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# Star with Coefficient a in the set of Real Numbers

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## Abstract

This paper gives two definitions: Star with coefficient a real and Star System with coefficient $\alpha$ in five unknowns. Examples of Star-System and Star-set are given, a relationship between two star-Systems is noted, and some general theorems are proven.

**Keywords:** System equation • Agebra • Linear equations • Matrix matrices

## Introduction

The aim of the present paper is to introduce and study a system of five equations in five unknowns, that will be called Star-System with coefficient α in five unknowns. Let a, b, c, d, e, α be elements of a R, and let T1, T2, T3, T4, T5 be unknowns (also called variables or indeterminates). Consider a star with α coefficient (– 1)

In addition to having the sum α in each line. The scalars α are called the star coefficient if α is a solution of equation α = T1(α) + T2(α) + T3(α) + T4(α) + T5(α) (Noted by α?), a vector (T1 , T2 , T3 , T4 , T5) is called a solution vector of this Star-System with coefficient α in five unknowns.

The present paper is organized as follows: In Section 2, we present some preliminary results and notations that will be useful in the sequel. In Section 3, we present some examples of Star-element. Section 4 is devoted to introduce and study a Star-function. In Section 5, we present one example of equivalent star-systems. Finally, in Section 6, we introduce the star-Differential operator and study some of their application.

## Some Basic Definitions and notations

In this section, we introduce some notations and star-system with coefficient α defined.

### Star with coefficient α in the set of real numbers.

**Definition 1.** A star with α coefficient is composed of five numbers outside a, b, c, d, e and five numbers inside T1, T2, T3, T4, T5, These last five numbers are written in the form of 5-tuple (T1, T2, T3, T4, T5) (Figure 2).

In addition to having the sum α in each line.

### A star-system with coefficient α:

**Definition 2.** Let a, b, c, d, e and α be real numbers, and let T1, T2, T3 , T4 , T5 be unknowns (also called variables or indeterminates). Then a system of the form is called a star-system with coefficient α in five unknowns. We have also noted ✯[a,b,c,d,e;α] = α. The scalars a, b, c, d, e are called the coefficients of the unknowns, and α is called the constant "Chaff" of the star-system in five unknowns. A vector (T1,T2,T3,T4,T5) in R5 is called a star-solution vector of this star-system if and only if ✯[a,b,c,d,e;α] = α.

The solution of a Star-system is the set of values for T1, T2, T3, T4 and T5 that satisfies five equations simultaneously.

**A star-element:** A star-element is a term of the five-tuple (T1, T2, T3, T4, T5) solution of a star-system ✯[a,b,c,d,e;α] = α, where (T1, T2, T3, T4, T5)∈ $R^5$.

**Star-Coefficient or Constant "Chaff":** The star-Coefficient or Constant "Chaff" is also noted by α? and is a solution of equation $\alpha_{\star}$ = T1(α) + T0 2(α) +

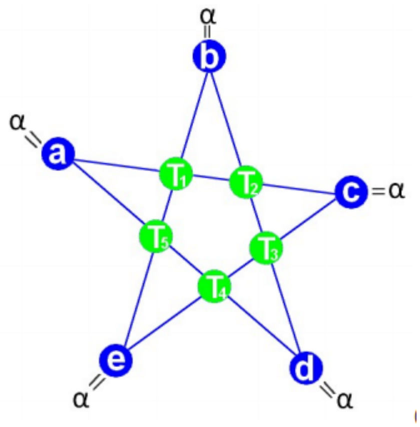


**Figure 1.**

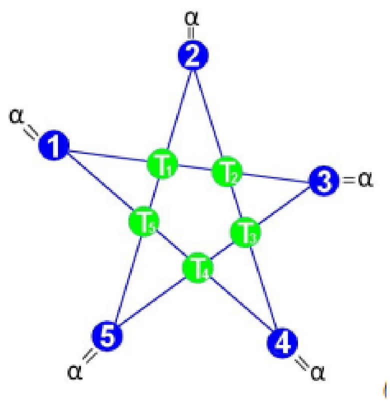


**Figure 2.**

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T3(α) + T4(α) + T5(α), wher (T1 , T2 , T3 , T4 , T5) is solution of a star-system ✯[a,b,c,d,e;α] = α.

**Star-Matrix:** The star-system with coefficient α can be written in matrix form

$M_* T = C_\alpha$ where $M_* = \begin{pmatrix} 11000 \\ 01100 \\ 00110 \\ 00011 \\ 10001 \end{pmatrix}$, vector T = (T1,T2,T3,T4,T5)

and $C_\alpha = \begin{pmatrix} \alpha - a - c \\ \alpha - b - d \\ \alpha - c - e \\ \alpha - a - d \\ \alpha - b - e \end{pmatrix}$

$M_*$ or $M_{Staris}$ called the star-Matrix of the star-system with coefficient α (✯[a,b,c,d,e;α] = α).

$M_★$ a matrix is said to be of dimension 5 × 5. A value called the determinant of M✯, that we denote by $|M_{Staris}|$ or |M✯|, corresponds to square matrix MF. Consequently, the determinant of M✯ is |M✯| = 1.

**Set-Star:** The set-star is constructed from the solution set of linear star-system with coefficient α ( [a,b,c,d,e;α] = α). The Set-star will be noted by $S_★$

**Star-System equivalent:** Equivalent Star-Systems are those systems having exactly same solution, i.e. Two star-systems are equivalent if solution of on starsystem is the solution of other, and vice-versa.

**Parametrized Curves:** A parametrized differentiable curve is simply a specific subset of $R^5$ with which certain aspects of differntial calculus can be applied.

**Definition 3.** A parametrized differentiable curve is a differentiable map α : I →$R^5$ of an open interval I = (a,b) of the real line R in to $R^5$

**Regular Curves:** A parametrized differentiable curve α : I →$R^5$, We call any point that satisfies α'(t) = 0 a singular point and we will ristrict our study to curves without singular points.

**Definition 4.** A parametrized differentiable curve is a differentiable α : I → $R^5$ is said to be regular if α'(t) ≠ 0 for all t ∈ I

**Parametric Arclength:** Generalized, a parametric arclength starts with a parametric curve in $R^5$. This is given by some parametric equations T1(t),T2(t),T3(t),T4(t),T5(t) , where the parameter t ranges over some given interval. The following formula computes the length of the arc between two points a, b.

**Lemma 1.** Consider a parametric curveT1(t),T2(t),T3(t),T4(t),T5(t), where t ∈ (a,b). The length of the arc traced by the curve as t ranges overt (a,b) is

$$\int_a^b \sqrt{\left(T_1'(t)\right)^2 + \left(T_2'(t)\right)^2 + \left(T_3'(t)\right)^2 + \left(T_4'(t)\right)^2 + \left(T_5'(t)\right)^2}\, dt$$

Thereafter I start with several examples with detailed solutions are presented.

## Examples of Star-element

This section will deal with solving problems with star-systems of five linear equations and five variables.

**Example 1.** A linear star-system with coefficient α composed of five linear equations in five variables T1 , T2 , T3 , T4 and T5 has the general form ✯[a,b,c,d,e;α] = α. In example 1: (a,b,c,d,e) = (1,2,3,4,5) When looking for the Solution of StarSystem with coefficient α✯[1,2,3,4,5;α] = α of Linear Equations, we can easily solve this using Star-Matrix $M_★$.

the star-systems of Linear Equations $\begin{cases} T_1 + T_2 = \alpha\text{-}4 \\ T_2 + T_3 = \alpha\text{-}6 \\ T_3 + T_4 = \alpha\text{-}8 \\ T_4 + T_5 = \alpha\text{-}5 \\ T_4 + T_5 = \alpha\text{-}7 \end{cases}$ → So the overall solution

is the set-star:$S_★$={α∈R,( $\frac{\alpha}{2} - 4 + \frac{\alpha}{2}, \frac{\alpha}{2} - 6, \frac{\alpha}{2} - 2, \frac{\alpha}{2} - 3$ )},

in a particular case if $\alpha = \frac{\alpha}{2} - 4 + \frac{\alpha}{2} + \frac{\alpha}{2} - 6 + \frac{\alpha}{2} - 2 + \frac{\alpha}{2} - 3$ that is to say α=10

We obtain the following results:

- The Star-coefficient: $α_★$ = 10
- The star-element is (1,5,−1,3,2) (Figure 3)

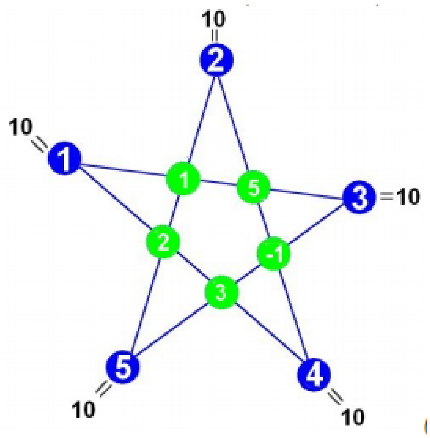


**Figure 3.**

**Note 1.** It is important to mention that a solution is made up of five values, (T1 , T2 , T3 , T4 , T5). A solution is made up of the set of values jointly taken by the variables to satisfy the system's equations.

**Example 2.** "Image of a five prime numbers"

Solve the following Star-system with coefficient α and five unknowns:

✯[3,7,11,13,17;α] = α.( Figure 4)

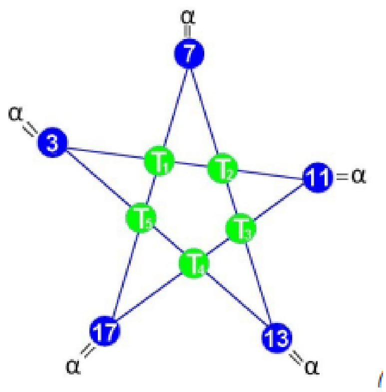


**Figure 4.**

The star-systems of linear equation: $\begin{cases} T_1 + T_2 = \alpha\text{-}14 \\ T_2 + T_3 = \alpha\text{-}20 \\ T_3 + T_4 = \alpha\text{-}28 \\ T_4 + T_5 = \alpha\text{-}16 \\ T_4 + T_5 = \alpha\text{-}24 \end{cases}$

The solution of the Star

System ✯[3,7,11,13,17;α] = α is therefore

$T_1=\frac{\alpha}{2}-15, T_2=\frac{\alpha}{2}+1, T_3=\frac{\alpha}{2}-21, T_4=\frac{\alpha}{2}-7, T_5=\frac{\alpha}{2}-9$

So the overall solution is the star-set: $S_\star$= $\alpha\in R,\left(\frac{\alpha}{2}-15,\frac{\alpha}{2}+1,\frac{\alpha}{2}-21,\frac{\alpha}{2}-7,\frac{\alpha}{2}-9\right)$

in a particular case if $\alpha=\frac{\alpha}{2}-4+\frac{\alpha}{2}+\frac{\alpha}{2}-6+\frac{\alpha}{2}-2+\frac{\alpha}{2}-3$ Then

• The Star-coefficient: $\alpha_\star$ = 34

• The star-element is (2,18,–4,10,8) (Figure 5)

More generally

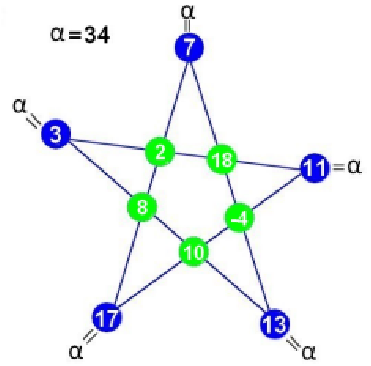


**Figure 5.**

**Theorem 1.** Let α ∈R, for all (a,b,c,d,e) ∈$R^5$ the star-system [a,b,c,d,e;α] = α has a unique solution $\left(\frac{\alpha}{2}-c+d-e,\frac{\alpha}{2}-a-d+e,\frac{\alpha}{2}+a-b-e,\frac{\alpha}{2}-a+b-c,\frac{\alpha}{2}-b+c-d\right)$ and the star-system we have unique star-cofficient: $\alpha_\star$ = 2/3(a+b+c+d+e)

For any star-system ✯[a,b,c,d,e;α] = α, the star-element is $\left(\frac{\alpha}{2}-c+d-e,\frac{\alpha}{2}-a-d+e,\frac{\alpha}{2}+a-b-e,\frac{\alpha}{2}-a+b-c,\frac{\alpha}{2}-b+c-d\right)$.

If α= $\frac{\alpha}{2}-c+d-e,\frac{\alpha}{2}-a-d+e,\frac{\alpha}{2}+a-b-e,\frac{\alpha}{2}-a+b-c,\frac{\alpha}{2}-b+c-d$

Then

• The Star-coefficient: $\alpha_\star$ = $\frac{2}{3}(a+b+c+d+e)$

• The star-element $\begin{pmatrix}T_1\\T_2\\T_3\\T_4\\T_5\end{pmatrix}=\begin{pmatrix}\frac{1}{3}&\frac{1}{3}&\frac{-2}{3}&\frac{5}{3}&\frac{-2}{3}\\ \frac{-2}{3}&\frac{1}{3}&\frac{1}{3}&\frac{-2}{3}&\frac{5}{3}\\ \frac{5}{3}&\frac{-2}{3}&\frac{1}{3}&\frac{1}{3}&\frac{-2}{3}\\ \frac{-2}{3}&\frac{5}{3}&\frac{-2}{3}&\frac{1}{3}&\frac{1}{3}\\ \frac{1}{3}&\frac{-2}{3}&\frac{5}{3}&\frac{-2}{3}&\frac{1}{3}\end{pmatrix}.\begin{pmatrix}a\\b\\c\\d\\e\end{pmatrix}$

## Examples of Star-function

In the following theorem, we give some useful result.

**Theorem 2**: Let α ∈ R, for all t ∈ R the Star-system ✯[t,t,t,t,t;α] = α has a unique solution, the Star-set containing only the vector ( $\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t$ ) and the Star-system we have unique star-coefficient : α= $\frac{10}{3}t$ .

Proof theorem (Figure 6)

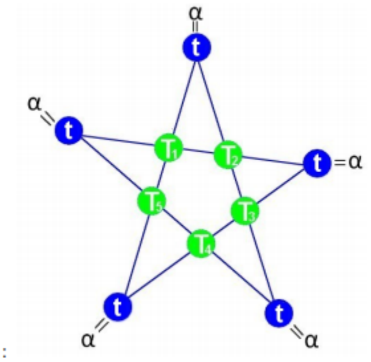


**Figure 6.**

The star with coefficient α:

Consider the following star-system of 5 equations in 5 unknowns:

$$\begin{cases}T_1+T_2=\alpha\text{-}2t\\T_2+T_3=\alpha\text{-}2t\\T_3+T_4=\alpha\text{-}2t\\T_4+T_5=\alpha\text{-}2t\\T_4+T_5=\alpha\text{-}2t\end{cases}$$

So the overall solution is the Star-set: $S_\star$ = $\left\{\left(\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t\right)\right\}$

in a particular case if $\alpha=\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t$ Then

• The Star-coefficient: $\alpha_\star$= $\frac{10}{3}t$

• The star-function: $\frac{2}{3}t,\frac{2}{3}t,\frac{2}{3}t,\frac{2}{3}t,\frac{2}{3}t$ (Figure 7)

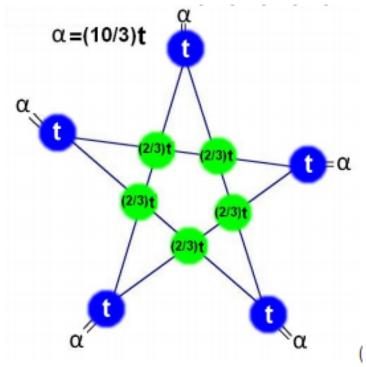


**Figure 7.**

**Theorem 3**: Let α ∈ R, for all t ∈ R the Star-system ✯[t, t+1, t+2, t+3, t+4,α] = α has a unique solution, the Star-set containing only the vector ( $\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t,\frac{\alpha}{2}-t$ ) and the Star-system we have unique star-coefficient : α= $\frac{10}{3}t$ .

Proof theorem (Figure 8)

Consider the following star-system of 5 equations in 5 unknowns:

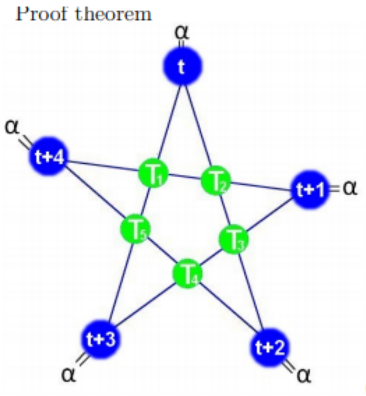


**Figure 8.**

$$\begin{cases} T_1+T_2=\alpha\text{-2t-2} \\ T_2+T_3=\alpha\text{-2t-4} \\ T_3+T_4=\alpha\text{-2t-6} \\ T_4+T_5=\alpha\text{-2t-3} \\ T_4+T_5=\alpha\text{-2t-5} \end{cases}$$

So the overall solution is the Star-set: $S_\star$ $\left\{\left(\frac{\alpha}{2}-t-3,\frac{\alpha}{2}-t+1,\frac{\alpha}{2}-t-5,\frac{\alpha}{2}-t-1,\frac{\alpha}{2}-t-4\right)\right\}$

if $\frac{\alpha}{2}-t-3,\frac{\alpha}{2}-t+1,\frac{\alpha}{2}-t-5,\frac{\alpha}{2}-t-1,\frac{\alpha}{2}-t-4$

Then

- The Star-coefficient: $\alpha_\star=\frac{10}{3}t+8$
- The star-function: $\frac{2}{3}t+1,\frac{2}{3}t+5,\frac{2}{3}t-1,\frac{2}{3}t+3,\frac{2}{3}t$ (Figure 9)

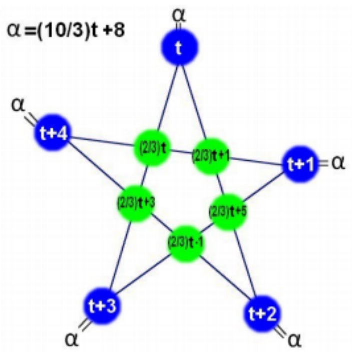


**Figure 9.**

**Theorem 4**: Let α ∈ R, for all t ∈ R the Star-system ✯$[x,2x,3x,4x,5x,\alpha]$ = α has a unique solution, the Star-set containing only the vector ( $\frac{\alpha}{2}-4t,\frac{\alpha}{2},\frac{\alpha}{2}-6t,\frac{\alpha}{2}-2t,\frac{\alpha}{2}-3t$ ) and the Star-system we have unique star-coefficient : α= $10t$ .

Proof theorem (Figure 10)

$$M_\star=\begin{pmatrix}11000\\01100\\00110\\00011\\10001\end{pmatrix}\text{, and } M_\star.\begin{pmatrix}T_1\\T_2\\T_3\\T_4\\T_5\end{pmatrix}=\alpha.\begin{pmatrix}1\\1\\1\\1\\1\end{pmatrix}$$

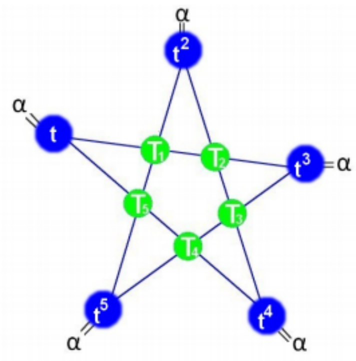


**Figure 10.**

So the overall solution is the Star-set: $S_\star$ : $\{(\frac{\alpha}{2}-4t,\frac{\alpha}{2},\frac{\alpha}{2}-6t,\frac{\alpha}{2}-2t,\frac{\alpha}{2}-3t)\}$.

if $\frac{\alpha}{2}-4t,\frac{\alpha}{2},\frac{\alpha}{2}-6t,\frac{\alpha}{2}-2t,\frac{\alpha}{2}-3t$ Then

- The Star-coefficient: $\alpha_\star=10t$.
- The star-function: $(t,5t,-t,3t,2t)$ (Figure 11)

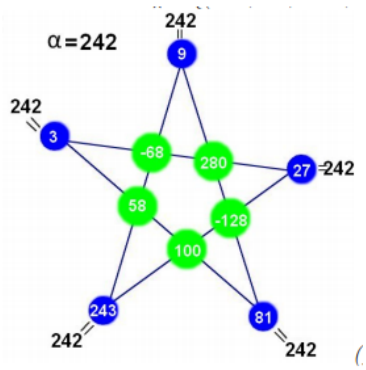


**Figure 11.**

In the special case t=3

•The Constant "Chaff": α = 30

•The Star-set:$S_\star$ = {3,15,–3,9,6)}. (Figure 12)

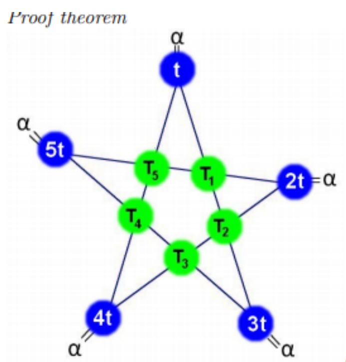


**Figure 12.**

**Theorem 5**: Let α ∈ R, for all t ∈ R the Star-system ✯$[t,t^2,t^3,t^4,t^5,\alpha]$ = α has a unique solution, $\left(\frac{\alpha}{2}-t^3+t^4-t^5,\frac{\alpha}{2}-t+t^4-t^5,\frac{\alpha}{2}-t+t^2-t^5,\frac{\alpha}{2}-t+t^2-t^3,\frac{\alpha}{2}-t^2+t^3-t^4\right)$ and the Star-system we have unique star-coefficient : α= $\frac{2}{3}(t+t^2+t^3+t^4+t^5)$ .

Proof theorem (Figure 13)

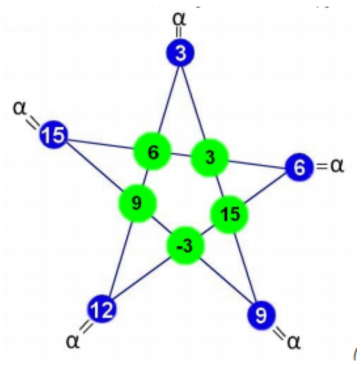


**Figure 13.**

$$M_{\star}=\begin{pmatrix}\frac{\alpha}{2}-t^3+t^4-t^5\\ \frac{\alpha}{2}-t\ -t^4+t^5\\ \frac{\alpha}{2}+t\ -t^2-t^5\\ \frac{\alpha}{2}-t\ +t^2-t^3\\ \frac{\alpha}{2}-t^2+t^3-t^4\end{pmatrix}=\alpha.\begin{pmatrix}1\\1\\1\\1\\1\end{pmatrix}$$

Star-set: $S_{\star}$: $\{(\frac{\alpha}{2}-t^3+t^4-t^5,\frac{\alpha}{2}-t\ -t^4+t^5,\frac{\alpha}{2}+t\ -t^2-t^5,\frac{\alpha}{2}-t\ +t^2-t^3,\frac{\alpha}{2}-t^2+t^3-t^4)\}$.

if $\frac{\alpha}{2}-t^3+t^4-t^5,\frac{\alpha}{2}-t\ -t^4+t^5,\frac{\alpha}{2}+t\ -t^2-t^5,\frac{\alpha}{2}-t\ +t^2-t^3,\frac{\alpha}{2}-t^2+t^3-t^4$

Then

- The Star-coefficient: $\alpha_{\star}=\frac{2}{3}(t+t^2+t^3+t^4+t^5)$ .
- The star-function: $f_{\star}$ : $R\to R^5$ defined by:

$f_{\star}$(t)=

$(\frac{1}{3}t+\frac{1}{3}t^2-\frac{2}{3}t^3+\frac{4}{3}t^4-\frac{2}{3}t^5,-\frac{2}{3}t\ +\frac{1}{3}t^2+\frac{1}{3}t^3-\frac{2}{3}t^4+\frac{4}{3}t^5,\frac{4}{3}t\ -\frac{2}{3}t^2+\frac{1}{3}t^3+\frac{1}{3}t^4-\frac{2}{3}t^5,$
$\frac{2}{3}t\ +\frac{4}{3}t^2-\frac{2}{3}t^3+\frac{1}{3}t^4+\frac{1}{3}t^5,\frac{1}{3}t-\frac{2}{3}t^2+\frac{4}{3}t^3-\frac{2}{3}t^4+\frac{1}{3}t^5)$

$$=\frac{1}{3}\begin{pmatrix}11-24-2\\ -211-24\\ 4-211-2\\ -24-211\\ 1-24-21\end{pmatrix}=\begin{pmatrix}t\\ t^2\\ t^3\\ t^4\\ t^5\end{pmatrix}$$

In the special case t=3

- The Star-coefficient: α = 242
- The Star-set:$S_{\star}$= {(–68,280,–128,100,58)}.(Figure 14)

**Theorem 6:** Let α ∈R, for all t ∈R the Star-system ✯[$e^{2t}$, $2e^{2t}$, $4e^{2t}$, $8e^{2t}$, $16e^{2t}$,α]= α has a unique solution $(\frac{\alpha}{2}-12e^{2t},\frac{\alpha}{2}+7e^{2t},\frac{\alpha}{2}-17e^{2t},\frac{\alpha}{2}-3e^{2t},\frac{\alpha}{2}-6e^{2t})$ and the Star-system we have unique star-coefficient:

$\alpha_{\star}=\frac{62}{3}e^{2t}$

Proof theorem (Figure 15)

Consider the following star-system of 5 equations in 5 unknowns:

$$\begin{cases}T_1+T_2=\alpha\text{-}5e^{2t}\\ T_2+T_3=\alpha\text{-}10e^{2t}\\ T_3+T_4=\alpha\text{-}20e^{2t}\\ T_4+T_5=\alpha\text{-}9e^{2t}\\ T_4+T_5=\alpha\text{-}18e^{2t}\end{cases}$$

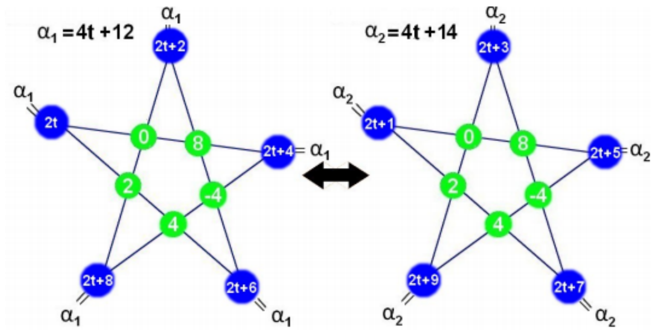


**Figure 14.**

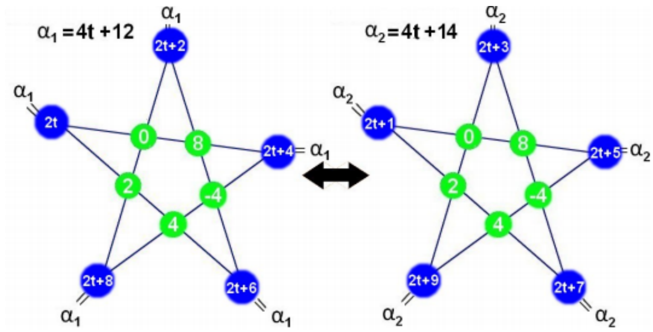


**Figure 15.**

$$2T_1=(\alpha-5e^{2t})-(\alpha-10e^{2t})-(\alpha-20e^{2t})-(\alpha-9e^{2t})-(\alpha-18e^{2t})=\alpha-23e^{2t}$$

So the overall solution is the Star-set: $S_{\star}$= $\left\{\left(\frac{\alpha}{2}-12e^{2t},\frac{\alpha}{2}+7e^{2t},\frac{\alpha}{2}-17e^{2t},\frac{\alpha}{2}-3e^{2t},\frac{\alpha}{2}-6e^{2t}\right)\right\}$

If α =T1 + T2 + T3 + T4 + T5 then the star coefficient: $\alpha_{\star}=\frac{62}{3}e^{2t}$

Let's find the length of t ∈ $\left[0;\frac{ln\left(\sqrt{2108}\right)}{2}\right]$ of the star-function $f$(t)= $\left(\frac{\alpha}{2}-12e^{2t},\frac{\alpha}{2}+7e^{2t},\frac{\alpha}{2}-17e^{2t},\frac{\alpha}{2}-3e^{2t},\frac{\alpha}{2}-6e^{2t}\right)$. We compute $f'$(t)= $(-24e^{2t},14e^{2t},-34e^{2t},-6e^{2t},-12e^{2t})$ and

$$|f'(t)|=\sqrt{(-24e^{2t})^2+(14e^{2t})^2+(-34e^{2t})^2+(-6e^{2t})^2+(-12e^{2t})^2}=\sqrt{2108e^{2t}},$$

So the length is $L=\int_0^{\frac{ln(\sqrt{2108})}{2}}\sqrt{2108e^{2t}}\,dt=1054-\sqrt{527}$

In the special case x= 0

- The star coefficient: $\alpha_{\star}=\frac{62}{3}$
- The Star-set:$S_{\star}=\left\{\left(\frac{-5}{3},\frac{52}{3},\frac{-20}{3},\frac{22}{3},\frac{13}{3}\right)\right\}$

## Examples of Star-systems equivalent

**Theorem 7.**

For all t ∈R if $\alpha_1$ = 4t + 12 and $\alpha_2$ = 4t + 12 then the two star-systems : $✯_1$[2t,2t+2,2t+4,2t+6,2t+8;α1] = α1 and $✯_2$[2t+1,2t+3,2t+5,2t+7,2t+9;α2] = $\alpha_2$ are equivalent.

Proof theorem

The star-system $✯_1$[2t,2t + 2,2t + 4,2t + 6,2t + 8;$\alpha_2$] = $\alpha_1$ can be written as:

$$\begin{cases} T_1+T_2=\alpha_1\text{-}4t\text{-}4 \\ T_2+T_3=\alpha_1\text{-}4t\text{-}8 \\ T_3+T_4=\alpha_1\text{-}4t\text{-}12 \\ T_4+T_5=\alpha_1\text{-}4t\text{-}6 \\ T_1+T_5=\alpha_1\text{-}4t\text{-}10 \end{cases} \Rightarrow \begin{cases} T_1=\frac{\alpha_1}{2}\text{-}2t\text{-}6 \\ T_2=\frac{\alpha_1}{2}\text{-}2t+2 \\ T_3=\frac{\alpha_1}{2}\text{-}2t\text{-}10 \\ T_4=\frac{\alpha_1}{2}\text{-}2t\text{-}2 \\ T_5=\frac{\alpha_1}{2}\text{-}2t\text{-}4 \end{cases}$$

The star-system $\star_2[2t+1, 2t+3, 2t+5, 2t+7, 2t+9; \alpha_2]=\alpha_2$ can be written as:

$$\begin{cases} T_1+T_2=\alpha_2\text{-}4t\text{-}6 \\ T_2+T_3=\alpha_2\text{-}4t\text{-}10 \\ T_3+T_4=\alpha_2\text{-}4t\text{-}14 \\ T_4+T_5=\alpha_2\text{-}4t\text{-}8 \\ T_1+T_5=\alpha_2\text{-}4t\text{-}12 \end{cases} \Rightarrow \begin{cases} T_1=\frac{\alpha_2}{2}\text{-}2t\text{-}7 \\ T_2=\frac{\alpha_2}{2}\text{-}2t+1 \\ T_3=\frac{\alpha_2}{2}\text{-}2t\text{-}11 \\ T_4=\frac{\alpha_2}{2}\text{-}2t\text{-}3 \\ T_5=\frac{\alpha_2}{2}\text{-}2t\text{-}5 \end{cases}$$

For all $t \in R$, if the tow Constants of the two star-systems $\alpha_1 = 4t + 12$ and $\alpha_2 = 4t+14$ then the Star-set of the two star-systems: $S_{\star 1} = S_{\star 2} = \{(0,8,\text{-}4,4,2)\}$ (Figure 16)

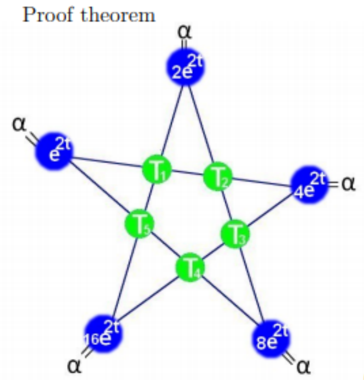


**Figure 16.**

## Examples of Star-operators

New Star-Differential operators: During our study of the construction of star-system, some new star-differential operators are required to be introduced.

Theorem 8. Let $\alpha \in R$, for all $t \in R$ and for all f be an n-times differentiable real function defined in interval I of R, the Star-system $\star\left[f, \frac{df}{dt}, \frac{d^2f}{dt}, \frac{d^3f}{dt}, \frac{d^4f}{dt}; \alpha\right] = \alpha$ has a unique solution and the Star-system we have unique star coefficient: $\alpha_\star = \frac{2}{3}f + \frac{2}{3}\frac{df}{dt}, \frac{2}{3}\frac{d^2f}{dt}, \frac{2}{3}\frac{d^3f}{dt}, \frac{2}{3}\frac{d^4f}{dt}$

**<u>Notation:</u>** A variety of notations are used to denote the n-times derivative.

$$\frac{df}{dt} = f', \frac{d^2f}{dt} = f^{(2)}, \frac{d^3f}{dt} = f^{(3)}, \frac{d^4f}{dt} = f^{(4)}$$

Proof theorem (Figure 17)

Consider the following star-system of 5 equations in 5 unknowns:

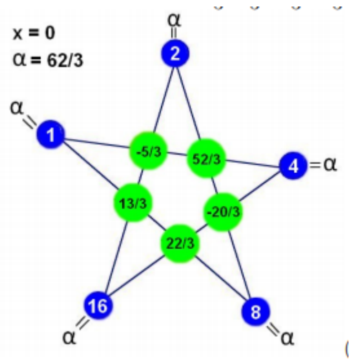


**Figure 17.**

$$\begin{cases} T_1+T_2=\alpha\text{-}f-f^{(2)} \\ T_2+T_3=\alpha\text{-}f'-f^{(3)} \\ T_3+T_4=\alpha\text{-}f^{(2)}-f^{(4)} \\ T_4+T_5=\alpha\text{-}f-f^{(3)} \\ T_4+T_5=\alpha\text{-}f'-f^{(4)} \end{cases} \Rightarrow \begin{cases} T_1=\frac{\alpha}{2}\text{-}f^{(2)}+f^{(3)}-f^{(4)} \\ T_2=\frac{\alpha}{2}\text{-}f-f^{(3)}-f^{(4)} \\ T_3=\frac{\alpha}{2}\text{-}f+f'-f^{(4)} \\ T_4=\frac{\alpha}{2}\text{-}f^{(2)}+f'-f^{(2)} \\ T_5=\frac{\alpha}{2}\text{-}f'+f^{(2)}-f^{(3)} \end{cases}$$

If $\alpha = \frac{\alpha}{2}\text{-}f^{(2)}+f^{(3)}-f^{(4)}+\frac{\alpha}{2}\text{-}f-f^{(3)}-f^{(4)}+\frac{\alpha}{2}\text{-}f+f'-f^{(4)}+\frac{\alpha}{2}\text{-}f^{(2)}+f'-f^{(2)}+\frac{\alpha}{2}\text{-}f'+f^{(2)}-f^{(3)}$ then the star coefficient: $\alpha_\star = \frac{2}{3}(f+f'+f^{(2)}+f^{(3)}+f^{(4)})$

In the special case $\alpha = \frac{2}{3}(f+f'+f^{(2)}+f^{(3)}+f^{(4)})$

The star-operator $\star_f$ defined by: $\star_f = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} & \frac{4}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{4}{3} & \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} & \frac{4}{3} & \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \cdot \begin{pmatrix} f \\ f' \\ f^{(2)} \\ f^{(3)} \\ f^{(4)} \end{pmatrix}$

## Applications

f(t)=sin(t): f is n-times differentiable at all $t \in R$

For all $t \in R$ the star-system $\star[\sin t, \cos t, -\sin t, -\cos t, \sin t; \alpha] = \alpha$

- The star coefficient: $\alpha_\star = \frac{2}{3}\sin(t)$
- The Star-functions: $\begin{cases} T_1 = \frac{1}{3}\sin t - cost \\ T_2 = \frac{1}{3}\sin t - cost \\ T_3 = \frac{1}{3}\sin t - cost \\ T_4 = \frac{1}{3}\sin t - cost \\ T_5 = -\frac{2}{3}\sin t \end{cases}$

• The Star-Set: $S_{\star} = \left\{\left(\frac{1}{3}\sin t - cost, \frac{1}{3}\sin t - cost, \frac{1}{3}\sin t - cost, \frac{1}{3}\sin t - cost, \frac{2}{3}\sin t\right)\right\}$ (Figure 18)

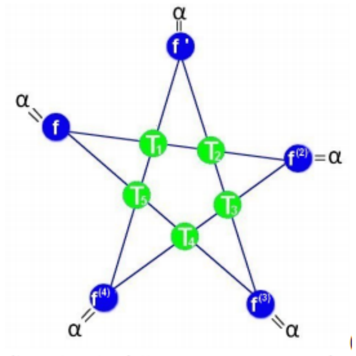


**Figure 18.**

f(t)=cos(t): f is n-times differentiable at all t ∈R

For all t ∈R the star-system ✯[cost,−sint,−cost,sint,cost;α] = α

• The star coefficient: $\alpha\star = \frac{2}{3}cos(t)$

• The Star-functions: $\begin{cases} T_1 = \frac{1}{3}cost + \sin t \\ T_2 = \frac{1}{3}cost - \sin t \\ T_3 = \frac{1}{3}cost + \sin t \\ T_4 = \frac{1}{3}cost - \sin t \\ T_5 = -\frac{2}{3}cost \end{cases}$

So the solution set is The Star-Set: $\left\{\left(\frac{1}{3}cost + \sin t, \frac{1}{3}cost - \sin t, \frac{1}{3}cost + \sin t, \frac{1}{3}cost - \sin t, -\frac{2}{3}\sin t\right)\right\}$

The star-function $f_{\star}(t) = \left(\frac{1}{3}cost + \sin t, \frac{1}{3}cost - \sin t, \frac{1}{3}cost + \sin t, \frac{1}{3}cost - \sin t, -\frac{2}{3}\sin t\right)$ defined in R to $R^5$

It's not possible to draw a 5*D* graphic, but in another world, a 5*D* world, it would be.

which can be written

$$f(t) = \left(\frac{1}{3}cost + \sin t\right).\begin{pmatrix}1\\0\\1\\0\\0\end{pmatrix} + \left(\frac{1}{3}cost - \sin t\right).\begin{pmatrix}1\\0\\1\\0\\0\end{pmatrix} - \frac{2}{3}\cos t.\begin{pmatrix}1\\0\\1\\0\\0\end{pmatrix}$$

The three vectors shown span the solution star-set. it is also not too hard to prove that they are linearly independent; therefore they form a basis for the solution starset (Figure 17).

In an abstract setting we can generally say that a projection is a mapping of a set, which means that a projection is equal to its composition with itself. (Figure 18)

in our case, we define by

$f(t) = (\frac{1}{3}cost + \sin t; \frac{1}{3}cost - \sin t; -\frac{2}{3}cost)$ the Star function in R to $R^3$.

In our world this Star-function is represent by (Figure 19):

**f(t)=sin(2t):** f is n-times differentiable at all t ∈R For all

t ∈R the star-system [sin2t,2cos2t,−4sin2t,−8cos2t,16sin2t;α] = α

• The star coefficient: α = 2 3 (14sin(2t)−6cos2t) (Figure 20)

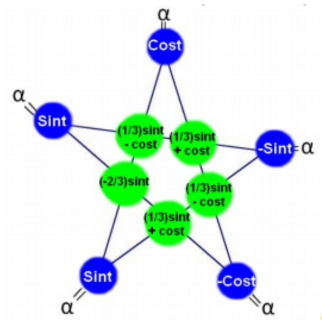


**Figure 19.**

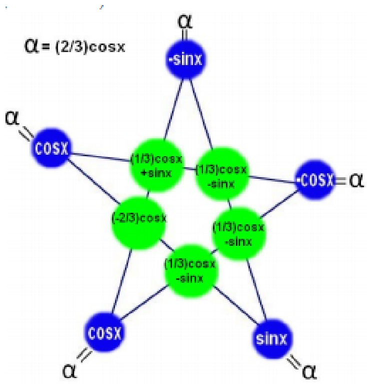


**Figure 20.**

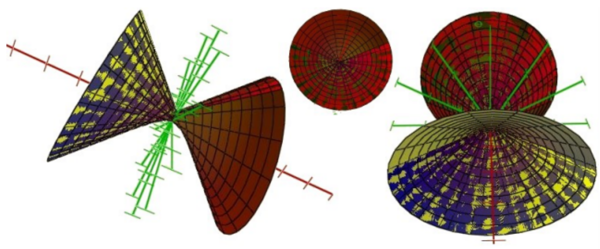

**Figure 21.**

• The Star-functions *f*: $\begin{cases} T_1 = \frac{-23}{3}sin2t - 10cos2t \\ T_2 = \frac{58}{3}sin2t + 6cos2t \\ T_3 = \frac{-32}{3}sin2t - 4cos2t \\ T_4 = \frac{22}{3}sin2t \\ T_5 = -\frac{1}{3}sin2t - 10cos2t \end{cases}$

**f(t)**= $e^t$: f is n-times differentiable at all t ∈R (Figure 21)

For all t ∈R the star-system [$e^t$, $e^t$, $e^t$, $e^t$, $e^t$;α] = α

• The star coefficient: $\alpha = \frac{10}{3}e^t$

• The Star-functions: $e^t = \frac{1}{3}e^t\begin{pmatrix}11-24-2\\-211-24\\4-211-2\\-24-211\\1-24-21\end{pmatrix}.\begin{pmatrix}1\\1\\1\\1\\1\end{pmatrix}$

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