From equation (10) we have,

$$\delta=\sqrt{2K\nu_\infty t}\quad+L \tag{11}$$

Where L is the integrating constant which is obtained through the condition that when $t=0$ then $\delta=L$. Here $K=0$ that is $\delta=L$. represents the length scale for steady flow and $K\neq 0$ implies that δ denotes the length scale for unsteady flow [16,17].

It thus appear from equation (10) that by puting $K=2$ in equation (11), the length scale $\delta(t)$ becomes $\delta=2\sqrt{\nu_\infty t}$. The usual scale factor $\delta(t)$ exactly suitable for unsteady viscous boundary layer flows considered by Schlichting [18], Maleque [19,20]. Since δ is a similarity parameter as well as scale factor, except that the scale would be different any value of K in equation (11) would not change the nature of solutions. Finally introducing $K=2$ (Maleque [21,22] in equation (10) , we have

$$\frac{\delta\,\delta'}{\nu_\infty}=2 \tag{12}$$

Using equation (12), the equations (8)-(9) yield

$$f''+A\,\gamma\,(1+\gamma\,\theta)^{-1}f'\,\theta'$$

$$+\left[2\eta\,f'+G_r\,\theta\right](1+\gamma\,\theta)^{-A}$$

$$+\left[M(1-f)+v_0\,f'\right](1+\gamma\,\theta)^{-A}=0 \tag{13}$$

$$\theta''+C\,\gamma\left(1+\gamma\,\theta\right)^{-1}\theta'^2+\Pr\,E_c\left(1+\gamma\,\theta\right)^{A-C}f'^2$$

$$+M\Pr\,E_c\left(1+\gamma\,\theta\right)^{-C}\left(1-f\right)^2$$

$$+2\Pr\left(1+\gamma\,\theta\right)^{-C}\eta\,\theta'\ \ +v_0\Pr\left(1+\gamma\,\theta\right)^{-C}\theta'=0 \tag{14}$$

The corresponding boundary conditions are obtained from equation (5) as,

$$f(0)=0,\ \theta(0)=1,\ f(\infty)=1,\ \theta(\infty)=0 \tag{15}$$

Prime denotes d/dn in the above equations.

The chief physical interests are the skin friction coefficient and the rate of heat transfer to the plate, are calculated out. The wall shearing stress (τ) is:

$$\tau = \left(\mu \frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{2} U_0^2 \, \rho \left(1+\gamma\right)^A R_e^{-\frac{1}{2}} f'(0)$$

Hence the skin friction (C_f) is

$$\left(1+\gamma\right)^{-A} R_e^{\frac{1}{2}} C_f = f'(0) \tag{16}$$

From the plate to the fluid the rate of heat transfer (q) is computed by the application of Fourier's law as given in the following:

$$R_e^{-\frac{1}{2}} \left(1+\gamma\right)^{-C} \; Nu = -\theta'(0) \, .$$

Hence the Nusselt number (Nu) is determined as $R_e^{-\frac{1}{2}} (1+\gamma)^{-C} \; Nu = -\theta'(0)$ (17)

Where,

$R_e^{\frac{1}{2}} = \left(\frac{\delta \, U_0}{v_\infty}\right)$ is the square root of local Reynolds number.

In equations (16)-(17), the gradient values of f and θ at the plate are evaluated when the corresponding differential equations are solved satisfying the convergence criteria.

Numerical Solutions

The set of coupled, nonlinear ordinary differential equations (13)-(14) are solved numerically using a standard initial value solver called the shooting method. For this purpose Nachtsheim and Swigert [23] iteration technique has been employed. A computer program was made for the solutions of the basic non-linear differential equations of our problem by adopting this numerical technique and six ordered Range-Kutta method of integration. In different phases various groups of parameters G_r, M, γ, E_c and v_0 were considered.

The step size $\Delta\eta=0.01$ was selected that satisfied a convergence criterion of 10^{-6} in almost all phases mentioned above in all computations. By setting $\eta_{\infty} = \eta_{\infty} + \Delta\eta$ the value of η_∞ was found to each iteration loop. Thus $(\eta_\infty)_{max}$, to each group of parameters, has been determined when the value of unknown boundary conditions at $\eta=0$ (that arise in the Shooting method) does not change to successful loop with error less than 10^{-6}. However, different step sizes such as $\Delta\eta=0.01$, $\Delta\eta=0.005$ and $\Delta\eta=0.001$ were also tried and the obtained solutions have been found to be independent of the step sizes observed in (Figure 1).

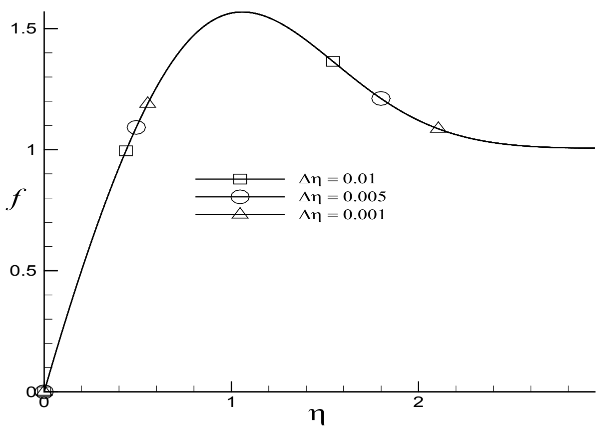

Figure 1: Velocity profiles for different step sizes.

Results and Discussion

As a result of the numerical calculation, the velocity and temperature distributions for the flow are obtained from equations (13)-(14) and are shown in (Figures 2-6) for different values G_r, M, γ, E_c and v_0. The fluid is considered as flue gas (P_r – 0.64) for the present study. The effects of G_r (Grashof number) on the velocity and temperature profiles are shown in (Figure 2). It appears from (Figure 2). That the temperature profile shows its usual trend of gradual decay when $G_r=0$ that is for forced convection. The profiles overshoot the uniform temperature close to the boundary when Grashof number G_r becomes larger. (Figure 2) also shows that the velocity profiles increase as G_r increases.

Figure 3 represents the effects of γ on the velocity and temperature profiles for $G_r=0$, M=0.5, $E_c=0.5$, $v_0=0.5$. In this figure comparison is made between constant property and variable property solutions. In (Figure 3), it is observed that the velocity profiles decreases for increasing values of temperature relative parameter γ, this figure also shows that for increase value of γ leads to the increase the values of temperature. It is also observed that the velocity profile increases and temperature profile decreases for negatively increasing values of relative temperature different parameter γ.

The act of imposing of a magnetic field to an electrically conducting fluid creates a drag like force called Lorentz force. The force has the tendency to slow down the flow around the plate at the expense of

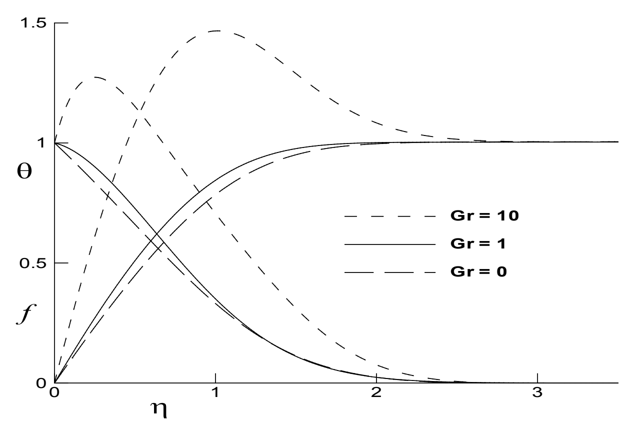

Figure 2: Effects of G_r on velocity and temperature profiles.

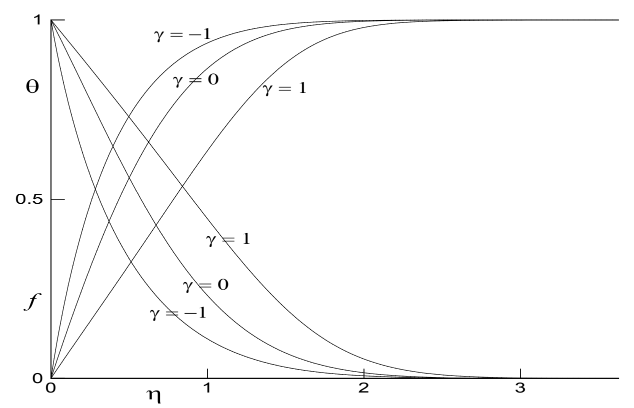

Figure 3: Effects of γ on velocity and temperature profiles for Gr=0.

increasing its temperature. This is depicted by decreases in velocity profiles and increases in the temperature profiles as M increases as shown in (Figure 4) In addition, the increases in the temperature profiles as M increases are accompanied by increases in the thermal boundary layer.

The effects of Eckert number (E_c) on velocity profiles and temperature profiles for G_r=10, M=0.5, γ=0.5, v_0=0.5 and P_r=0.64 shown in (Figure 5). In the equation of continuity and momentum equation, the dimensionless Eckert number (E_c) does not enter directly its influence come through the energy equation. It has been found that the Eckert number (E_c) has marked effect on velocity profiles in presence of buoyancy force. The velocity profiles increase as Eckert number increases shown in (Figure 5).

It has also been observed from this figure that the parameter E_c has remarkable effect on the temperature profiles. The increasing values of E_c leads to the increasing value of temperature profile. It indicates that the thermal boundary layer thickness increases with increasing values of Eckert number. For G_r=10, M=0.5, γ=0.5, E_c=0.5, (Figure 6) shows the effects of suction/injection parameter v_0 on the velocity profiles and the temperature profiles. It has been observed from this figure that the velocity profile decreases for increasing values of injection (v_0>0) while the velocity profile increases for increasing value of suction parameter (v_0<0). It implies that the boundary layer is increasingly blown away from the plate to form an interlayer between the suction and the outer flow regions. It has also been observed from this figure that the temperature profiles decrease for increasing values of v_0 and increase for negatively increasing values of v_0. The thermal boundary layer increases rapidly for strong suction while the temperature usual decays for strong injection.

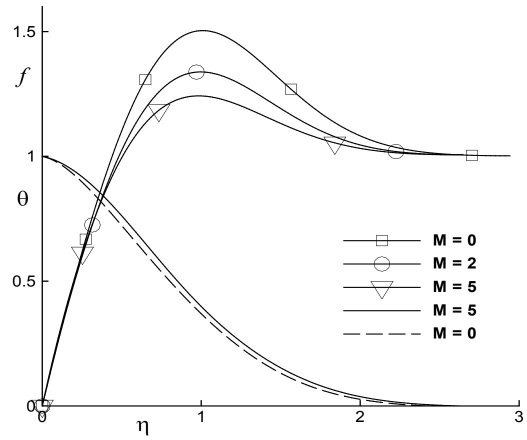

Figure 4: Effects of *M* on velocity and temperature profiles.

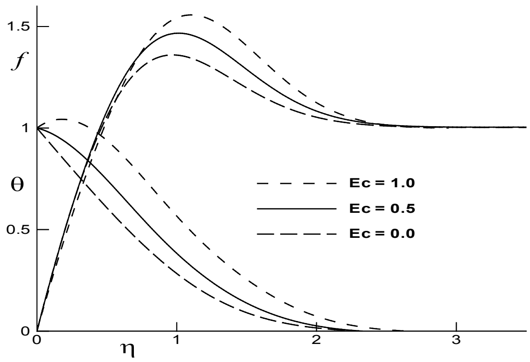

Figure 5: Effects of Ec on velocity and temperature profiles.

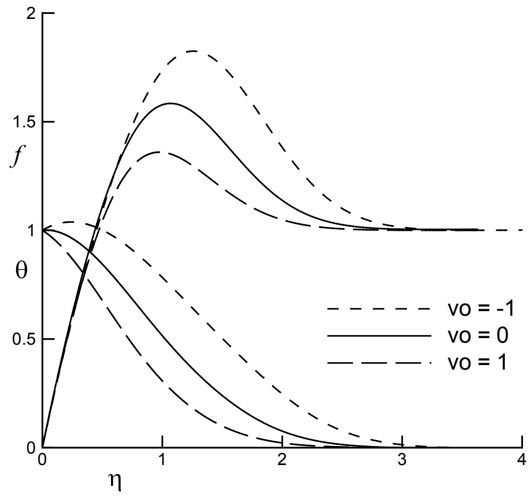

Figure 6: Effects of v_0 on velocity and temperature profiles.

γ	f′(0)	−θ′(0)
-1.5	6.4013	18.9261
-1.0	3.2132	9.1721
-0.5	1.9042	5.1378
0.0	1.3142	3.2430
0.5	1.0245	2.2272
1.0	0.8638	1.6206
1.5	0.7573	1.2322

Table 1: The skin-friction coefficient and Nusselt number for different values of γ.

Table 1 shows the effects of temperature related parameter γ on the skin-friction coefficient and the heat transfer co-efficient. From Table 1, it is investigated that the increase of relative temperature difference parameter γ leads to decrease both the shearing stress (skin-friction coefficient) and the heat flux (Nusselt number). These are found to in agreement with the effects of γ on the boundary layer (velocity profiles) and thermal boundary layer thicknesses (temperature profiles) (Table 1).

Conclusions

In the present investigation, the effects of suction/injection along with the variable properties on MHD combined convective unsteady laminar incompressible boundary layer flow were studied. We obtained the nonlinear ordinary differential equations by introducing the similarity variables in basic equations. Using Range-Kutta six order integration schemes and Nachtsheim-Swigert iteration technique differential equations are then solved numerically.

As a result of the computations the following conclusions can be made:

1. Grashof number (G_r) has marked effects on velocity and temperature fields. Both velocity and temperature profiles increase as G_r increases.

2. Variable properties (γ) have marked effects on the velocity and temperature profiles. It is shown increasing value relative temperature different parameter γ leads the decrease of the non-dimensional velocity profile and increase of temperature profile.

3. We investigate that due to the application of a transverse magnetic field normal to the flow direction will result in a resistive force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reduces its velocity. As a result the magnetic interaction parameter M the boundary layer thickness decreases and thermal boundary layer thickness increases.
4. It has been observed that although Eckert number E_c does not involve directly in boundary layer equation but it has marked effect on velocity profile through thermal boundary layer equation by buoyancy force. Both the velocity and the temperature profiles increase with the increasing values of E_c.
5. It is observed that the temperature profiles decrease for increasing values of v_0 and increase for negatively increasing values of v_0. The temperature usually decays for strong injection while the thermal boundary layer increases rapidly for strong suction.
6. Increasing the values of γ (-1, to 1) lead to the decrease in skin friction coefficient and heat transfer coefficient for fixed values of M, E_c and v_0.

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