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Study on the Model of Insurer's Solvency Ratio under Lévy Process

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Abstract

This paper studies the insurer's solvency ratio model with the Lévy process in the presence of financial distress cost. By an option pricing formula for the Lévy process, the explicit formula for the expected present value of shareholder's terminal payoff is given.

Keywords: Lévy process; Solvency ratio; Financial distress cost; Option pricing formula; Girsanov's theorem

Introduction

The studies for the insurer's risk management incentives under the financial distress have a long history, and many elegant results have been established. Opler and Titman [1], Briys and De Varenne [2], Ma et al. [3] and the references therein. A solvency model in the presence of costs of financial distress has been introduced in references [4-6]. Based on the Markov-modulated market [7], the solvency ratio model is further discussed in Xia et al. [8].

In this paper, we will using the pricing formula for European options to study the insurer's solvency ratio, as we know, in 1973, Black and Scholes [9] provided the famous pricing formula for European options under the assumptions that the risk-free bond price $S_t^0 : t \in [0,T]$ and the price of the stock $S_t^1 : t \in [0,T]$ are described as

$$\begin{cases} dS_t^0 = rS_t^0 dt, \\ dS_t^1 = \mu S_t^1 dt + \sigma S_t^1 dW_t, \end{cases}$$

where r, μ, σ; are constants called risk-free interest rate, expected return rate and volatility of the stock respectively, $W=\{W_t : t \in [0,T]\}$ is a standard Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, T is called maturity $(0 < T < \infty)$. However, in real markets the expected return rate and the volatility usually are not constants but they can vary with time [10,11]. Thus, following their work, many authors discussed various option pricing problems under the more general model

$$dS_t^1 = \mu S_t^1 dt + \sigma S_t^1 dW_t,$$

where $\{\mu_t : t \in [0,T]\}$ and $\{\sigma_t : t \in [0,T]\}$ are two given stochastic processes with some integrable conditions, and the risk-free bond is described as

$$dS_t^0 = r_t S_t^0 dt \text{ or } S_t^0 = e^{\int_0^t r_u du},$$

where r_t is deterministic interest rate function on [0,T] with $S_0^0 = 1$ and $\int_0^T |r_t| dt < \infty$. The pricing theory of Shreve [11] options shows that, if the market is arbitrage-free and complete, there exists a unique risk neutral martingale measure $\mathbb{Q}$ defined by

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\left\{ -\frac{1}{2}\int_0^T \left(\frac{\mu_s - r_s}{\sigma_s} \right) ds + \int_0^T \left(\frac{\mu_s - r_s}{\sigma_s} \right) dW_s \right\}$$

such that for any contingent claim η at time T, the value of η at any time $t \in [0; T)$ is given by

$$V_t(\eta) = S_t^0 \mathbb{E}_{\mathbb{Q}}\left[\eta \left(S_T^0 \right)^{-1} \mid \mathcal{F}_t \right] = \mathbb{E}_{\mathbb{Q}}\left[e^{\int_t^T r_u du} \eta \mid \mathcal{F}_t \right],$$

and, in particular, the current price is

$$V_0(\eta) = \mathbb{E}_{\mathbb{Q}}\left[e^{\int_t^T r_u du} \eta \right],$$

where the notation $\mathbb{E}_{\mathbb{Q}}$ is the expectation with respect to the probability measure $\mathbb{Q}$.

Lévy processes are processes with stationary and independent increments and are thus, in a way, generalizations of a Brownian motion. Unlike the latter, their increments are not normally distributed, the distribution of their increments belong to the wide class of infinitely divisible distributions. Lévy processes can be decomposed as the sum of three independent processes. One component is linear deterministic, the second a Brownian motion and the third a pure jump process.

Lévy processes have recently become an object of interest in finance modeling because they have diffusion-like and jump properties at the same time [12,13]. In finance, as well as in insurance, this has been achieved by adding extra components into the model. In finance, large fluctuations are incorporated via a jump process and in insurance small fluctuations are incorporated via diffusion. Lévy processes account for both types of structures.

Because of Lévy´ models provide a better fit to empirical asset price distributions that typically have fatter tails than Gaussian ones, and can reproduce volatility smile phenomena in option prices. It has been shown by Cont et al. [14] and Bjork et al. [15], that Lévy processes are relevant in mathematical finance, in particular in model of stock prices. Pricing the continuously sampled geometric average options in exponential Lévy models is easy and quite straightforward [16].

Following in reality, the prices of assets depend on Lévy processes, and then because of the relation between the insurer's solvency ratio and the prices of assets, it is interesting to discuss the insurer's solvency ratio model with Lévy´ processes. The dynamic model of solvency ratio with Lévy process in the presence of financial distress cost is addressed in this paper. By using the options pricing formula on a stock whose

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price processes are modeled by a Lévy process, which is introduced in Cont et al. [14] and Sato [17], we obtain the expected present value of shareholders' terminal payoff. The model extends the existing results. The arrangement of the paper is as follows.

Insurer's Solvency Ratio Model with Lévy Process

Since the variability in financial outcomes is costly when a corporation operates in an environment with frictional costs, the risk management strategies depend on the nature of the frictional costs. In general, there are essentially three sources of frictional capital costs: tax asymmetry, costs of financial distress and agency costs. In the insurance industry, an insurer experiences an additional two costs of corporate risk: cost of double taxation and cost of regulatory restrictions [5]. This article investigates the strategies of an insurer's risk management in the presence of financial distress costs.

We suppose that there is no claim payment made other than at the end of the period. A_T denotes the value of the company assets and L_T the value of company liabilities at the end of a period T, $\Lambda_T := A_T/L_T$ is called an insurer's solvency ratio. b is a pre-specified threshold (financial distress (FD) barrier). If the terminal value of the solvency ratio exceeds b (i.e., $\Lambda_T > b$), then the insurer is financially healthy. An insurer becomes financially distressed if the terminal value of the solvency ratio falls below b (i.e., $\Lambda_T \le b$). In the state of the FD, the insurer experiences deadweight loss proportional to the terminal value of assets A_T with the proportional coefficient (1 - w), w ∈ [0, 1). Further, if the net terminal value of assets under the FD costs exceeds the terminal value of liabilities, i.e., $wA_T > L_T$ or $\Lambda_T > 1/w$, then we call the insurer is financially distressed but solvent; in contrast, when wb<1 or b<1/w, we call the insurer is insolvent.

In sum, there are three different economic states of an insurer: financially healthy; financially distressed and solvent; insolvent.

Assume that at time t the value of the insurer's solvency ratio Λ_t is driven by a Lévy process. Under the original measure $\mathbb{P}$, dynamics of Λ_t is

$$d\Lambda_t = \Lambda_{t-}\left[\mu_\Lambda(t)dt + \sigma_\Lambda(t)dW_t^\Lambda + \int_{-1}^{\infty} y\left(N(dy,dt) - \lambda m(dy)dt\right)\right], t \ge 0, \quad (1)$$

where $\mu_\Lambda(t)$ denotes mean rate of return, $\sigma_\Lambda(t)$ the volatility of Λ_t, both are deterministic. $\left(W_t^\Lambda, 0 \le t \le T\right)$ is a Brownian motion on a filted probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$. N(dy, dt) is a homogeneous Poisson measure for Poisson process $(N_t, (U_j)_{j\ge1})$, λm(dy) is the compensator of N(dy, dt), λ is the intensity of a Poisson process N_t, m(dy) is probability measure of square-integrable and independent identically distributed random variables $(U_j)_{j\ge1}$ with $U_j > -1$. Let also $(W_t)_{t\ge0}$, $(N_t)_{t\ge0}$ and $(U_j)_{j\ge1}$ be independent of each other.

The expected present value of shareholders' terminal payoff is defined by

$$V_0^{SR} = \mathbb{E}_{\mathbb{P}}\left\{\exp\left\{-\int_0^T r(s)ds\right\}\left[(wA_T - L_T)^+ 1_{\{\Lambda_T \le b\}} + (A_T - L_T)1_{\{\Lambda_T > b\}}\right]\right\}$$

$$= \mathbb{E}_{\mathbb{P}}\left\{\exp\left\{-\int_0^T r(s)ds\right\} L_T\left[w\left(\Lambda_T - \frac{1}{w}\right)^+ + (1-w)\Lambda_T 1_{\{\Lambda_T > b\}}\right]\right\},$$

where $\Lambda_0 = \dfrac{A_0}{L_0} > 1, r(t)$ is a deterministic riskless interest rate.

Now a risk-neutral measure $\mathbb{Q}$ is defined as

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = Z_T = \exp\left\{-\frac{1}{2}\int_0^T \delta_s^2 ds + \int_0^T \delta_s dW_s^\Lambda\right\},$$

where $\delta_t = (r(t) - \mu_\Lambda(t))/\sigma_\Lambda(t)$. Thus, we have

$$\mathbb{E}_{\mathbb{P}}\left[Z_T \mid \mathcal{F}_t\right] = Z_t$$

By Girsanov's theorem Karatzas et al. [18] and Rogers et al. [19] the process $\hat{W}_t^\Lambda = W_t^\Lambda - \int_0^t \delta_s ds$ is a $\mathbb{Q}$-Wiener process. We suppose that $L_t = L_0 e^{\int_0^t \mu_L(s)ds} Z(t), 0 \le t \le T$ where $\mu_L(t)$ is deterministic. Then for any function of the terminal value of Λ_T, we get

$$\mathbb{E}_{\mathbb{Q}}\left[\exp\left\{-\int_t^T r(s)ds\right\} f(\Lambda_T) \mid \mathcal{F}_t\right] = \mathbb{E}_{\mathbb{P}}\left[\exp\left\{-\int_t^T r(s)ds\right\}\frac{Z(T)}{Z(t)} f(\Lambda_T) \mid \mathcal{F}_t\right]$$

Therefore, we have

$$\mathbb{E}_{\mathbb{Q}}\left[\exp\left\{-\int_t^T r(s)ds\right\} L_T f(\Lambda_T) \mid \mathcal{F}_t\right] = Z(t) L_0 e^{\int_0^T \mu_L(s)ds}\mathbb{E}_{\mathbb{Q}}\left[\exp\left\{-\int_t^T r(s)ds\right\} f(\Lambda_T) \mid \mathcal{F}_t\right]$$

For our aim, we can set $P_b = \mathcal{Q}(\Lambda_T > b)$ as known.

Hence, under the risk-neutral measure $\mathbb{Q}$ dynamics of Λ_t is

$$d\Lambda_t = \Lambda_{t-}\left[r(t)dt + \sigma_\Lambda(t)d\hat{W}_t^\Lambda + \int_{-1}^{\infty} y\left(N(dy,dt) - \lambda m(dy)dt\right)\right], t \ge 0 \cdot$$

Results

Now, we give a useful lemma appeared in Terence [20] and Xiong [21].

Lemma 1: Under the risk-neutral measure $\mathbb{Q}$, if the stock price S = (S_t) follows

$$dS_t = S_{t-}\left[r(t)dt + \sigma(t)dW_t + \int_{-1}^{\infty} y\left(N(dy,dt) - \lambda m(dy)dt\right)\right],$$

Where W_t denotes a standard Brownian motion on a filted probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{Q})$, σ(t) is deterministic. Then for the European call option with the cost function $(S_T - K)^+$, we have

$$C(K,T) = \sum_{n=1}^{\infty} e^{-\lambda T}\frac{(-\lambda T)^n}{n!} \times \mathbb{E}_{\mathbb{Q}}\left[SN(d_2^n)e^{-\lambda T\mathbb{E}_{\mathbb{Q}}(U_1)}\prod_{j=1}^{n}(1+U_j) - K\exp\left\{-\int_0^T r(s)ds\right\}N(d_1^n)\right],$$

Where

$$d_1^n = \frac{1}{\sqrt{\int_0^T \sigma^2(s)ds}}\left[\log\left(\frac{Se^{-\lambda T\mathbb{E}_{\mathbb{Q}}(U_1)}\prod_{j=1}^{n}(1+U_j)}{K}\right) + \int_0^T\left(r(s) - \frac{1}{2}\sigma^2(s)\right)ds\right],$$

$$d_2^n = d_1^n + \sqrt{\int_0^T \sigma^2(s)ds},$$

and $N(y) = \dfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{y} e^{-\frac{1}{2}x^2}dx$ is the cumulative normal distribution function.

Theorem 1: The dynamics of Λ_t is given in (1). Then, under above conditions, the expected present value of shareholders' terminal payoff is given.

$$V_0^{SR} = wL_0 e\int_0^T r(s)ds\sum_{n=1}^{\infty} e^{-\lambda T}\frac{(-\lambda T)^n}{n!} \times \mathbb{E}_{\mathbb{Q}}\left[\Lambda_0 N(g_2^n)e^{-\lambda T\mathbb{E}_{\mathbb{Q}}(U_1)}\prod_{j=1}^{\infty}(1+U_j) - \frac{1}{w}\exp\left\{-\int_0^T r(s)ds\right\}N(g_1^n)\right]$$

$$+(1-w)L_0 e\int_0^T r(s)ds\sum_{n=1}^{\infty} e^{-\lambda T}\frac{(-\lambda T)^n}{n!} \times \mathbb{E}_{\mathcal{Q}}\left[\Lambda_0 N(h_2^n)e^{-\lambda T\mathbb{E}_{\mathbb{Q}}(U_1)}\prod_{j=1}^{n}(1+U_j) - b\exp\left\{-\int_0^T r(s)ds\right\}N(h_1^n)\right]$$

$$+L_0(1-w)bP_b, \quad (2)$$

Where

$$g_1^n = \frac{1}{\sqrt{\int_0^T \sigma_\Lambda^2(s)ds}}\left[\log\left(\frac{\Lambda_0 e^{-\lambda T\mathbb{E}_{\mathbb{Q}}(U_1)}\prod_{j=1}^{n}(1+U_j)}{1/w}\right) + \int_0^T\left(r(s) - \frac{1}{2}\sigma_\Lambda^2(s)ds\right)\right],$$

$$g_2^n = g_1^n + \sqrt{\int_0^T \sigma_\Lambda^2(s)ds},$$

$$h_1^n = \frac{1}{\sqrt{\int_0^T \sigma_\Lambda^2(s)ds}}\left[\log\left(\frac{\Lambda_0 e^{-\lambda T \mathbb{E}_{\mathbb{Q}}(U_1)}\prod_{j=1}^{n}\left(1+U_j\right)}{b}\right)+\int_0^T\left(r(s)-\frac{1}{2}\sigma_\Lambda^2(s)\right)ds\right]$$

$$h_2^n = h_1^n + \sqrt{\int_0^T \sigma_\Lambda^2(s)ds}.$$

Proof: The expected present value of shareholders' terminal payoff is

$$V_0^{SR} = \mathbb{E}_{\mathbb{P}}\left\{e^{-\int_0^T r(s)ds} L_T\left[w\left(\Lambda_T - \frac{1}{w}\right)^+ + (1-w)\Lambda_T 1_{\Lambda_T > b}\right]\right\}$$

$$= L_0 e^{\int_0^T \mu_L(s)ds}\left\{w\mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^T r(s)ds}\left(\Lambda_T - \frac{1}{w}\right)^+\right] + (1-w)\mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^T r(s)ds}\Lambda_T 1_{\Lambda_T > b}\right]\right\}.$$

Let

$$C_1 = \mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^T r(s)ds}\left(\Lambda_T - \frac{1}{w}\right)^+\right],$$

$$C_2 = \mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^T r(s)ds}\Lambda_T 1_{\Lambda_T > b}\right]$$

$$= \mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^T r(s)ds}\left(\Lambda_T - b\right)^+ + b1_{\Lambda_T > b}\right]$$

$$= \mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^T r(s)ds}\left(\Lambda_T - b\right)^+\right] + e^{-\int_0^T r(s)ds} bP_b.$$

By Lemma 1, we deduce

$$C_1 = \sum_{n=1}^{\infty} e^{-\lambda T}\frac{(-\lambda T)^n}{n!}\times\mathbb{E}_{\mathbb{Q}}\left[\Lambda_0 N\left(g_2^n\right)e^{-\lambda T \mathbb{E}_{\mathbb{Q}}(U_1)}\prod_{j=1}^{n}\left(1+U_j\right) - \frac{1}{w}\exp\left\{-\int_0^T r(s)ds\right\}N\left(g_1^n\right)\right],$$

$$C_2 = \sum_{n=1}^{\infty} e^{-\lambda T}\frac{(-\lambda T)^n}{n!}\times\mathbb{E}_{\mathbb{Q}}\left[\Lambda_0 N\left(h_2^n\right)e^{-\lambda T \mathbb{E}_{\mathbb{Q}}(U_1)}\prod_{j=1}^{n}\left(1+U_j\right) - b\exp\left\{-\int_0^T r(s)ds\right\}N\left(h_1^n\right)\right] + e^{-\int_0^T r(s)ds} bP_b.$$

Similar to discussion in reference [4], we know $\mu_L(t) = r(t)$. Hence we have

$$V_0^{SR} = L_0 e^{\int_0^T r(s)ds}\left[wC_1 + (1-w)C_2\right],$$

from which (2) is derived. The proof is complete.

Corollary 1: When an insurer becomes financially distressed, i.e., $\Lambda_T \le J$, then the expected present value of shareholders' terminal payoff is given by

$$V_0^{SR} = wL_0 e^{\int_0^T r(s)ds}\sum_{n=1}^{\infty} e^{-\lambda T}\frac{(-\lambda T)^n}{n!}\times\mathbb{E}_{\mathbb{Q}}\left[\Lambda_0 N\left(g_2^n\right)e^{-\lambda T \mathbb{E}_{\mathbb{Q}}(U_1)}\prod_{j=1}^{n}\left(1+U_j\right) - \frac{1}{w}\exp\left\{-\int_0^T r(s)ds\right\}N\left(g_1^n\right)\right].$$

Remark: When w=0, the insurer losses all terminal value of assets A_T, we say the insurer is bankruptcy. At this time, $\Lambda_T \le b$, $\Lambda_T \to 0$, and $L_T \to \infty$, we can easily prove that $V_0^{SR} = 0$.

Conclusion

This paper mainly introduces the insurer's solvency ratio model with Lévy processes in the presence of financial distress cost, where an insurer's solvency ratio is characterized by a Lévy process. By the option pricing formula for a Lévy process, the expected present value of shareholders' terminal payoff is explicitly provided.

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