Journal of Generalized Lie Theory and Applications
ISSN: 1736-4337
Muktibodh and Santilli, J Generalized Lie Theory Appl 2017, 11:1
DOI: 10.4172/1736-4337.1000251
Research Article Open Access

Studies of the Regular and Irregular Isorepresentations of the Lie-Santilli Isotheory

Muktibodh AS[1]* and Santilli RM[2]

[1]Institute for Basic Research, Palm Harbor, Florida, USA
[2]Thunder Energies Corporation, Tarpon Springs, Florida, USA

Abstract

As it is well known, the Lie theory is solely applicable to dynamical systems consisting of point-like particles moving in vacuum under linear and Hamiltonian interactions (systems known as exterior dynamical systems). One of the authors (R.M. Santilli) has proposed an axiom-preserving broadening of the Lie theory, known as the Lie-Santilli isotheory, that is applicable to dynamical; systems of extended, non-spherical and deformable particles moving within a physical medium under Hamiltonian as well as non-linear and non-Hamiltonian interactions (broader systems known as interior dynamical systems). In this paper, we study apparently for the first time regular and irregular isorepresentations of Lie-Santilli isoalgebras occurring when the structure quantities are constants or functions, respectively. A number of applications to particle and nuclear physics are indicated. It should be indicated that this paper is specifically devoted to the study of isorepresentations under the assumption of a knowledge of the Lie-Santilli isotheory, as well as of the isotopies of the various branches of 20th century applied mathematics, collectively known as isomathematics, which is crucial for the consistent formulation and elaboration of isotheories.

Keywords: Isoalgebra, Isorepresentation, Lie-Santilli isotheory, Isomathematics

Introduction

Despite historical achievements in atomic physics [1], nuclear physics [2] and other fields, a number of authoritative scientists expressed doubts on the final character of quantum mechanics for the representation of all possible conditions existing in the universe.

For instance, Einstein, Podolsky and Rosen [3] expressed doubts on the "completeness" of quantum mechanics. Fermi [4] states in regard to the structure of strongly interacting particles "..... there are some doubts as to whether the usual concepts of geometry hold for such small region of space." Blatt and Weisskopf [2] have indicated then possibility that the deviations of the predictions of quantum mechanics from experimental data on nuclear magnetic moments are due to deformations of the charge distributions of protons and neutrons when under strong nuclear interactions.

One of us (Santilli) has conducted systematic studies on the above historical legacies over a number of decades [5-8] and papers quoted therein suggesting that the indicated limitations originate from the mathematics underlying quantum mechanics, rather than on its basic axioms.

More particularly, the limitations originated from the local-differential character of 20th century mathematics because such a calculus is solely applicable to dynamical systems of point-like particles moving in vacuum under Hamiltonian interactions, which systems are known as exterior dynamical systems.

The resulting mathematics has been established as being valid for mutual distances of particles much bigger than their wavepackets and/or charge distributions, as it is the case for the atomic structure, particles in accelerators, the structural crystallography, and numerous other systems.

However, the same point-like approximation results to be insufficient for extended, non-spherical and deformable particles moving within a physical medium under potential-Hamiltonian as well as contact non-Hamiltonian interactions (Broader systems known as interior dynamical systems) as it is the case for the structure of hadrons, nuclei and stars.

Therefore, Santilli submitted in 1978 [5] an axiom-preserving broadening of 20th century mathematics called isotopic lifting, based on the associativity-preserving generalization of the conventional product $A \times B$ between generic quantities A, B (such as numbers, functions, matrices, operators, etc.) called isoproduct:

$$A \times B \rightarrow A \hat{\times} B = A \times \hat{T} \times B, \quad \hat{T} > 0, \tag{1}$$

where $\hat{T}$, called the isotopic element, is restricted to be positive-definite, but possess otherwise an unrestricted functional dependence on local quantities, such as coordinates x, velocities v, energy E, density, temperature τ, pressure ξ, wavefunction ψ, their derivatives $_x\psi$, etc.

$$\hat{T} = \hat{T}(x, v, E, \mu, \tau, \xi, \psi, \partial_x \psi, \ldots) > 0. \tag{2}$$

The above assumptions immediately allowed an explicit representation of protons and neutrons as being extended, non-spherical and deformable charge distributions via realizations of the isotopic element of the type:

$$\hat{T} = Diag.(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2})e^{-\Gamma}, \tag{3a}$$

$$n_\mu = n(x, v, E, \mu, \tau, \xi, \psi, \partial_x \psi, \ldots)0, \ \mu = 1,2,3,4, \tag{3b}$$

where: $n_k^2, k = 1,2,3$ represent the semi-axes of the particle considered (assumed to be a spheroid ellipsoid for simplicity) normalized to the values n_K=1 for the vacuum; n_4^2 represents the density of the particle considered also normalized to the value n_4=1 for the vacuum; and Γ

***Corresponding author:** Muktibodh AS, Institute for Basic Research, Palm Harbor, Florida, USA, Tel: +1-727-688 3992; E-mail: amukti2000@yahoo.com

Received March 12, 2017; **Accepted** April 13, 2017; **Published** April 17, 2017

Citation: Muktibodh AS, Santilli RM (2017) Studies of the Regular and Irregular Isorepresentations of the Lie-Santilli Isotheory. J Generalized Lie Theory Appl 11: 251. doi:10.4172/1736-4337.1000251

Citation: Muktibodh AS, Santilli RM (2017) Studies of the Regular and Irregular Isorepresentations of the Lie-Santilli Isotheory. J Generalized Lie Theory Appl 11: 251. doi:10.4172/1736-4337.1000251

represents non-linear, non-local and non-Hamiltonian interactions due to mutual penetrations of particles (interactions known as variationally non-selfadjoint) [5].

Subsequently, Santilli realized that an isomathematics solely based on isoproduct (1) would have other shortcomings because the emerging time evolution would be non-unitary, with ensuing lack of preservation of Hermiticity (and observability) over time, violation of causality, etc. [8].

In order to resolve this impasse, memoir [6] of 1993, Santilli introduced the isotopies $\hat{F}(\hat{n},\hat{\times},\hat{I})$ of conventional numeric fields $F(n,\times,I)$ characterized by isoproduct (1), generalized multiplicative unit called isounit,

$$\hat{1} = 1/\hat{T} > 0, \tag{4}$$

and generalized numbers, called isoreal, isocomplex and isoquaternionic isonumbers, and related isoaxiom:

$$\hat{n} = n \times \hat{1}, \tag{5a}$$

$$\hat{I} \hat{\times} \hat{n} = \hat{n} \hat{\times} \hat{I} = \hat{n},\ \forall \hat{n} \in \hat{F} \tag{5b}$$

Despite the reformulation of all preceding works from their original formulation on conventional fields to new formulations on isofields, the emerging mathematics was still unable to achieve the crucial invariance possessed by 20th century theories, namely, the prediction of the same numerical values under the same conditions at different times.

Following a number of attempts, Santilli achieved the resolution of the latter impasse via the following isotopies of the ordinary differential calculus, today known as Santilli isodifferential,

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(r,\ldots)] = dr + r\hat{T}d\hat{I}, \tag{6}$$

with corresponding isoderivative [6],

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\frac{\partial\hat{f}(\hat{r})}{\partial\hat{r}}. \tag{7}$$

It essentially emerged that the isotopic generalization of the product, Eq. (1), and its intrinsic representation of extended particles, Eq. (3), were incompatible with the local-differential character of the Newton-Leibnitz differential calculus that, consequently, to be lifted into a form eqs (6) and (7) admitting extended particles in their very structure.

Following the achievement of mathematical maturity in memoir [7], monographs [8] presented isotopic images of various branches of 20th century mathematics and their application to the isotopies of quantum mechanics known as hadronic mechanics, while monographs [9] presented applications and experimental, verifications in quantitative sciences.

Vol. I of ref. [9] presents a large bibliography of independent contributions in the new field, among which we indicate the works of: Chun-Xuan Jiang; RM Falcon Ganfornina and J Nunez Valdes; J. V. Kadeisvili; S. Georgiev; and the general review by Gandzha and Kadeisvili [10-14].

It should be noted that this paper is devoted to one single application of isomathematics, the initiation of isorepresentations of the Lie-Santilli isoalgebras. Therefore, a technical understanding of this paper requires a knowledge of the Lie-Santilli isotheory at large, as well as of isomathematics since the latter is essential for the consistent formulation and elaboration of isotheories.

Besides the above rudimentary outline of the most salient aspects of isomathematics, and an equally rudimentary outline of the Lie-Santilli isotheory in the next section, the authors regret being unable to review the various branches of isomathematics to prevent a prohibitive as well as non-necessary length of the paper, since the foundations of isomathematics are available in the original proposals [5-9] as well as in numerous mathematical treatments [10-15].

The reader not aware of this rather large literature should be alerted that the mixing of mathematics and isomathematics generally leads to insidious inconsistencies. Fir instance, the formulation of the Lie-Santilli isotheory over conventional numeric fields, even though seemingly acceptable on mathematical grounds, carries exactly the same-inconsistencies of the formulation of the conventional Lie theory over isofields, both formulations implying the violation of causality in physical applications.

Hence, the correct formulation and elaboration of the Lie-Santilli isotheory requires the isotopies of the totality of mathematical notions used for the Lie theory with no exception known to the authors.

Rudiments of the Lie-Santilli Isotheory

Santilli's primary motivation for the isotopes of 20th century mathematics was its application to the isotopies of the various branches of Lie's theory that were first presented in ref. [5] on conventional spaces over conventional field and elaborated via the conventional differential calculus. Said isotopic theory was then reformulated [7-9] on isospaces over isofield and elaborated via the isofunctional analysis and the isodifferential calculus, in which form it is nowadays called the Lie-Santilli isotheory.

The main elements of the Lie-santilli isotheory can be outlined as follows. Let $E=E(L)$ be the universal enveloping associative algebra of an N-dimensional Lie algebra L with ordered (Hermitean) generators X_k, $k=1,2,\ldots,N$, and attached antisymmetric algebra isomorphic to the Lie algebra, $[E(L)]^{-}\approx L$ over a field F (of characteristic zero), and let the infinite-dimensional basis I, X_k, $X_i \times X_j$, $i\leq j,\ldots$ of $E(L)$ be characterized by the Poincaré-Birkhoff-Witt theorem. We then have the following.

Theorem 1 [5b] (Poincaré-Birkhoff-Witt-Santilli theorem)

The isocosets of the isounit and of the standard isomonomials:

$$\hat{I},\ X_k,\ \hat{X}_i \hat{\times} \hat{X}_j, i \le j,\ \hat{X}_i \hat{\times} \hat{X}_j \hat{\times} \hat{X}_k, i \le j \le k, \ldots, \tag{8}$$

form an (infinite dimensional) basis of the universal enveloping isoassociative algebra $\hat{E}(\hat{L})$ (also called isoenvelope for short) of a Lie-Santilli isoalgebra $\hat{L}$.

The first application of the above theorem, also formulated [4] and then reexamined by various authors, is a rigorous characterization of the isoexponentiation, i.e.,

$$\hat{e}^{\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}}$$

$$= \hat{I} + \hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X} \hat{/} \hat{1}! + (\hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X}) \hat{\times} (\hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X}) \hat{/} \hat{2}! + \ldots =$$

$$\hat{I} \times (e^{i\times w\times T\times X}) = (e^{i\times w\times X\times T}) \times \hat{I}, \tag{9a}$$

$$\hat{i} = i \times \hat{I}, \hat{w} = w \times \hat{I} \in \hat{F}. \tag{9b}$$

where we continue to use the notation of Paper I according to which quantities with a "hat" are formulated on isospaces over isofields and those without are formulated on conventional spaces over conventional fields.

The non-triviality of the Lie-Santilli isotheory is illustrated by the emergence of the nonlinear, nonlocal and noncanonical or nonunitary

Citation: Muktibodh AS, Santilli RM (2017) Studies of the Regular and Irregular Isorepresentations of the Lie-Santilli Isotheory. J Generalized Lie Theory Appl 11: 251. doi:10.4172/1736-4337.1000251

isotopic element T directly in the exponent, thus ensuring the desired generalization.

As it is well known, Lie algebras are the antisymmetric algebras $L\approx[\xi(L)]^-$ attached to the universal enveloping algebras $\xi(L)$. This main characteristic is preserved although enlarged under isotopies as expressed by the following.

Theorem 2 [5b] (Lie-Santilli Second theorem)

The antisymmetric isoalgebras $\hat{L}$ attached to the isoenveloping algebras $\hat{E}(\hat{L})$ verify the isocommutation rules:

$$[\hat{X}_i \hat{,} \hat{X}_j] = \hat{X}_i \hat{\times} \hat{X}_j - \hat{X}_j \hat{\times} \hat{X}_i =$$

$$= X_i \times T(x,v,\xi,\omega,\psi,\partial\psi,...) \times X_j - X_j \times T(x,v,\xi,\omega,\psi,\partial\psi,...) \times X_i =$$

$$= \hat{C}_{ij}^k(x,v,\xi,\omega,\psi,\partial\psi,...) \hat{\times} \hat{X}_k = C_{ij}^k(x,v,\xi,\omega,\psi,\partial\psi,...) \times X_k,, \quad (10)$$

where the C's, called the "structure isofunctions" of $\hat{L}$, generally have an explicit dependence on local variables, and are restricted by the conditions (Lie-Santilli Third Theorem)

$$[X_i \hat{,} X_j] + [X_j \hat{,} X_i] = 0,$$

$$[[X_i \hat{,} X_j] \hat{,} X_k] + [[X_j \hat{,} X_k] \hat{,} X_i] + [[X_k \hat{,} X_i] \hat{,} X_j] = 0. \quad (11)$$

It was stated in the original proposal [4] that all isoalgebras $\hat{L}$ are isomorphic to the original algebra L for all positive-definite isotopic elements. In other words, the isotopies cannot characterize any new Lie algebra because all possible Lie algebras are known from Cartan classification. Therefore, Lie-Santilli isoalgebras merely provide new nonlinear, nonlocal and noncanonical or nonunitary realizations of existing Lie algebras.

Under certain integrability and smoothness conditions hereon assumed, Lie algebras L can be "exponentiated" to their corresponding Lie transformation groups G and, vice-versa, Lie transformation groups G admit corresponding Lie algebras L when computed in the neighborhood of the unit I.

These basic properties are preserved under isotopies although broadened to the most general possible, axiom-preserving nonlinear, nonlocal and noncanonical transformation groups according to the following.

Theorem 3 [5b] (Lie-Santilli isogroups)

The isogroup characterized by finite (integrated) form $\hat{G}$ of isocommutation rules (1.12) on an isospace $\hat{S}(\hat{x},\hat{F})$ over an isofield $\hat{F}$ with common isounit $\hat{I} = 1/\hat{T} > 0$ is a group mapping each element $\hat{x} \in \hat{S}$ into a new element $\hat{x}' \in \hat{S}$ via the isotransformations:

$$\hat{x}' = \hat{g}(\hat{w}) \hat{\times} \hat{x}, \ \hat{x},\hat{x}' \in \hat{S}, \ \hat{w} \in \hat{F}, \quad (12)$$

with the following isomodular action to the right.

1) The map $\hat{g} \hat{\times} \hat{S}$ into $\hat{S}$ is isodifferentiable $\forall \hat{g} \in \hat{G}$;

2) $\hat{I}$ is the left and right unit:

$$\hat{I} \hat{\times} \hat{g} = \hat{g} \hat{\times} \hat{I} \equiv \hat{g}, \ \forall \hat{g} \in \hat{G}; \quad (13)$$

3) the isomodular action is isoassociative, i.e.,

$$\hat{g}_1 \hat{\times} (\hat{g}_2 \hat{\times} \hat{x}) = (\hat{g}_1 \hat{\times} \hat{g}_2) \hat{\times} \hat{x}, \ \forall \hat{g}_1, \hat{g}_2 \in \hat{G}; \quad (14)$$

4) in correspondence with every element $\hat{g}(\hat{w}) \in \hat{G}$ there is the inverse element $\hat{g}^{-\hat{I}} = \hat{g}(-\hat{w})$ such that:

$$\hat{g}(\hat{0}) = \hat{g}(\hat{w}) \hat{\times} \hat{g}(-\hat{w}) = \hat{I}; \quad (15)$$

5) the following composition laws are verified:

$$\hat{g}(\hat{w}) \hat{\times} \hat{g}(\hat{w}') = \hat{g}(\hat{w}') \hat{\times} \hat{g}(\hat{w}) = \hat{g}(\hat{w} + \hat{w}'), \forall \hat{g} \in \hat{G}, \ \hat{w} \in \hat{F}; \quad (16)$$

with corresponding isomodular action to the left, and general expression:

$$\hat{g}(\hat{w}) = \prod_k \hat{e}^{\hat{i} \hat{\times} \hat{w}_k \hat{X}_k} \hat{\times} \hat{g}(0) \hat{\times} \prod_k \hat{e}^{-\hat{i} \hat{\times} \hat{w}_k \hat{X}_k}, \quad (17)$$

Another important property is that conventional group composition laws admit a consistent isotopic lifting, resulting in the following.

Theorem 4 [5b] (Baker-Campbell-Hausdorff-Santilli Theorem)

$$(\hat{e}^{\hat{X}_1}) \hat{\times} (\hat{e}^{\hat{X}_2}) = \hat{e}^{\hat{X}_3}, \quad (18a)$$

$$\hat{X}_3 = \hat{X}_1 + \hat{X}_2 + [\hat{X}_1 \hat{,} \hat{X}_2] \hat{/} \hat{2} + [(\hat{X}_1 - \hat{X}_2) \hat{,} [\hat{X}_1 \hat{,} \hat{X}_2]] \hat{/} \hat{1}2 + \ldots \quad (18b)$$

Let $\hat{G}_1$ and $\hat{G}_2$ be two isogroups with respective isounits $\hat{I}_1$ and $\hat{I}_2$. The direct isoproduct $\hat{G}_1 \hat{\times} \hat{G}_2$ is the isogroup of all ordered pairs:

$$(\hat{g}_1, \hat{g}_2), \ \hat{g}_1 \in \hat{G}_1, \hat{g}_2 \in \hat{G}_2, \quad (19)$$

with isomultiplication:

$$(\hat{g}_1, \hat{g}_2) \hat{\times} (\hat{g}_1', \hat{g}_2') = (\hat{g}_1 \hat{\times} \hat{g}_1', \hat{g}_2 \hat{\times} \hat{g}_2'), \quad (20)$$

total isounit $(\hat{I}_1, \hat{I}_2)$ and inverse $(\hat{g}_1^{-\hat{I}_1}, \hat{g}_2^{-\hat{I}_2})$.

The following particular case is important for the isotopies of inhomogeneous groups. Let $\hat{G}$ be an isogroup with isounit $\hat{I}$ and $\hat{G}_{\hat{a}}$ the group of all its inner automorphisms. Let $\hat{G}_{\hat{a}}^o$ be a subgroup of $\hat{G}_{\hat{a}}$ with isounit $\hat{I}^o$, and let $\Lambda(\hat{g})$ be the image of $\hat{g} \in \hat{G}$ under $\hat{G}_{\hat{a}}$. The semi-direct isoproduct $\hat{G} \hat{\times} \hat{G}_{\hat{a}}^o$ is the isogroup of all ordered pairs $(\hat{g}, \hat{\Lambda}) \hat{\times} (\hat{g}^o, \hat{\Lambda}^o)$ with total isounit:

$$I_{tot} = \hat{I} \times \hat{I}^o. \quad (21)$$

The studies of the isotopies of the remaining aspects of the structure of Lie groups is then consequential. It is hoped the reader can see from the above elements that the entire conventional Lie theory does indeed admit a consistent and nontrivial lifting into the covering Lie-Santilli formulation.

Among a considerable number of mathematical papers on the Lie-Santilli isotheory listed in the Comprehensive Bibliography of Volume [9], we quote in particular the readable review by Kadeisvili [15], an excellent presentation of the all fundamental isotopology by Falcon Ganfornina and Nunez Valdes [11], and the unification of all simple Lie algebras of a given dimension (excluding exceptional algebras) into one single Santilli isotope of the same dimension by Tsagas [16].

Theorem 5 [5b]

The structure functions $\hat{L}$ of a Lie-isotopic algebra $\hat{L}$ verify the conditions:

$$\hat{C}_{ij}^k = -\hat{C}_{ji}^k \quad (22)$$

and the property (when commuting with the generators)

$$\hat{C}_{ij}{}^p \star \hat{C}_{pk}{}^q + \hat{C}_{jk}{}^p \star \hat{C}_{pi}{}^q + \hat{C}_{ki}{}^p \star \hat{C}_{pj}{}^q = 0 \quad (23)$$

Isorepresentations of Lie-Santilli isoalgebras is classified into;

1. Regular isorepresentations which occur due to C's of the rules(10) are constants; and,

2. Irregular Isorepresentation occurring when the *C*'s of the rules (10) are functions of the local variables (an occurrence solely possible for the Lie-Santill isotheory).

Following the construction of the above isotopies, Santilli applied the resulting isotheory to the step-by-step construction of the isotopes of all primary aspects of the Lorentz and Poincaré symmetries for the above identified regular case [17-24] and representative independent contributions [25,26]. A recent general outline of the applications of the Lie-Santilli isotheory to various branches of physics and chemistry is available from memoir [27].

General Rules for Construction of Regular Isorepresentations

General construction

Regular isorepresentation of Lie-Santilli isoalgebras $\hat{L}$ over an isofield of characteristic zero can be constructed via non-unitary transformations of the conventional representations of the conventional Lie algebra *L*.

Therefore, the classification of of said isorepresentations can be done by classification of all possible non-unitary transforms of dimensions 2,3,4,...

The general rule for mapping Lie algebras into regular Lie-Santilli isoalgebras were identified for the first time by Santilli in [7] and then studied systematically in monographs [8]. They can be written as follows;

$$U \times U^{\dagger} = \hat{I} \neq I \tag{24}$$

This non-unitary transformation is applied to the entire mathematics of Lie's theory leading to Santilli's isomathematics. We get the following important fundamental transformations;

$$I \to \hat{I} = U \times I \times U^{\dagger} = \frac{1}{\hat{T}} \tag{25}$$

$$a \to \hat{a} = U \times a \times U^{\dagger} = a \times U \times U^{\dagger} = a \times \hat{I} \in \hat{F}, a \in F \tag{26}$$

$$e^{A} \to U \times e^{A} \times U^{\dagger} = \hat{I} \times e^{\hat{T} \times \hat{A}} = (e^{\hat{A} \times \hat{T}}) \times \hat{I} \tag{27}$$

$$A \times B \to U \times (A \times B) \times U^{\dagger} =$$

$$(U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = \hat{A} \hat{\times} \hat{B} \tag{28}$$

$$[X_i, X_j] \to U \times [X_i, X_j] \times U^{\dagger} =$$

$$= [\hat{X}_i \hat{,} \hat{X}_j] = U \times (C_{ij}^{k} \times X_k) \times U^{\dagger} =$$

$$C_{ij}^{k} \times \hat{X}_k \tag{29}$$

$$< \psi | \times | \psi > \to U \times < \psi | \times | \psi > \times U^{\dagger} =$$

$$= < \psi | \times U^{\dagger} \times (U \times U^{\dagger})^{-1} \times U \times | \psi > \times (U \times U\ \dagger) =$$

$$= < \hat{\psi} | \hat{\times} | \hat{\psi} > \times \hat{I} \tag{30}$$

$$H \times | \psi > \to U \times (H \times | \psi >) = (U \times H \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times | \psi >) =$$

$$= \hat{H} \hat{\times} | \hat{\psi} > \tag{31}$$

etc.

Invariance

The extremely important Invariance of the isorepresentation over time requires non-unitary transforms identically rewritten on Hilbert-Myung-Santilli isospaces over isofields.

$$W \times W^{\dagger} \neq I \tag{32}$$

to eqs (25)-(31) above, causes the lack of invariance, with consequential activation of the catastrophic inconsistency theorems reviewed in ref. [8] such as the change of basic isounit;

$$\hat{I} \to \hat{I}' = W \times \hat{I} \times W^{\dagger} \neq \hat{I} \tag{33}$$

This identical reformulation gives;

$$W \times W^{\dagger} = \hat{I}, W = \hat{W} \times \hat{T}^{\frac{1}{2}} \tag{34}$$

$$W \times W^{\dagger} = \hat{W} \hat{\times} \hat{W}^{\dagger} = \hat{W}^{\dagger} \hat{\times} \hat{W} = \hat{I} \tag{35}$$

under which we have the invariance of the isounit and isoproduct as;

$$\hat{I} \to \hat{I}' = W \times \hat{I} \times W^{\dagger} \neq \hat{I} \tag{36}$$

$$\hat{A} \hat{\times} \hat{B} \to \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^{\dagger} = (\hat{W} \times \hat{T} \times \hat{A} \times \hat{T} \times \hat{W}^{\dagger})$$

$$\times (\hat{T} \times \hat{W}^{\dagger})^{-1} \times \hat{T} \times (\hat{W} \times \hat{T})^{-1}$$

$$\times (\hat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \hat{W}^{\dagger})$$

$$= \hat{A}' \times (\hat{W}^{\dagger} \times \hat{T} \times \hat{W})^{-1} \times \hat{B}'$$

$$= \hat{A}' \times \hat{T} \times \hat{B}'$$

$$= \hat{A}' \hat{\times} \hat{B}' \tag{37}$$

etc.

Namely, the numerical value of the isounit and isotopic element are invariant.

$$\hat{I} \to \hat{I}' \equiv \hat{I} \tag{38}$$

$$A \hat{\times} B \to \hat{A} \hat{\times} 'B' \equiv \hat{A}' \hat{\times} B' \tag{39}$$

This is crucial since the isounit or the isotopic element represent non-Hamiltonian interactions.

Classification of regular isounitary isoirreducible isorepresentations of the Lie-Santilli $\widehat{SU}(2)$ isoalgebras over isofields of characteristic zero

Santilli [19,20] identified and constructed the following regular isorepresentation of Lie-Santilli isoalgebra $\widehat{SU(2)}$, from the conventional two-dimensional irreducible representation of the *SU*(2) Lie algebra defined by the well known Pauli's matrices.

This Classification is merely given by either the nonunitary transform $U\text{–}Diag(n_1,n_2),n_k$ real>0, or by $U\text{–}OffDiag(n_1,n_2)$.

Conventional Paulli matrices σ_k [2] satisfy the rules $\sigma_i\ \sigma_j = i\varepsilon_{ijk}, i, j, k=1,2,3$. We present the identification and classification [19,20] of these matrices due to isoalgebra $S\hat{U}_Q(2)$.

In general Lie-isotopic algebras are the image of Lie algebras under nonunitary transformations [15,28]. Under the transformation $UU^{\dagger} = \hat{I} \neq I$ a Lie commutator among the matrices acquires the Lie-isotopic form:

$$U(AB - BA)U^{\dagger} = A'QB' - B'QA', A' = UAU^{\dagger}, B' = UBU^{\dagger}, Q = (UU^{\dagger})^{-1} = Q^{\dagger} \tag{40}$$

As a result, a first class of fundamental (adjoint) isorepresentations called as regular adjoint isorepresentations are characterized by the maps $J_k = \frac{1}{2}\sigma_k \to \hat{J}_k = UJ_kU^{\dagger}, UU^{\dagger}\hat{I} \neq I$ with isotopic contributions that are factorizable in the spectra, $\pm\frac{1}{2} \to +\frac{1}{2}f(\Delta), \frac{3}{4} \to (\frac{3}{4}f^2(\Delta))$

Citation: Muktibodh AS, Santilli RM (2017) Studies of the Regular and Irregular Isorepresentations of the Lie-Santilli Isotheory. J Generalized Lie Theory Appl 11: 251. doi:10.4172/1736-4337.1000251

where $\Delta=det\mathbf{Q}$ and $f(\Delta)$ is a smooth nowhere-null function such that $f(1)=1$.

Santilli constructed the following example of regular iso-Pauli matrices:

$$\hat{\sigma}_1=\Delta^{-\frac{1}{2}}\begin{pmatrix}0 & g_{11}\\ g_{22} & 0\end{pmatrix},\hat{\sigma}_2=\Delta^{-\frac{1}{2}}\begin{pmatrix}0 & -ig_{11}\\ ig_{22} & 0\end{pmatrix}\hat{\sigma}_3=\Delta^{-\frac{1}{2}}\begin{pmatrix}g_{22} & 0\\ 0 & -g_{11}\end{pmatrix} \quad (41)$$

where $\Delta=det\mathbf{Q}=g_{11}g_{22}>0$. These representations verify the isotopic rules $\hat{\sigma}_i Q\hat{\sigma}_j=i\Delta^{\frac{1}{2}}\varepsilon_{ijk}\hat{\sigma}_k$, and consequently the following isocommutator rules and generalized isoigenvalues for $f(\Delta)=\Delta^{\frac{1}{2}}$ and,

$$[\hat{\sigma}_1,\hat{}\hat{\sigma}_j]=\hat{\sigma}_i Q\hat{\sigma}_j-\hat{\sigma}_j Q\hat{\sigma}_i=2i\Delta^{\frac{1}{2}}_{ijk}\hat{\sigma}_k\hat{\sigma}_3*|\hat{b}_i^2\rangle=\pm\Delta^{\frac{1}{2}}|\hat{b}_i^2\rangle\hat{\sigma}^{\hat{2}}*|\hat{b}_i^2\rangle=3\Delta|\hat{b}_i^2\rangle,i=1,2 \quad (42)$$

This confirms the 'regular' character of the generalization considered here. The isonormalized isobasis is then given by a trivial extension of the conventional basis $|\hat{b}\rangle=Q^{-\frac{1}{2}}|b\rangle$.

In fact, regular iso-Pauli matrices (41) admit the conventional eigenvalues $\frac{1}{2}$ and $\frac{3}{4}$ for $\Delta=1$ which can be verified by putting $g_{11}=g_{22}^{-1}=\lambda$.

It is important to emphasize the condition of isounitarity, i.e. $UU^{\dagger}=\hat{I}\neq I$ for which $n_1^2=1/n_2^2=\lambda>0$. This degree of freedom has major fundamental implications presented in ref. [20] as well as for the spin component of the first known representation of nuclear magnetic moments presented in the paper [29].

Classification of Regular Isounitary Isoirreducible Isorepresentations of the Lie-Santilli Isoalgebra $\hat{SO}_3$ on a Santilli Isofield of Characteristic Zero

The regular isounitary isoirreducible isorepresentations of the Lie-Santilli isoalgebra $\hat{SO}_3$ on a Santilli isofield of characteristic zero were first studied by Santilli in [17,30]. In this case we have the trivial isorepresentation characterized by $U=Diag(n_1,n_2,n_3)$ with $n_k>0$. There exist only two compact Lie-isoalgebras:

$$\hat{SO}_1(3): sign \quad g=(+,+,+) \quad (43)$$

$$\hat{SO}_2(3): sign \quad g=(-,-,-) \quad (44)$$

while the other remaining six algebras are noncompact.

The Lie-isotopic commutation rules for the compact algebras are:

$$\hat{SO}_1(3):[\widetilde{J_1};\widetilde{J_2}]=\widetilde{J_3},[\widetilde{J_2};\widetilde{J_3}]=\widetilde{J_1},[\widetilde{J_3};\widetilde{J_1}]=\widetilde{J_2} \quad (45)$$

For the case of isotope $\hat{SO}_1(3)$ we consider the compact isotopic rotation about the third axis. Then the group element:

$$\hat{R}(\theta_3)=S_g(\theta_3)\hat{I}=\begin{pmatrix}\cos\theta_3 & \frac{b_2}{b_1}\sin\theta_3 & 0\\ -\frac{b_1}{b_2}\sin\theta_3 & \cos\theta_3 & 0\\ 0 & 0 & 1\end{pmatrix}\hat{I} \quad (46)$$

with underlying transformations:

$$r'=\hat{R}(\theta_3)*r=S_g(\theta_3)r=\begin{pmatrix}x\cos\theta_3+y\frac{b_2}{b_1}\sin\theta_3\\ -x\frac{b_1}{b_2}\sin\theta_3+y\cos\theta_3\\ z\end{pmatrix} \quad (47)$$

which leave invariant the hyperboloids:

$$r'^t g_{(1)}r'=x'b_1^2x'+y'b_2^2y'+z'b_3^2z'=xb_1^2x+y(b_2)^2y+zb_3^2z=r^t g_{(1)}r \quad (48)$$

It is important to note that the isotopic commutation rules of $\hat{SO}_1(3)$ and those of the conventional $SO(3)$ coincide at abstract level of realization-free treatment of rotations. The same situation occurs for all other aspects of the theory, such as enveloping algebra, Casimir invariants, etc.

We start reformulation of:

$$SO(3): R(\theta)=e^{J_1\theta_1}|_\xi\, e^{J_2\theta_2}|_\xi\, e^{J_3\theta_3}|_\xi \quad (49)$$

verifying the conditions:

$$R^t R=RR^t=I \quad (50)$$

$$R^t=R^{-1} \quad (51)$$

$$detR=1 \quad (52)$$

where the θ's are the Euler's angles; the skew Hermitian generators are given by:

$$J_1=J_{23}=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{pmatrix},J_2=J_{31}=\begin{pmatrix}0 & 0 & -1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix},$$

$$J_3=J_{12}=\begin{pmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},J_k'=-J_k \quad (53)$$

leading to universal enveloping algebra ξ with conventional associative product of matrices and unit:

$$\xi: J_iJ_j=\text{associative product} \quad (54)$$

$$IJ_i=J_iI=J_i,\ I=diag(+1,+1,+1) \quad (55)$$

The attached Lie algebra is characterized by the familiar commutation rules:

$$SO(3):[J_i,J_j]=J_i,J_j]-J_jJ_i=-\varepsilon_{ijk}J_k,\ i,j,k=1,2,3 \quad (56)$$

while the second order Casimir invariant is given by:

$$J^2=\sum_{k=1}^{3}J_kJ_k=-2I \quad (57)$$

Now the isotopic rotation can be readily computed to give:

$$S_g(\theta)=\begin{pmatrix}cos(\theta_3g_{11}^{1/2}g_{22}^{1/2}) & g_{22}(g_{11}g_{22})^{-1/2}sin(\theta_3g_{11}^{1/2}g_{22}^{1/2}) & 0\\ -g_{11}(g_{11}g_{22})^{-1/2}sin(\theta_3g_{11}^{1/2}g_{22}^{1/2}) & cos(\theta_3g_{11}^{1/2}g_{22}^{1/2}) & 0\\ 0 & 0 & 1\end{pmatrix} \quad (58)$$

This is the most general possible isotopic lifting of rotations, which includes as particular cases all other forms under consideration. Following illustration are the particular cases.

Assume that the elements g_{11}=g_{33}=+1 and g_{22} is given by a function of the local variables $t, r, \dot{r}, ...$ that interconnects smoothly the values +1 and −1. Then it can be seen that;

Case I

For g_{11}=g_{22}=g_{33}=1 the transformations (58)reduce to the familiar compact rotations:

$$S_g(\theta_3) = \begin{pmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{59}$$

whereas,

Case-II

For g_{11}=− g_{22}=g_{33}=1 we get the familiar noncompact Lorentz transformations:

$$S_g(\theta_3) = \begin{pmatrix} \cosh\theta_3 & -\sinh\theta_3 & 0 \\ -\sinh\theta_3 & \cosh\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{60}$$

Irregular isorepresentations

The following example suggests the irregular representations of the Lie-Santilli isoalgebras [31,32], although there is no method of systematic construction for these algebras.

Consider the $SU(2)$ Lie-algebra. Then the following covering of Paulli matrices called as Paulli-Santilli irregular isomatrices, define an irregular two-dimensional isorepresentation of the Lie-Santilli isoalgebras $\hat{SU}(2)$.

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & g_1^2 \\ g_2^2 & 0 \end{pmatrix}, \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times g_1^2 \\ i \times g_2^2 & 0 \end{pmatrix}, \hat{\sigma}_3 = \begin{pmatrix} \omega \times g_1^2 & 0 \\ 0 & \omega \times g_2^2 \end{pmatrix} \tag{61}$$

The commutation rules for this irrgular isoalgebra are characterized by the following structure functions:

$$[\hat{\sigma}_1, \hat{\sigma}_2] = i \times \omega^{-1} \times \hat{\sigma}_3, [\hat{\sigma}_2, \hat{\sigma}_3] = i \times \omega \times \hat{\sigma}_1, [\hat{\sigma}_3, \hat{\sigma}_2] = i \times \omega \times \hat{\sigma}_1 \tag{62}$$

with generalized $SU(2)$ -spin eigenvalues given by:

$$\hat{\sigma}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle = (\hat{\sigma}_1 \times \hat{T} \times \hat{\sigma}_1 + \hat{\sigma}_2 \times \hat{T} \times \hat{\sigma}_2 + \hat{\sigma}_3 \times \hat{T} \times \hat{\sigma}_3) \times \hat{T} \times |\hat{\psi}\rangle = (2 + \omega^2) \times |\hat{\psi}\rangle, \tag{63}$$

$$\hat{\sigma}_3 \hat{\times} |\hat{\psi}\rangle = \hat{\sigma}_3 \times \hat{T} \times |\hat{\psi}\rangle = \pm\omega \times |\hat{\psi}\rangle, \omega \neq 1 \tag{64}$$

and we have the generalization of the conventional constant values of the spin 1/2 to locally variable eigenvalues that are important for the representation of the spin of a particle when under the extremely high pressures in the core of a star.

Concluding Remarks

In this paper, we have presented the apparently first study of the regular and irregular isorepresentations of the Lie-Santilli isoalgebras. The significance of the isorepresentations is established by their applications, such as: the reconstruction of exact parity in nuclear physics under electromagnetic interactions [20]; the first known numerically exact and time invariant representation of the anomalous magnetic moment of the deuteron [29], as well as of stable nuclei at large; the first known exact and time invariant representation of the spin of stable nuclei [33-35]; and other applications [8,9]. It is hoped that this study will stimulate additional research by independent colleagues in this important new field of mathematics and physics.

Acknowledgments

The authors have no words to thank all participants of the sessions New Sciences for a New Era of the ICNPAA Conference, University of La Rochelle, France, July 4-8, 2016, and of the SIPS 2-16 Summit, Hainan island, China, November 5 to 11, 2016. Particular thanks are due to Prof. S. Georgiev for deep mathematical insights, and Prof. A. A. Bhalekar for deep insight on applications to nuclear physics.

References

1. Bransden BH, Joachain CJ (2003) Physics of Atoms and Molecules, (2ndedn), Prentice Hall(Pearson Prentice Hall).
2. Blatt JM, Weiskopf VF (1963) Theoretical Nuclear Physics, Wiley, New York.
3. Einstein A, Rosen N (1935) The Particle Problem in General Theory of Relativity. Phy Rev 48: 73.
4. Fermi E (1950) Nuclear Physics. Univ of Chicago Pres.
5. Santilli RM (1983) Foundations of Theoretical Mechanics I. Springer-Verlag, Heidelberg, Germany.
6. Santilli RM (1993) Isonumbers and Genonumbers of Dimensions 1, 2, 4, 8, their Isoduals and Pseudoduals, and "Hidden Numbers" of Dimension 3, 5, 6, 7,. Algebras, Groups and Geometries 10: 273.
7. Santilli RM (1996) Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries, " in Isotopies of Contemporary Mathematical Structures, 42: 7-82.
8. Santilli RM (1995) Elements of Hadronic Mechanics, Volumes I and II Ukraine Academy of Sciences, Kiev, (2ndedn).
9. Santilli RM (2008) Hadronic Mathematics, Mechanics and Chemistry, Vol.s. I, II, III, IV, and V, International Academic Press.
10. Jiang CX (2001) Foundations of Santilli Isonumber Theory. International Academic Press.
11. Ganfornina RMF, Valdes JN (2001) Fundamentos de la Isdotopia de Santilli, International Academic Press. English translations Algebras, Groups and Geometries 32: 135-308.
12. Kadeisvili JV (1997) Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities. Ukraine Academy of Sciences, (2ndedn).
13. Georgiev S (2014) Foundation of IsoDifferential Calculus, Nova Publisher, USA.
14. Gandzha I, Kadeisvili J (2011) New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli, Sankata Printing Press, Nepal
15. Kadeisvili JV (1996) Foundations of Lie-Santilli isotheory. In: Vetro P (ed.) Isotopies of Contemporary Mathematical Structures, Rendiconti Circolo Matematico Palermo, Suppl 42: 7-82.
16. Sourlas DS, Tsagas GT (1993) Mathematical Foundation of the Lie-Santilli Theory. Ukraine Academy of Sciences.
17. Santilli RM (1985) Isotopies of Lie Symmetries. I (basic theory) and II (isotopies of the rotational symmetry), Hadronic J 8: 36.
18. Santilli RM (1983) Lie-isotopic lifting of special relativity for extended and deformable particles. Lettere Nuovo Cimento 37: 545-555.
19. Santilli RM (1993) Isotopic Lifting of the SU(2) Symmetry with Applications to Nuclear Physics. JINR rapid Comm 6: 24-38.
20. Santilli RM (1998) Isorepresentation of the Lie-isotopic SU(2) Algebra with Application to Nuclear Physics and Local Realism. Acta Applicandae Mathematicae 50: 177-190.
21. Santilli RM (1983) Lie-isotopic Lifting of Unitary Symmetries and of Wigner's Theorem for Extended and Deformable Particles. Lettere Nuovo Cimento 38: 509-521.
22. Santilli RM (1993) Nonlinear, Nonlocal and Noncanonical Isotopies of the Poincare' Symmetry. Moscow Phys. Soc 3: 255.

23. Santilli RM (1995) Recent theoretical and experimental evidence on the synthesis of the neutron. Chinese J. System Eng. and Electr 6: 177.

24. Santilli RM (1998) Isominkowskian Geometry for the Gravitational Treatment of Matter and its Isodual for Antimatter. Intern J Modern Phys D **7**: 351-407.

25. Aringazin AK, Aringazin KM (1995) Universality of Santilli's iso-Minkowskian geometry. in Frontiers of Fundamental Physics, M. Barone and F. Selleri, Editors, Plenum.

26. Kadeisvili JV (1996) An introduction to the Lie-Santilli isotopic theory. Mathematical Methods in Applied Sciences 19: 1349.

27. Santilli RM (2016) An introduction to new sciences for a new era. Clifford Analysis, Clifford Algebras and their Applications In press December (Cambridge, U.K).

28. Santilli RM (1991) Isotopic Generalization of Galilei's and Einstein's Relativities, Vol. I, Mathematical Foundations, 1-st edition Hadronic Press, Palm Harbor, FL (1991): 2-nd edition Ukrain Academy of Sciences, Kiev.

29. Santilli RM (1994) A quantitative isotopic representation of the dueteron magnetic moment, Proceedings of International Symposium, Dubna Deuteron-93, Joint Institute of Nuclear Research, Dubna, Russia.

30. Santilli RM (1991) Rotational isotopic symmetries. ICTP communication No. IC/91/261.

31. Santilli RM (1993) Isotopic Lifting of SU(2)-Symmetry with Applications to Nuclear Physics. JINR Rapid Comm 6: 93.

32. Santilli RM (1998) Isorepresentations of the Lie-Isotopic SU(2) Algebra with Applications to Nuclear Physics and to Local Realism. Acta Applicande Mathematicae 50: 177-190.

33. Santilli RM (1998) Neuclear realization of hadronic maechnics and the exact representation of neuclear magnetic moments. Intrn J of Physics 4: 1-70.

34. Santilli RM (2010) Experimental verification of IsoRedshift with possible absence of Universe expansion, Big Bang, Dark matter, and Dark energy. The open Astronomy Journal 3: 124.

35. Bhalekar AA, RM Santilli (2016) Exact and Invariant Representation of Nuclear Magnetic Moments and Spins According to Hadronic Mechanics. American Journal of Modern Physics 5(2-1): 56-118.