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Supplier Selection of Foreign Trade Sourcing Company using ANP-VIKOR Method in Hesitant Fuzzy Environment

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Abstract

International supplier selection which includes different criteria can be regarded as a kind of multi-criteria decision making (MCDM) problems. By combining the analytic network process (ANP) with the Vlsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method in hesitant fuzzy (HF) environment, this paper proposes a HF-ANP-VIKOR method. First, a novel HF-ANP approach is presented to determine the weight of each criterion. In this approach, the preference relations between criteria are hesitant fuzzy preference relations (HFPRs) whose elements are hesitant fuzzy elements (HFEs). According to the distance between two HFPRs, a new compatibility measure for HFPRs is proposed to measure the compatibility degrees of HFPRs. If the HFPRs are acceptable compatibility, they are converted into fuzzy preference relations by which the weights of sub-criteria are determined. Subsequently, extending the classical VIKOR method into HF environment, a new HF-VIKOR method is put forward to rank the alternatives. Finally, a case of Nantong uasia import and export limited company is studied to illustrate the practicability and effectiveness of the HF-ANP-VIKOR method proposed in this paper.

Keywords: Supplier selection; MCDM; ANP; VIKOR; HFS

Introduction

With the development of globalization, lots of companies choose multinational operations to help them seek profits from development differences among nations. Foreign trade sourcing company provides foreign customers agency service, including labor-intensive, low value-added primary products. According to customer demand, foreign trade sourcing company seeks sources, production, arrange transportation, export process and other value-added services. International sourcing process is quite complicated because it includes many procedures, such as selecting suppliers, confirming and sending the sample, examining the goods, booking cargo space, applying to the customs, exchange settlement and tax return. Figure 1 intuitively depicts these procedures.

In these procedures, the supplier selection may directly impact the purchase cost of enterprises. Usually the purchase cost reaches about 60% of the total cost in the enterprise, sometimes reaches more than 80%. It can be seen that an appropriate supplier can decrease the costs of the company. Therefore, supplier selection is a very important procedure. However, selecting an appropriate suppler is very complicated because many indicators have to be involved. Hence, supplier selection could be considered as a kind of multi-criteria decision making (MCDM) problems.

In general, existing research on selecting suppliers by using MCDM methods mainly focuses on two crucial issues: the evaluation criteria determination and the MCDM methods, which are briefly reviewed as follows.

Determination the criteria for supplier selection

To solve international supplier selection problems, first, the evaluation indices should be determined. Dickson [1] introduced price, quality, technology level and management indices. Eliram [2] suggested some hard targets and the soft targets. For example, product cost, quality and delivery are hard targets, whereas organization and management are soft targets. Choi [3] proposed price, technology status, financial and service indicators, and then used them to select the vendor of the US auto industry. Dowlatshahi [4] provided management, service and product development indictors. Sarkis and Talluri [5] evaluated suppliers using the quality, technology status, product cost and culture. The comparisons of the criteria studied in above works are shown in Table 1.

However, different kinds of companies will select diverse indicators according to their own scale and management tactics. Therefore, selecting criteria should be more cautious and evaluated by the authorities.

MCDM methods for supplier selection

The MCDM methods are mainly to rank the finite alternatives based on multiple criteria. Many researchers presented various MCDM

Criteria	Dickson [1]	Eliram [2]	Choi [3]	Dowlatshahi [4]	Sarkis and Talluri [5]
Price	√	√	√		
Quality	√	√			√
Technology status	√	√	√		√
Credit status	√				
Financial	√		√		
Product cost		√			√
Management	√			√	
service			√	√	
Product development				√	
Culture					√

Table 1: The criteria selection in different works.

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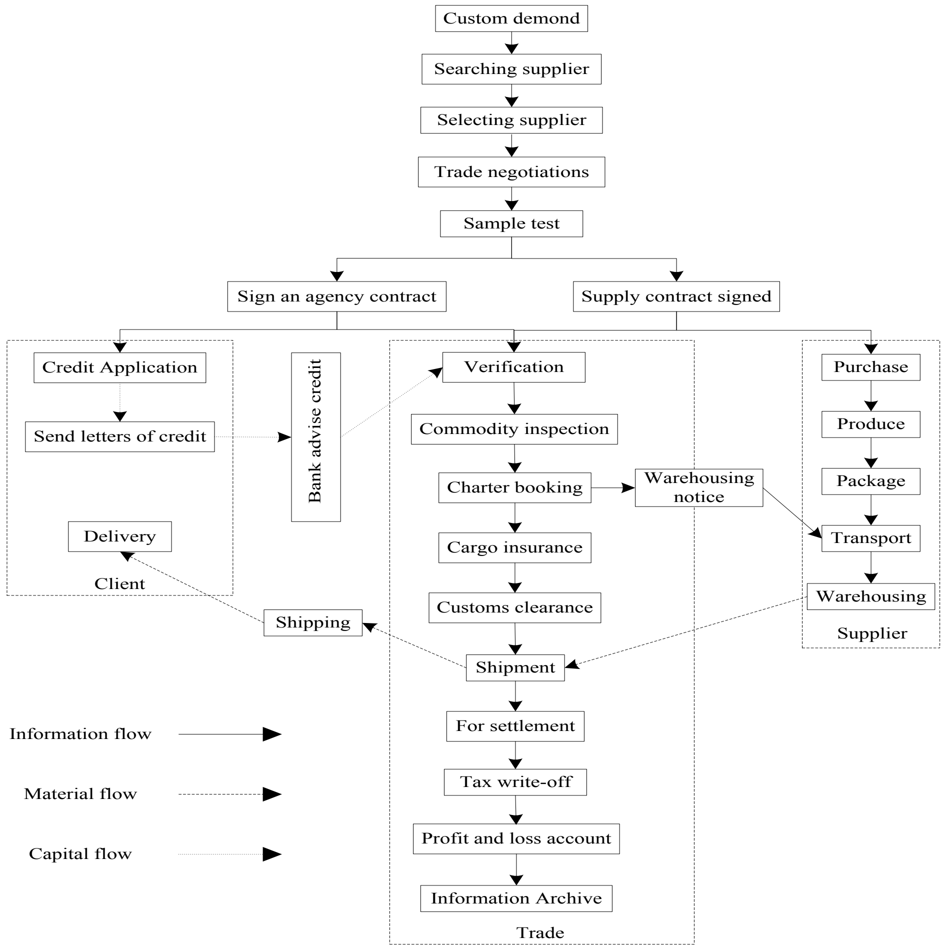

Figure 1: The flow of main business processes.

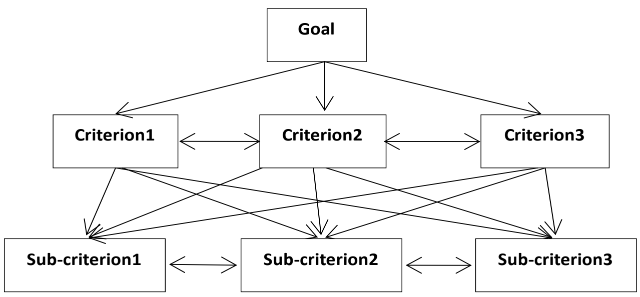

Figure 2: The network of ANP.

methods for selecting supplier [1-24]. These methods mostly are divided into two categories.

The first category is Single methods. The most popular MCDM method for selecting supplier is analytic hierarchy process (AHP) method [6]. The applications of AHP method in MCDM problem are briefly introduced as follows:

Hill and Nydick [7] employed AHP to rank alternatives by pair-wise comparisons. Although this method is feasible, the workloads are very large. To simplify the calculation, Yahya and Kingsman [8] improved method [7] and only used AHP to determine the criterion weights. The alternatives are ranked based on comprehensive scores of alternatives which are obtained by weighting sum of the scores of alternatives on each criterion. Liu and Hai [9] proposed a voting-based AHP to select alternatives.

In addition, other single MCDM methods for supplier selection are often employed, such as the analytic network process (ANP) (Figure 2), the multiple attribute utility theory (MAUT) method, the outranking method, the technique for order preference by similarity to ideal solution (TOPSIS) method. Sarkis and Talluri [10] proposed an ANP model for strategic supplier selection. Min [11] presented a MAUT-based analytical approach to evaluate various international sourcing strategies under dynamically changing scenarios. De et al. [12] took an outranking approach as a suitable decision making tool for selecting supplier. Chen et al. [13] presented a closeness coefficient to determine the ranking order of all suppliers by calculating the distances between the alternatives and the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS).

The second category is hybrid methods. Jasmine [25] proposed an analytical approach combining quality function deployment (QFD) and ANP for guiding shipping companies' design. Kaya and Kahraman [26] presented an integrated the VIKOR-AHP methodology, in which the weights of the selection criteria were derived by fuzzy AHP. Hsu et al. [27] discussed the recycled material vender selection problems and

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built a new MCDM model combining DEMATEL (decision-making trial and evaluation laboratory)-based ANP (DANP) with VIKOR. Nilashi et al. [28] used a DEMATEL-ANP based MCDM approach to evaluate the critical success factors in construction projects.

Although the above methods have advantages for supplier selection, there are some drawbacks which limit the applications of these methods. For example, these methods are unable to deal with the MCDM problems where several possible values may be supplied by decision makers (DMs) due to the ambiguity of human thinking. Hesitant fuzzy set (HFS) [29] is a proper tool to handle such a type of MCDM problems.

In this paper, a novel ANP-VIKOR method is proposed to solve MCDM problems in hesitant fuzzy (HF) environment. First, a HF-ANP approach is presented to determine the weights of criteria. A notable characteristic of this approach is that the elements of preference relation matrices are hesitant fuzzy elements (HFEs) which can more flexibly express the preferences of experts. When the values of criteria are in the form of HFEs, a new HF-VIKOR method is proposed and applied to rank the alternatives .The key features of the proposed method in this paper are listed as follows:

(1) Considering the interactions among criteria, we firstly extend ANP method into HF environment and propose HF-ANP to determine the weight of each criterion in supplier selection. Due to the fact that HFS permits the membership has a set of possible values, it is more suitable to use HFEs to describe the preferences of experts.

(2) In HF-ANP approach, we first propose a new measure to calculate the compatibility degree between hesitant fuzzy preference relations (HFPRs). If a HFPR is acceptable compatibility, we use the score function to convert the HFPRs to fuzzy preference relations (FPRs). Thus, using FPRs is much easier than using HFPRs to determine the weights of criteria.

(3) We propose a novel HF-VIKOR approach to rank alternatives. Concretely, we generalize the scope of the applications of the VIKOR method, which makes the VIKOR method solve more MCDM problems in different environments.

The paper is organized as follows. In Section 2, we briefly review the concepts, such as the score function, some operations of HFS and HFPR. In Section 3, we present the method of HF-ANP, HF-VIKOR and the HF-ANP-VIKOR method. In Section 4, a practical example of international supplier selection for foreign trade sourcing company with the ANP-VIKOR method in HFS environment. Finally, the conclusion is presented in Section 5.

Preliminaries

In this section, we briefly review some basic concept and properties about HFS and HFPR.

Definition 1 [29,30]. Let X be a fixed set. It is defined as a HFS on X in terms of a function that $h_E(x)$ when applied to X returns a subset of $[0,1]$

$$E = \{< x, h_E(x) > | x \in X\}, \tag{1}$$

where $h_E(x)$ is a set of some different values in $[0,1]$, denoting the possible membership degrees of the element $x \in X$ to the set E. For convenience, Xu and Xia [31] called $h_E(x)$ HFEs denoted by $h = \{\gamma_1, \gamma_2, \ldots, \gamma_{l(h)}\}$, where $l(h)$ is the number of all elements in h. The elements in a HFE h are in an increasing order.

For any two HFEs h_1 and h_2, if $l(h_1) \neq l(h_2)$, we extend the shorter one by adding the minimum element of it until both of HFEs have the same length. For example, let $h_1 = \{0.2, 0.3\}$ and $h_2 = \{0.3, 0.4, 0.5\}$. We can extend h_1 to $h_1' = \{0.2, 0.2, 0.3\}$.

For three HFEs h, h_1 and h_2, the following operations are defined as [29,30]:

1) Lower bound: $h^-(x) = \min h(x)$;

2) Upper bound: $h^+(x) = \max h(x)$;

3) Complement: $h^c = \bigcup_{\gamma \in h} \{1 - \gamma\}$;

4) $h^\lambda = \bigcup_{\gamma \in h} \{\gamma^\lambda\}$;

5) $\lambda h = \bigcup_{\gamma \in h} \{1 - (1-\gamma)^\lambda\}$;

6) $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2\}$;

7) $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 \gamma_2\}$;

Let $h_j \ (j = 1, 2, \ldots, n)$ be a collection of HFEs. Liao et al. [32] generalized 6) and 7) to the following forms:

8) $\overset{n}{\underset{j=1}{\oplus}} h_j = \bigcup_{\gamma_j \in h_j} \{1 - \prod_{j=1}^{n} (1 - \gamma_j)\}$;

9) $\overset{n}{\underset{j=1}{\otimes}} h_j = \bigcup_{\gamma_j \in h_j} \{\prod_{j=1}^{n} \gamma_j\}$.

Definition 2 [33]. Let $h_1 = \{\gamma_1^{(1)}, \gamma_2^{(1)}, \ldots, \gamma_{l(h_1)}^{(1)}\}$ and $h_2 = \{\gamma_1^{(2)}, \gamma_2^{(2)}, \ldots, \gamma_{l(h_2)}^{(2)}\}$ be two HFEs. The Manhattan distance between HFEs is defined as:

$$d(h_1, h_2) = \frac{1}{L(h)} \sum_{i=1}^{L(h)} | \gamma_i^{(1)} - \gamma_i^{(2)} | \tag{2}$$

where $L(h) = \max\{l(h_1), l(h_2)\}$.

Example 1. Let $h_1 = \{0.2, 0.6\}$ and $h_2 = \{0.1, 0.5, 0.6\}$ be two HFEs. First, extend the h_1 to $h_1' = \{0.2, 0.2, 0.6\}$, then the Manhattan distance between h_1 and h_2 is calculated as

$$d(h_1, h_2) = \frac{|0.2 - 0.1| + |0.2 - 0.5| + |0.6 - 0.6|}{3} \approx 0.1333 \cdot$$

It can be clear that the Manhattan distance between two HFEs h_1 and h_2 is a crisp number, Eq. (2) is a useful tool for defuzzying two HFEs into a crisp number.

Definition 3 [34]. Let $h = \{\gamma_1, \gamma_2, \ldots, \gamma_{l(h)}\}$ be a HFE, where $l(h)$ denotes the number of all elements in h. A score function S of the HFE h is defined as

$$S(h) = \frac{\sum_{i=1}^{l(h)} i \gamma_i}{\sum_{i=1}^{l(h)} i} \tag{3}$$

For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then h_1 h_2.

Theorem 1 [34]. Let $h = \{\gamma_1, \gamma_2, \ldots, \gamma_{l(h)}\}$, $h_1 = \{\gamma_1^{(1)}, \gamma_2^{(1)}, \ldots, \gamma_{l(h_1)}^{(1)}\}$ and $h_2 = \{\gamma_1^{(2)}, \gamma_2^{(2)}, \ldots, \gamma_{l(h_2)}^{(2)}\}$ be three HFEs, then

1) $S(h) + S(h^c) = 1$;

2) $S(h_1 \cup h_2) = 1 - S(h_1^c \cap h_2^c)$;

3) $S(\lambda h) = 1 - S((h^c)^\lambda)$;

4) $S(h_1 \oplus h_2) = S(h_1) + S(h_2) - S(h_1 \otimes h_2)$;

5) $S(h_1 \oplus h_2) = S(h_1) + S(h_2) + S(h_1^c \oplus h_2^c) - 1$;

6) $S(h_1 \otimes h_2) = S(h_1) + S(h_2) + S(h_1^c \otimes h_2^c) - 1$.

Definition 4. Let $h_1 = \{\gamma_1^{(1)}, \gamma_2^{(1)}, \ldots, \gamma_{l(h_1)}^{(1)}\}$ and $h_2 = \{\gamma_1^{(2)}, \gamma_2^{(2)}, \ldots, \gamma_{l(h_2)}^{(2)}\}$ be two HFEs, Then the compatibility degree of h_1 and h_2 is defined as:

$$c(h_1, h_2) = \frac{\gamma_1^{(1)}\gamma_1^{(2)} + \gamma_2^{(1)}\gamma_2^{(2)} + \cdots + \gamma_{l(h_1)}^{(1)}\gamma_{l(h_2)}^{(2)}}{\max\{(\gamma_1^{(1)})^2 + (\gamma_2^{(1)})^2 + \cdots + (\gamma_{l(h_1)}^{(1)})^2, (\gamma_1^{(2)})^2 + (\gamma_2^{(2)})^2 + \cdots + (\gamma_{l(h_2)}^{(2)})^2\}} \quad (4)$$

The compatibility degree is used to express the similarity degree between two HFEs. It is obvious that the larger the value of $c(h_1,h_2)$, the greater the compatibility degree between h_1 and h_2.

Remark 2: In real-life decision making, all elements in h_1 or h_2 cannot be zero simultaneously. Therefore, we suppose that at most one HFE be zero in Definition 4.

Theorem 2. The compatibility degree $c(h_1,h_2)$ satisfies the properties:

i) $0 \le c(h_1, h_2) \le 1$; $c(h_1, h_2) = 1$, if and only if $h_1=h_2$;

ii) $c(h_1,h_2)=c(h_2,h_1)$.

Proof. i) Since $\gamma_1^{(1)}, \gamma_1^{(2)}, \gamma_2^{(1)}, \gamma_2^{(2)}, \cdots, \gamma_{l(h_1)}^{(1)}, \gamma_{l(h_2)}^{(2)} \in [0,1]$, we get $c(h_1,h_2) \ge 0$. In the following, we only have to prove $c(h_1,h_2) \le 1$.

Using Cauchy-Schwarz inequality, we have

$$\gamma_1^{(1)}\gamma_1^{(2)} + \gamma_2^{(1)}\gamma_2^{(2)} + \cdots + \gamma_{l(h_1)}^{(1)}\gamma_{l(h_2)}^{(2)}$$

$$\le \sqrt{(\gamma_1^{(1)})^2 + (\gamma_2^{(1)})^2 + \cdots + (\gamma_{l(h_1)}^{(1)})^2 \cdot (\gamma_1^{(2)})^2 + (\gamma_2^{(2)})^2 + \cdots + (\gamma_{l(h_2)}^{(2)})^2}.$$

$$\le \sqrt{(\max\{(\gamma_1^{(1)})^2 + (\gamma_2^{(1)})^2 + \cdots + (\gamma_{l(h_1)}^{(1)})^2 \cdot (\gamma_1^{(2)})^2 + (\gamma_2^{(2)})^2 + \cdots + (\gamma_{l(h_2)}^{(2)})^2\})^2}$$

$$= \max\{(\gamma_1^{(1)})^2 + (\gamma_2^{(1)})^2 + \cdots + (\gamma_{l(h_1)}^{(1)})^2, (\gamma_1^{(2)})^2 + (\gamma_2^{(2)})^2 + \cdots + (\gamma_{l(h_2)}^{(2)})^2\}$$

Therefore,

$$c(h_1, h_2) = \frac{\gamma_1^{(1)}\gamma_1^{(2)} + \gamma_2^{(1)}\gamma_2^{(2)} + \cdots + \gamma_{l(h_1)}^{(1)}\gamma_{l(h_2)}^{(2)}}{\max\{(\gamma_1^{(1)})^2 + (\gamma_2^{(1)})^2 + \cdots + (\gamma_{l(h_1)}^{(1)})^2, (\gamma_1^{(2)})^2 + (\gamma_2^{(2)})^2 + \cdots + (\gamma_{l(h_2)}^{(2)})^2\}} \le 1$$

If $\gamma_1^{(1)} = \gamma_1^{(2)}, \gamma_2^{(1)} = \gamma_2^{(2)}, \cdots, \gamma_{l(h_1)}^{(1)} = \gamma_{l(h_2)}^{(2)}$, i.e., $h_1=h_2$, then $c(h_1,h_2) =1$.

ii) From Eq. (4), we have

$$c(h_1, h_2) = \frac{\gamma_1^{(1)}\gamma_1^{(2)} + \gamma_2^{(1)}\gamma_2^{(2)} + \cdots + \gamma_{l(h_1)}^{(1)}\gamma_{l(h_2)}^{(2)}}{\max\{(\gamma_1^{(1)})^2 + (\gamma_2^{(1)})^2 + \cdots + (\gamma_{l(h_1)}^{(1)})^2, (\gamma_1^{(2)})^2 + (\gamma_2^{(2)})^2 + \cdots + (\gamma_{l(h_2)}^{(2)})^2\}}$$

$$= \frac{\gamma_1^{(2)}\gamma_1^{(1)} + \gamma_2^{(2)}\gamma_2^{(1)} + \cdots + \gamma_{l(h_1)}^{(1)}\gamma_{l(h_2)}^{(2)}}{\max\{(\gamma_1^{(2)})^2 + (\gamma_2^{(2)})^2 + \cdots + (\gamma_{l(h_2)}^{(2)})^2, (\gamma_1^{(1)})^2 + (\gamma_2^{(1)})^2 + \cdots + (\gamma_{l(h_1)}^{(1)})^2\}}$$

$$= c(h_2, h_1)$$

Namely, $c(h_1,h_2)=c(h_2,h_1)$

This proof is completed.

Definition 5 [32]. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set. A HFPR $\boldsymbol{H}$ on X is presented by a matrix $\boldsymbol{H} = (h_{ij})_{n\times n} \subset X \times X$, where $\boldsymbol{h}_{ij} = \{\gamma_{ij1}, \gamma_{ij2}, \ldots, \gamma_{ijl(h_{ij})}\}$ is a HFE indicating all the possible degrees to which x_i is preferred to x_j. Moreover, h_{ij} should satisfy the following conditions:

$\gamma_{ijl(h_{ij})} + \gamma_{jil(h_{ji})} = 1$, $\boldsymbol{h}_{ii} = \{0.5, 0.5, \ldots, 0.5\}$, $l(h_{ij}) = l(h_{ji})$, $i, j = 1, 2, \ldots, n$.

Definition 6. Let $\boldsymbol{H}^{(1)} = (\boldsymbol{h}_{ij}^{(1)})_{n\times n}$ and $\boldsymbol{H}^{(2)} = \{\boldsymbol{h}_{ij}^{(2)}\}_{n\times n}$ be two HFPRs, where $\boldsymbol{h}_{ij}^{(1)} = \{\gamma_{ij1}^{(1)}, \gamma_{ij2}^{(1)}, \ldots, \gamma_{ijl(h_{ij})}^{(1)}\}$ and $\boldsymbol{h}_{ij}^{(2)} = \{\gamma_{ij1}^{(2)}, \gamma_{ij2}^{(2)}, \ldots, \gamma_{ijl(h_{ij})}^{(2)}\}$. Then the compatibility degree of $\boldsymbol{H}^{(1)}$ and $\boldsymbol{H}^{(2)}$ is defined as:

$$c(\boldsymbol{H}^{(1)}, \boldsymbol{H}^{(2)}) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij1}^{(1)}\gamma_{ijl(h_{ij})}^{(2)} + \gamma_{ij2}^{(1)}\gamma_{ij(l(h_{ij})-1)}^{(2)} + \cdots + \gamma_{ijl(h_{ij})}^{(1)}\gamma_{ij1}^{(2)}}{\max\{\sum_{i=1}^{n}\sum_{j=1}^{n}(\gamma_{ij1}^{(1)})^2 + (\gamma_{ij2}^{(1)})^2 + \cdots + (\gamma_{ijl(h_{ij})}^{(1)})^2, \sum_{i=1}^{n}\sum_{j=1}^{n}(\gamma_{ij1}^{(2)})^2 + (\gamma_{ij2}^{(2)})^2 + \cdots + (\gamma_{ijl(h_{ij})}^{(2)})^2\}} \quad (5)$$

Theorem 3. The compatibility degree $c(\boldsymbol{H}^{(1)}, \boldsymbol{H}^{(2)})$ satisfies the properties:

i) $0 \le c(\boldsymbol{H}^{(1)}, \boldsymbol{H}^{(2)}) \le 1$;

ii) $c(\boldsymbol{H}^{(1)}, \boldsymbol{H}^{(2)}) = 1$, if and only if $\boldsymbol{H}^{(1)} = \boldsymbol{H}^{(2)}$;

iii) $c(\boldsymbol{H}^{(1)}, \boldsymbol{H}^{(2)}) = c(\boldsymbol{H}^{(2)}, \boldsymbol{H}^{(1)})$.

The proof procedure is similar to that of Theorem 3.

Definition 7. Let $\boldsymbol{H}^{(k)} = (\boldsymbol{h}_{ij}^{(k)})_{t\times t}$ $(k = 1, 2, \ldots, t)$ be t individual HFPRs, the HFPR $\overline{\boldsymbol{H}} = (\overline{\boldsymbol{h}}_{ij})_{t\times t}$ is called the collective HFPR.

where $\overline{\boldsymbol{h}}_{ij} = \{\overline{\gamma}_{ij1}, \overline{\gamma}_{ij2}, \ldots, \overline{\gamma}_{ij\rho}\}$ $(\rho = 1, 2, \ldots, l(h_{ij}^k))$, the values of $\overline{\gamma}_{ij\rho}$ calculate as:

$$\overline{\gamma}_{ij\rho} = \frac{1}{l(h_{ij})}\sum_{\rho=1}^{l(h_{ij})}\gamma_{ij\rho}^{(k)} \quad (6)$$

Definition 8. $c(\boldsymbol{H}^{(k)}, \overline{\boldsymbol{H}})$ is called the compatibility measure of HFPR $\boldsymbol{H}^{(k)}$. If $c(\boldsymbol{H}^{(k)}, \overline{\boldsymbol{H}}) = 1$, we call $\boldsymbol{H}^{(k)}$ perfect compatibility. If $c(\boldsymbol{H}^{(k)}, \overline{\boldsymbol{H}}) \ge \delta_0$, we call $\boldsymbol{H}^{(k)}$ acceptable compatibility, where δ_o is the threshold value of acceptable compatibility. Usually, we take $\delta_o \in [0.5, 1]$ in real decision making.

Theorem 4. Each individual HFPR and the collective HFPR are perfectly compatible if and only if any two individual HFPRs are perfectly compatible. i.e.,

$c(\boldsymbol{H}^{(k)}, \overline{\boldsymbol{H}}) = 1$ if and only if $c(\boldsymbol{H}^{(k)}, \boldsymbol{H}^{(l)}) = 1$, for all $(k, l = 1, 2, \ldots, t)$.

Proof. Sufficiency:

From Theorem 4, if $c(\boldsymbol{H}^{(k)}, \overline{\boldsymbol{H}}) = 1$ for any $k = 1, 2, \ldots, t$, then we know that $\boldsymbol{H}^{(k)} = \overline{\boldsymbol{H}}$, Therefore, for any $k, l = 1, 2, \ldots, t$, we have $\boldsymbol{H}^{(k)} = \overline{\boldsymbol{H}} = \boldsymbol{H}^{(l)}$. Hence, $c(\boldsymbol{H}^{(k)}, \boldsymbol{H}^{(l)}) = 1$.

Necessity:

If $c(\boldsymbol{H}^{(k)}, \boldsymbol{H}^{(l)}) = 1$ for any $k, l = 1, 2, \ldots, t$, then we have $\boldsymbol{H}^{(k)} = \boldsymbol{H}^{(l)}$. Therefore, for any $i, j = 1, 2, \ldots, n$, we acquire $\boldsymbol{h}_{ij}^k = \boldsymbol{h}_{ij}^l$. i.e., $\gamma_{ij\rho}^{(k)} = \gamma_{ij\rho}^{(l)}$ $(\rho = 1, 2, \ldots, l(h_{ij}^k))$. Thus, from Eq. (6), we get $\overline{\gamma}_{ij\rho} = \frac{1}{t}\sum_{k=1}^{t}\gamma_{ij\rho}^{(k)} = \overline{\gamma}_{ij\rho}^{k}$ $(\rho = 1, 2, \ldots, l(h_{ij}^k))$. Hence, for any $k = 1, 2, \ldots, t$, $\boldsymbol{h}_{ij}^k = \overline{\boldsymbol{h}}_{ij}$. Accordingly, we obtain $\boldsymbol{H}^{(k)} = \overline{\boldsymbol{H}}$.

Thereby, $c(\boldsymbol{H}^{(k)}, \overline{\boldsymbol{H}}) = 1$.

This completes the proof of the Theorem 5.

A Novel Method for Solving MCDM Problems in HF Environment

For a MCDM problem, suppose that G is the goal, C_i. $(i = 1, 2, \ldots, n)$ are the criteria and c_{ij} is the j-th sub-criterion of the i-th criterion, where

$j = 1,2,\ldots,i_q$ and i_q is the number of the sub-criteria of the criterion C_i. A_l $(l = 1,2,\ldots,m)$ are the alternatives, indicate the k-th expert. The ratings of the alternative $A_l(l = 1,2,\ldots,m)$ regarding to the sub-criterion $c_o(o = 1,2,\ldots,n)$ are give as , $\boldsymbol{w}$ is the weight vector of the sub-criteria with respect to the goal.

Determining the criterion weights by HF-ANP approach in HF environment

In this sub-section, we firstly introduce the classical ANP method, and then extend the ANP into the HF environment.

The classical ANP method: The ANP method [35] is generalized from AHP method. The AHP method claims that criteria in the same layer are mutually independent. However, the ANP method allows that the criteria in the same layer are interactive. The procedure of ANP is introduced step by step as follows:

(1) The network construction. The problem should be stated clearly and be decomposed into a network structure. An example of the network is shown in Figure 2. The network model is composed of three levels, including goal level, criterion level and sub-criterion level.

(2) Weighting matrix determination. Similar to the comparisons performed in the AHP, the weighting matrix expresses the degrees of interaction between criteria in the network, which means the relative importance of each C_i $(i$=1,2,…,$n)$ with respect to the goal. To obtain the weighting matrix $\boldsymbol{A}$ we can calculate the eigenvalues and eigenvectors for pair-wise matrices.

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

where column vectors in $\boldsymbol{A}$ are the weight vectors expressing the influence degree of C_i on C_j $(i,j$=1,2,…,$n)$ under the goal.

(3) Super-matrix formation. The super-matrix concept is similar to the Markov chain process [35]. It denotes the degree of mutual effect between sub-criteria under the criterion level. In other words, it expresses the degree to which one sub-criterion is preferred to the other. The calculation process for super-matrix $\boldsymbol{W}$ is the same as that of weighting matrix.

$$\boldsymbol{W} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & w_{nn} \end{bmatrix}$$

where column vectors in $\boldsymbol{W}$ are the weight vectors expressing the impact degrees of the sub-criteria in C_i on the sub-criteria in C_j.

(4) Weighting super-matrix. The degree of mutual influence among the criteria in ANP can be expressed by weighting super-matrix $\overline{\boldsymbol{W}}$, which is computed as

$$\overline{\boldsymbol{W}} = \boldsymbol{A} \cdot \boldsymbol{W} \tag{7}$$

(5) Determine the sub-criteria weights. Calculate the limit of weighting super-matrix. The elements in the each row of weighting super-matrix will tend to the same value which denotes the corresponding weight of each sub-criterion over the goal.

$$\boldsymbol{W}^{\infty} = \lim_{n\to\infty} \boldsymbol{W}^{n} \tag{8}$$

The HF-ANP approach: In most situations, crisp data are insufficient to model real-life situations. Since human judgments or preferences are often vague and cannot evaluate his preferences with real numbers. Due to the fact that the HFS can express the degree of fuzzy clearly, it is more suitable to use HFSs to deal with the human judgments or preferences.

The HF-ANP is composed of six steps:

Step 1: For criterion C_k $(k = 1,2,...,n)$, expert construct the following HFPR by comparing criteria.

$$\boldsymbol{H}^{(k)} = \begin{bmatrix} \boldsymbol{h}_{11}^{(k)} & \boldsymbol{h}_{12}^{(k)} & \cdots & \boldsymbol{h}_{1j}^{(k)} \\ \boldsymbol{h}_{21}^{(k)} & \boldsymbol{h}_{22}^{(k)} & \cdots & \boldsymbol{h}_{2j}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{h}_{i1}^{(k)} & \boldsymbol{h}_{i2}^{(k)} & \cdots & \boldsymbol{h}_{ij}^{(k)} \end{bmatrix}$$

Step 2: Measure the compatibility degree of HFPR $\boldsymbol{H}^{(k)}$ $(k = 1,2,\ldots,n)$ by Definition 7 and Definition 8.

Step 3: Determine weighting matrix $\boldsymbol{A} = (a_{ij})_{n\times n}$.

First, using Eq. (3), HFPR $\boldsymbol{H}^{(k)}$ is converted into a FPR $\boldsymbol{S}^k = (s_{ij}^k)_{n\times n}$. The weights of FPR $\boldsymbol{S}^k$ can be calculated as:

$$\hat{\boldsymbol{s}}_{ij}^{k} = \frac{\boldsymbol{h}_{ij}^{(k)}}{\sum_{i=1}^{n} \boldsymbol{h}_{ij}^{(k)}} \tag{9}$$

$$\boldsymbol{w}_i^k = \tfrac{1}{n} \sum_{j=1}^{n} \hat{\boldsymbol{s}}_{ij}^{k} \quad (0 \le w_i^k \le 1) \tag{10}$$

Then, the weighting matrix $\boldsymbol{A} = (a_{ij})_{n\times n}$ can be determined, where $\boldsymbol{A} = (\boldsymbol{w}^1, \boldsymbol{w}^2, ..., \boldsymbol{w}^n)$ and $\boldsymbol{w}^k = (\boldsymbol{w}_1^k, \boldsymbol{w}_2^k, ..., \boldsymbol{w}_n^k)$. As the similar way in determining $\boldsymbol{A}$, we can determine the super-matrix $\boldsymbol{w}$.

Step 4: By Eq. (7), the weighted super-matrix $\overline{\boldsymbol{W}}$ is derived.

Step 5: Utilizing Eq. (8), the limit of weighted super-matrix can be calculated to obtain the weight of sub-criteria over alternatives. Then the weight vector $\boldsymbol{w}^{\mathrm{T}}$ is acquired.

A HF-VIKOR approach to ranking alternatives

VIKOR is a compromise MCDM technique proposed by Opricovic and Tzeng [36]. This method determines compromise solutions to rank alternatives and select best one(s). The compromise solution is a feasible solution which is the closest to the ideal solution, and the "compromise" means an agreement established by mutual concessions [37].

The basic measure for compromise ranking is developed from the L_p-metric function which is used as an aggregation function in the compromise programming (Yu 1973) [38].

The compromise ranking procedure of the VIKOR method can be set up as follows:

(i) Determine the ideal solution $\boldsymbol{h}_o^+$ and negative solution $\boldsymbol{h}_o^-$ as

$$\boldsymbol{h}_o^+ = \begin{cases} \max\limits_{l}\{\boldsymbol{h}_{lo}\} & o \in \mathrm{F}_1 \\ \min\limits_{l}\{\boldsymbol{h}_{lo}\} & o \in \mathrm{F}_2 \end{cases} \tag{11}$$

$$\boldsymbol{h}_o^- = \begin{cases} \min\limits_l \{\boldsymbol{h}_{lo}\} & o \in \mathrm{F}_1 \\ \max\limits_l \{\boldsymbol{h}_{lo}\} & o \in \mathrm{F}_2 \end{cases} \tag{12}$$

where F_1 and F_2 are respectively the sets of benefit criteria and cost criteria.

Then the form of L_p-metric distance measure over the alternatives $A_l\ (l = 1,2,\ldots,m)$ in compromise programming was developed as:

$$L_{p,l} = ((\sum_{o=1}^{n} (w_o \frac{h_o^+ - \boldsymbol{h}_{lo}}{h_o^+ - h_o^-})^p)^{\frac{1}{p}}, (l = 1,2,...,m;\ o = 1,2,\ldots,n)\ ;\ p \ge 1 \tag{13}$$

where w_o is the weight vector of the criteria.

(ii) Compute the group utility and individual regret values of the alternatives as:

$$S_l = L_{1,l} \sum_{o=1}^{n} w_o \frac{d(h_o^+ - \boldsymbol{h}_{lo})}{d(h_o^+ - h_o^-)} \tag{14}$$

$$R_l = L_{\infty,l} = \max_l w_o \frac{d(h_o^+ - \boldsymbol{h}_{lo})}{d(h_o^+ - h_o^-)} \tag{15}$$

(iii) Calculate the value of Q_i as follows:

$$Q_l = v\frac{S_l - S^-}{S^+ - S^-} + (1-v)\frac{R_l - R^-}{R^+ - R^-} \tag{16}$$

where $S^- = \min\limits_l S_l$, $S^+ = \max\limits_l S_l$, $R^- = \min\limits_l R_l$ and $R^+ = \min\limits_l R_l$.

(iv) Rank the $A_l\ (l = 1,2,\ldots,m)$ by the values of S_i, R_i and Q_i.

(v) Determine a compromise solution the alternatives A' which is the best ranked by the measure Q (minimum) if the following two conditions should be satisfied:

C1. Acceptable advantage:

$Q(A'') - Q(A') \ge DQ$,

where A'' is the alternative with second position in the ranking list by Q; $DQ = 1/(m-1)$; m is the number of alternatives.

C2. Acceptable stability in decision making: The alternative A' must also be the best ranked by S or/and R. This compromise solution could be "voting by majority rule" (when v>0.5 is needed), or "by consensus" $v \approx 0.5$, or "with veto" (v<0.5). Here, v is the weight of decision making strategy "the majority of criteria" (or "the maximum group utility").

If one of the conditions is not satisfied, then a set of compromise solution is proposed, which consist of:

- Alternatives A' and A' if only condition **C2** is not satisfied, or

- Alternatives $A', A'', ..., A^{(M)}$ if condition **C1** is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A') < DQ$ for maximum M (the positions of these alternatives are "in closeness").

A novel ANP-VIKOR method to MCDM in HF environment

In this section, we extend the ANP-VIKOR in the HF environment to solve the MCDM problem.

The HF-ANP-VIKOR method is composed of eight major steps:

Step 1: Specify the criteria, sub-criteria and alternatives.

Step 2: Construct a network structure according to the relations among criteria.

Step 3: Construct the HFPRs $\boldsymbol{H}^{(k)}\ (k = 1,2,\ldots,n)$ and decision matrix.

Step 4: Measure the compatibility degree of HFPR $\boldsymbol{H}^{(k)}$ $(k = 1,2,\ldots,n)$ by Definition 7 and Definition 8.

Step 5: Use Eqs. (9)-(10), determine weighting matrix $\boldsymbol{A} = (a_{ij})_{n\times n}$ and super-matrix $\overline{\boldsymbol{W}}$.

Step 6: By Eqs. (7)-(8), calculate the weights of sub-criteria over total goal.

Step 7: Convert the $\boldsymbol{H} = (\boldsymbol{h}_{lo})\ (l = 1,2,\ldots,m; o = 1,2,\ldots,n)$ to a FPR matrix.

Step 8: Determine the ideal solution h_o^+ and negative ideal solution h_o^- using Eqs. (11)-(12).

Step 9: Calculate the group utility and individual regret values of S_i, R_i and the value of Q_i by Eqs. (14)-(16).

Step 10: Rank alternatives according to the conditions **C1** and **C2**.

An Application of the HF-ANP-VIKOR for Supplier Selection

In this section, we gave an application of the HF-ANP-VIKOR for supplier selection.

Nantong uasia import and export limited company intends to select a supplier for artificial flavors. In order to select the best supplier, four potential suppliers (A_1, A_2, A_3, A_4) are required assessment. The object indicators, which are confirmed by the procurement department in the company, are shown in Table 2.

Step 1: The network construction. The network construct by DMs for the MCDM problems is showed in Figure 3.

Step 2: Determine the weights of sub-criteria.

First, by comparing criteria, the HFPRs matrices are constructed as Tables 3-5.

Second, measure the compatibility of each preference relation matrix by Definition 7 and Definition 8 (Table 6).

Third, determine the weighting matrix and super-matrix.

Using Eq. (2), the HFPRs are converted to FPRs (Tables 7-9). By Eqs. (9)-(10), the weights of FPRs are computed. These weights compose the following weighting matrix.

$$\boldsymbol{A} = \begin{bmatrix} 0.273 & 0.31 & 0.379 \\ 0.352 & 0.355 & 0.283 \\ 0.375 & 0.335 & 0.338 \end{bmatrix}$$

Similarly, the super-matrix is calculated as

$$\boldsymbol{W} = \begin{bmatrix} 0.331 & 0.34 & 0.269 & 0.268 & 0.268 & 0.367 & 0.32 & 0.357 & 0.267 \\ 0.31 & 0.37 & 0.366 & 0.327 & 0.327 & 0.308 & 0.313 & 0.293 & 0.358 \\ 0.359 & 0.29 & 0.365 & 0.405 & 0.405 & 0.325 & 0.367 & 0.35 & 0.375 \\ 0.249 & 0.27 & 0.299 & 0.324 & 0.271 & 0.281 & 0.341 & 0.261 & 0.25 \\ 0.38 & 0.39 & 0.342 & 0.317 & 0.393 & 0.348 & 0.333 & 0.369 & 0.38 \\ 0.371 & 0.34 & 0.359 & 0.359 & 0.336 & 0.371 & 0.326 & 0.37 & 0.37 \\ 0.341 & 0.29 & 0.245 & 0.349 & 0.274 & 0.292 & 0.343 & 0.248 & 0.281 \\ 0.311 & 0.35 & 0.376 & 0.3 & 0.397 & 0.346 & 0.311 & 0.361 & 0.357 \\ 0.348 & 0.37 & 0.379 & 0.351 & 0.329 & 0.362 & 0.346 & 0.391 & 0.362 \end{bmatrix}$$

	Police risk e_{11}	Due to the changes of political to bring the possibility of economic losses.
Risk C_1	Tariff e_{12}	Import and export commodities after the declaration of tax levied.
	Credit risk e_{13}	Borrowers, issuers for various reasons are unwill or unable to fulfill the contract conditions.
	product price ratio e_{21}	The average price ratio of products (and services) between the supplier and other suppliers, in order to evaluate the price level between suppliers.
Price C_2	exchange rate e_{22}	The currency exchange rate of another currency, a currency that the price is based on another currency.
	payment e_{23}	Payment for the performance of debt instruments adopted specific practices.
	product quality pass rate e_{31}	The ratio the number of quality products to that of total products.
Quality C_3	quality certification e_{32}	The authority to prove that product complies with the standards and the technical requirements through the issuance of the certificates or certification marks.
	technical level e_{33}	Technical level and updated equipment usage.

Table 2: The criteria and sub-criteria.

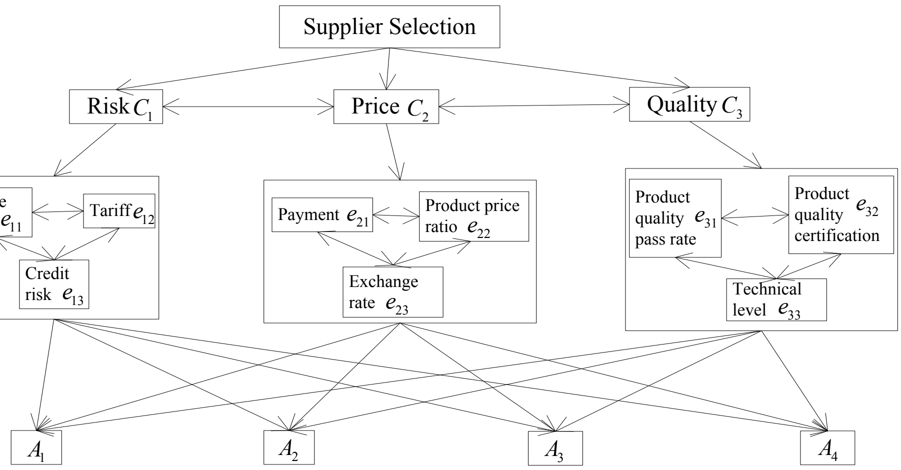

Figure 3: Network tree.

C_1	C_1	C_2	C_3
C_1	{0.5,0.5,0.5}	{0.2,0.3,0.5}	{0.3,0.3,0.4}
C_2	{0.5,0.7,0.8}	{0.5,0.5,0.5}	{0.1,0.3,0.7}
C_3	{0.6,0.7,0.7}	{0.3,0.7,0.9}	{0.5,0.5,0.5}

Table 3: Pair-wise comparison matrix under C_1 .

C_2	C_1	C_2	C_3
C_1	{0.5,0.5,0.5}	{0.1,0.4,0.5}	{0.3,0.4,0.7}
C_2	{0.5,0.7,0.9}	{0.5,0.5,0.5}	{0.2,0.4,0.6}
C_3	{0.3,0.6,0.7}	{0.4,0.6,0.8}	{0.5,0.5,0.5}

Table 4: Pair-wise comparison matrix under C_2 .

According to Eqs. (7)-(8), we compute the limit matrix as follows:

$$\overline{W}=\begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.106 & 0.106 & 0.106 & 0.106 & 0.106 & 0.106 & 0.106 & 0.106 & 0.106 \\ 0.116 & 0.116 & 0.116 & 0.116 & 0.116 & 0.116 & 0.116 & 0.116 & 0.116 \\ 0.093 & 0.093 & 0.093 & 0.093 & 0.093 & 0.093 & 0.093 & 0.093 & 0.093 \\ 0.119 & 0.119 & 0.119 & 0.119 & 0.119 & 0.119 & 0.119 & 0.119 & 0.119 \\ 0.117 & 0.117 & 0.117 & 0.117 & 0.117 & 0.117 & 0.117 & 0.117 & 0.117 \\ 0.102 & 0.102 & 0.102 & 0.102 & 0.102 & 0.102 & 0.102 & 0.102 & 0.102 \\ 0.121 & 0.121 & 0.121 & 0.121 & 0.121 & 0.121 & 0.121 & 0.121 & 0.121 \\ 0.126 & 0.126 & 0.126 & 0.126 & 0.126 & 0.126 & 0.126 & 0.126 & 0.126 \end{bmatrix}$$

C_3	C_1	C_2	C_3
C_1	{0.5,0.5,0.5}	{0.5,0.7,0.9}	{0.3,0.3,0.6}
C_2	{0.1,0.3,0.5}	{0.5,0.5,0.5}	{0.3,0.5,0.7}
C_3	{0.4,0.7,0.7}	{0.3,0.5,0.7}	{0.5,0.5,0.5}

Table 5: Pair-wise comparison matrix under C_3 .

	Cd		Cd		Cd		Cd		Cd
C_1	0.96	e_{11}^3	0.96	e_{21}^3	0.98	e_{31}^2	0.98	e_{11}^1	0.99
C_2	0.96	e_{12}^3	0.96	e_{22}^3	0.99	e_{32}^2	0.97	e_{12}^1	0.97
C_3	0.96	e_{13}^3	0.96	e_{23}^3	0.98	e_{33}^2	0.99	e_{13}^1	0.97
e_{11}^2	0.96	e_{21}^1	0.97	e_{31}^1	0.99	e_{21}^2	0.99	e_{31}^3	0.99
e_{12}^2	0.95	e_{22}^1	0.98	e_{32}^1	0.99	e_{22}^2	0.96	e_{32}^3	0.97
e_{13}^2	0.99	e_{23}^1	0.98	e_{33}^1	0.95	e_{23}^2	1	e_{33}^3	0.99

Table 6: Compatible degree (Cd).

From the limit matrix, the weights of sub-criteria are obtained as $\boldsymbol{w}=(0.1,0.106,0.116,0.093,0.119,0.117,0.102,0.121,0.126)^{\mathrm{T}}$.

Step 3: Elicit the hesitant fuzzy decision matrix, see Table 10.

Step 4: By Eqs. (11)-(12), the ideal and negative ideal solutions are seen in Table 11.

C_1	C_1	C_2	C_3
C_1	0.5	0.3833	0.35
C_2	0.6167	0.5	0.467
C_3	0.65	0.533	0.5

Table 7: FPR matrix of C_1.

C_2	C_1	C_2	C_3
C_1	0.5	0.367	0.533
C_2	0.633	0.5	0.467
C_3	0.467	0.533	0.5

Table 8: FPR matrix of C_2.

C_3	C_1	C_2	C_3
C_1	0.5	0.767	0.45
C_2	0.233	0.5	0.567
C_3	0.55	0.433	0.5

Table 9: FPR matrix of C_3.

	e_{11}	e_{12}	e_{13}	e_{21}	e_{22}	e_{23}	e_{31}	e_{32}	e_{33}
A_1	{0.1,0.2,0.3}	{0.2,0.2,0.3}	{0.3,0.3,0.4}	{0.1,0.1,0.2}	{0.2,0.4,0.6}	{0.2,0.3,0.4}	{0.3,0.5,0.7}	{0.2,0.2,0.3}	{0.1,0.3,0.5}
A_2	{0.2,0.2,0.5}	{0.2,0.3,0.4}	{0.1,0.3,0.5}	{0.3,0.3,0.5}	{0.2,0.4,0.6}	{0.1,0.2,0.3}	{0.1,0.1,0.3}	{0.3,0.4,0.5}	{0.2,0.4,0.6}
A_3	{0.2,0.4,0.6}	{0.3,0.4,0.5}	{0.1,0.3,0.5}	{0.3,0.7,0.9}	{0.4,0.5,0.6}	{0.2,0.4,0.7}	{0.3,0.4,0.5}	{0.1,0.2,0.3}	{0.1,0.2,0.3}
A_4	{0.3,0.5,0.7}	{0.2,0.4,0.6}	{0.1,0.2,0.4}	{0.2,0.4,0.6}	{0.2,0.2,0.3}	{0.2,0.5,0.7}	{0.1,0.1,0.3}	{0.3,0.5,0.7}	{0.3,0.3,0.4}

Table 10: Hesitant fuzzy decision matrix.

	e_{11}	e_{12}	e_{13}	e_{21}	e_{22}	e_{23}	e_{31}	e_{32}	e_{33}
h^+	{0.1,0.2,0.3}	{0.2,0.2,0.3}	{0.1,0.2,0.4}	{0.1,0.1,0.2}	{0.2,0.2,0.3}	{0.1,0.2,0.3}	{0.3,0.5,0.7}	{0.3,0.5,0.7}	{0.2,0.4,0.6}
h^-	{0.3,0.5,0.7}	{0.2,0.4,0.6}	{0.3,0.3,0.4}	{0.3,0.7,0.9}	{0.4,0.5,0.6}	{0.2,0.5,0.7}	{0.1,0.1,0.3}	{0.1,0.2,0.3}	{0.1,0.2,0.3}

Table 11: The values of ideal and negative ideal solutions.

d_{lo}	e_{11}	e_{12}	e_{13}	e_{21}	e_{22}	e_{23}	e_{31}	e_{32}	e_{33}
A_1	0	0	0.1	0	0.167	0.1	0	0.267	0.1
A_2	0.1	0.067	0	0.233	0.167	0	0.333	0.1	0
A_3	0.2	0.167	0.067	0.5	0.267	0.233	0.1	0.3	0.2
A_4	0.3	0.167	0	0.267	0	0.267	0.333	0	0.067

Table 12: The values of $d(h_o^+, \boldsymbol{h}_{lo})$.

	S_l	R_l	Q_l	Rank
A_1	0.405	0.116	0.373	3
A_2	0.335	0.102	0	4
A_3	0.841	0.126	1	1
A_4	0.516	0.117	0.499	2

Table 13: The values of S_l, R_l and Q_l.

Step 5: Calculate $d(h_o^+, \boldsymbol{h}_{lo})$ and $d(h_o^+, h_o^-)$ (Table 12).

The values of $d(h_o^+, h_o^-)$ over each sub-criterion are show as:

C	C_1	C_2	C_3
C_1	{0.5,0.5,0.5}	{0.27,0.43,0.63}	{0.3,0.33,0.57}
C_2	{0.43,0.57,0.73}	{0.5,0.5,0.5}	{0.2,0.4,0.67}
C_3	{0.43,0.67,0.7}	{0.33,0.6,0.8}	{0.5,0.5,0.5}

Table 14: The collective HFPR matrix.

$$d(h_o^+, h_o^-) = \{0.3, 0.167, 0.1, 0.5, 0.267, 0.267, 0.333, 0.3, 0.2\}$$

Step 6: By Eqs. (14)-(15), calculate the group utility S_l and individual regret values R_l, which are shown as Table 13.

Step 7: Take $v = 0.5$ in Eq. (16), the Q_l are computed and listed in Table 14.

Step 8: The rank by HF-ANP-VIKOR is $A_3 \succ A_4 \succ A_1 \succ A_2$. Therefore, the best supplier is A_3.

Conclusions

In this paper, we proposed an ANP-VIKOR method for solving supplier selection problems with HFSs. First, we developed a HF-ANP approach and applied to calculate the weights of sub-criteria. In this calculation process, we not only presented a new measure to calculate the compatibility degree, but also converted the HFPRs to FPRs by the score function, which can save the calculations. Then, we presented a HF-VIKOR approach to rank alternatives. The main idea of this approach is to determine the values of group utility and individual regret over the alternatives, integrate the maximum group utility and the minimum individual regret to rank the alternatives. Finally, the numeral analysis indicated that the HF-ANP-VIKOR method is practicable and valid to solve the MCDM problem with HFEs.

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