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Stochastic Differential Games with Applications

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Due to the nature of uncertainty, implementing stochastic differential games is of importance for managing risk and maximizing investors' returns in financial markets. Furthermore, stochastic differential games have extensive applications in many fields such as engineering, economics, and management science. The theory has been well developed for stochastic differential games with deterministic coefficients over the past decades. While studying financial investment models, however, one has to cope with term-structure interest rates or stochastic volatilities. In this research project, we investigate stochastic differential game problems with random coefficients in the dynamic systems and the cost functional. Stochastic differential games, one of the most complex forms of decision making with uncertainty, have been implemented in everyday life and given rise to enriched research problems in academic areas such as science, engineering, economics and finance. In stochastic differential games, interactions among strategic decisions, dynamic evolution and stochastic characteristics have to be taken into consideration simultaneously. The complexity of stochastic differential games gives rise to significant difficulties in deriving solutions. A good example is the strategic game-type problem in economics and finance in which the leader firm moves first and then the follower firms move sequentially. The leader takes higher risk, while the followers give up the first-mover advantage but observe more valuable information before making investment decisions. This highlights the need for research on stochastic differential games. To the best of our knowledge, while there has been some research in this stream of literature which considers financial investment problems via stochastic differential games, none of the prior studies have involved random coefficients. This field intends to investigate a family of stochastic differential problems where both the coefficients in the dynamic systems and the cost functional are random, and will attempt to solve the problems using various delicate stochastic analysis techniques. Game theory is the study of decision making in an interactive environment. The foundations of modern game theory were laid by von Neumann-Morgenstern [1]. One fruitful branch of game theory is the theory of differential games, which originated from Isaacs [2] who investigated interactive decision making over time. In financial markets, due to the nature of uncertainty, considering stochastic differential games is of importance if one wants to control risk and to maximize returns for investors. In recent years, researchers have expended significant effort toward the theory of stochastic differential games. The study of Fleming-Souganidis is one of the main representatives along this line of research. In the field of financial economics, furthermore, stochastic differential game problems have been extensively studied in the principal-agent framework over a finite investment horizon. Nevertheless, the theory of stochastic differential games has not yet been developed to the point where it could be applied to a more general set-up with random coefficients, as would be required to solve more complex financial investment questions with two or more players. Therefore, in this field, we are interested in how to make more reasonable decisions when decision-makers face a more complex uncertainty environment. In the financial economics literature, the study of stochastic differential games is usually based on a forward stochastic system governed by an Itˆo stochastic differential equation. (SDE, for short) with deterministic coefficients. Fleming-Souganidis analyze (two player) zero-sum stochastic differential games in a rigorous way, and prove that the upper and lower the value functions of such games satisfy the dynamic programming principle and are the unique viscosity solutions of the associated Hamilton-Jacobi-Bellman-Isaacs equations. Lim et al, [3,4] introduces a special stochastic differential game with HARA utility. Yong and Mou-Yong [5,6] studied a class of linear-quadratic stochastic differential games with explicit strategies. Huang-Li-Wang considers risk-sensitive models with nearly optimal controls and partial information. Another direction for study is that Buckdahn-Li [7] apply the technique of backward stochastic differential equations (BSDEs, for short) to investigate two-player zero-sum stochastic differential games, while Yong [8] further paves the road for coupled forward-backward stochastic differential equations (FBSDEs, for short) with mixed initial-terminal conditions motivated by leader-follower stochastic differential games.

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