Synergies Between Computer Science and Mathematical Logic

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Description

This is a look at some of the many ways that computer science and mathematical logic interact. The general idea is that computer science provides new ways of looking at logical problems and emphasizes the importance of areas of logic that might otherwise have been overlooked. Computer science has strong connections with many aspects of mathematical logic, but those aspects are sometimes different from those that are typically studied for pure mathematical purposes. Additionally, mathematical logic provides tools for understanding and unifying topics in computer science. This paper examines a few of those interactions in detail. The clarification of computational concepts by logic and the incorporation of new concepts and questions into logic by computation are its two primary themes. This section talks about some of the first interactions. Because they do not involve resource limitations, they are referred to as computability theory or recursion theory rather than computer science. Nonetheless, I think it's important to mention them to emphasize the role that mathematical logic plays in a wide range of mathematics-related issues [1].

This section discusses first-order logic, in particular the idea of structure that serves as the foundation for the semantics of first-order logic. It makes the point that the same idea of structure is suitable for numerous roles in computer science. In addition, I present a computational perspective on first-order logic's expressive potential. The central non-first-order concept in computation, iteration, is added to first-order logic to create logical systems in this section. Fixed-point operators, which are involved in the logical characterizations of various complexity classes, provide the logical manifestation of iteration. This paper's subsequent sections all involve fixed-point operators. The very broad hypothesis of Yuri Gurevich that polynomial-time computation on general finite structures cannot be precisely captured by any logic (in the broad sense of "logic") serves as the foundation for this section. I also describe a logic called "choiceless polynomial time" that is surprisingly close to capturing PTime and discuss the possibility that it actually serves as a counterexample to Gurevich's hypothesis.

A rather distinct type of logic is the focus of this section. It is derived from first-order logic by omitting the universal quantifier and adding a fixed-point operator simultaneously. It turns out that this existential fixed-point logic is mathematically elegant and has strong connections to computation. Although a reasonable set of axioms and rules governing the behavior of a fixed-point operator can be written down in a reasonable way, fixed-point logics do not permit a complete deductive system in the traditional sense. The inquiry then emerges whether the undeniable inadequacy of such a framework influences just muddled, Gödel-style sentences or whether exceptionally straightforward bits of insight may be unprovable. In this section, I talk about the system I'm thinking of and end with a question about a simple fact that might be hard to prove. In each of these instances, the initial challenge was to locate an algorithm. After describing the equations under consideration, Hilbert's tenth problem states, "One is to give a procedure by which it can be determined by a finite number of operations whether the equation is solvable in rational integers," for instance. In a similar vein, the word problem is stated by Dehn as "one is to give a method to decide, with a finite number of steps, whether [a given] element is equal to the identity or not [4].

The decision problem was described by Hilbert and Ackermann as "The decision problem is solved if one knows a procedure which, given a logical expression, allows one to decide its validity resp. by finitely many operations." its satisfiability Logicians provided a brand-new perspective on such issues. They substituted "Is there an algorithm...?" for "Find an algorithm..." That is, they introduced the idea that an algorithm might not exist and, more importantly, that such a result might be able to be proven. This indicates that one requires a precise mathematical concept of algorithm, or at least of algorithmic computability, as opposed to the prevalent notion of algorithm at the time, which was "I'll know it when I see it." These definitions were independently provided by Church employing the -calculus and by Turing employing what are now referred to as Turing machines. The results of undecidability, also known as theorems claiming that there are no algorithms for various problems, came next. The decision problem's undecidability for first-order logic was first demonstrated by Turing [3-5].

Acknowledgement

Conflict of Interest

None.

References

1. Pardé, Mickaël, J-P. Wigneron, Philippe Waldteufel and Yann H. Kerr et al. "N-parameter retrievals from L-band microwave observations acquired over a variety of crop fields." *IEEE Trans Geosci Remote Sens* 42 (2004): 1168-1178.

2. Njoku, Eni G., Thomas J. Jackson, Venkat Lakshmi and Tsz K. Chan et al. "Soil moisture retrieval from AMSR-E." *IEEE Trans Geosci remote sens* 41 (2003): 215-229.

3. Scharlemann, Jörn PW, David Benz, Simon I. Hay and Bethan V. Purse et al. "Global data for ecology and epidemiology: a novel algorithm for temporal fourier processing MODIS data." *PloS one* 3 (2008)

4. Paloscia, Simonetta, Giovanni Macelloni, Emanuele Santi and Toshio Koike. "A multifrequency algorithm for the retrieval of soil moisture on a large scale using microwave data from SMMR and SSM/I satellites." *IEEE Trans Geosci Remote Sens* 39 (2001): 1655-1661.

5. Owe, Manfred, Richard de Jeu and Jeffrey Walker. "A methodology for surface soil moisture and vegetation optical depth retrieval using the microwave polarization difference index." *IEEE Trans Geosci Remote Sens* 39 (2001): 1643-1654.

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