Journal of Lasers, Optics & Photonics
ISSN: 2469-410X

Safaei et al., J Laser Opt Photonics 2016, 3:2
DOI: 10.4172/2469-410X.1000141

Research Article | Open Access

Stability of Temporal Dark Soliton in *PT*-Symmetric Nonlinear Fiber Couplers in Normal Dispersion Regime

Lida Safaei[1]*, Mohsen Hatami[2] and Mahmood Borhani Zarndi[1]

[1]*Faculty of Science, Department of Physics, Yazd University, Iran*
[2]*Faculty of Physics, Shiraz University of Technology, Iran*

Abstract

In this paper, we present analytical soliton solutions in a nonlinear *PT*-symmetric coupler with gain in one fiber and loss in the other one in the normal dispersion regime. As usual, we derive a temporal dark soliton solution for the coupler in the normal dispersion regime. We study the stability of the soliton solution by imposing a perturbation in the system by using the eigenvalue method. The results show that there exist two forms of perturbed soliton solutions, bright and dark. The numerical calculations show that the perturbed solutions in the bright form are stable and the dark form are unstable. Previous works in bright solitons show that there exists only one perturbed bright soliton solution which is stable only in particular regions.

Keywords: Fibers; Nonlinear optics; Instabilities and chaos; Guided waves; Dark solitons; *PT*-symmetric

Introduction

There has been a revolution in nonlinear physics over the past 20 years. Existence of solitons in the field of nonlinear optics was considered in 1973 [1].

Optical solitons are pulses which can be formed due to the balance between the group velocity dispersion and the self-phase modulation (SPM), traveling without distortion due to the dispersion or other effects. Solitons are classified in two forms [2,3].

Spatial Solitons is pulses which can propagate for long distances with an invariant transverse profile.

Temporal Solitons is Optical pulses that can propagate through a dispersive non-linear optical medium with an invariant shape.

These solitons, depending on the dispersion regime, can be found in two forms:

- Bright solitons for the anomalous dispersion regime.
- Dark solitons, for normal dispersion regime.

Optical dark solitons have been investigated in many theoretical and experimental papers [4]. Recently the applications of dark solitons have been interested in many achievements [5-7]. For example pulse propagation in nonlinear optical media a their applications in waveguides are some of intersting fields in new papers [8,9]. Also soliton transmission in optical fibers has been considered in nonlinear physics, electronics, photonics and communication [10].

Wave propagation modes of physical media are naturally separated into two generic classes, conservative and dissipative. Recently, it was recognized that a more particular species of *PT* (parity-time)-symmetric systems may be identified at the boundary between these generic types which remain invariant under the combination of parity and time-reversal symmetry operation [11-14].

The concept has its roots in quantum mechanics where a *PT*-symmetric non-Hermitian Hamiltonian may have an entirely real spectrum of eigenvalues [11,15-17].

The demonstrations of the *PT*-symmetric effects in optics in two waveguide directional linear couplers composed of waveguides with gain and loss were recently proposed in [17-19] and for bright optical solitons, which exist when the Kerr nonlinearity is obeyed by the couplers, were reported in [19-21]. Such systems are described by non Hermitian Hamiltonians. In quantum mechanics, the PT-symmetric potential satisfies the condition $V(x)=V^*(-x)^*$, denoting complex conjugation[15,20]. In nonlinear optics, the PT-symmetric potential is introduced by a complex refractive-index distribution combined with gain and loss regions, that obey the condition [17,20]. Nonlinear effects in directional couplers studied started in 1982 [22]. Theoretical analyses have suggested that such couplers operating in the nonlinear regime can be used for the all signal controls [23,24]. Arrays of the *PT*-symmetric couplers were proposed as a means of control of the spatial beam dynamics including the formation and switching of spatial solitons [17,25]. Investigating the properties of these solitary waves is interesting. Hatami et al. studied properties and behaviors of temporal dark solitons in different medias [26-29].

Solitons propagating in the *PT*-symmetric nonlinear couplers, composed of wave guides with gain and loss, were theoretically described in [30]. Stable bright spatial solitons with *PT*-symmetric potentials have recently been reported [17]. However, the stability of temporal dark solitons in PT-symmetric potentials in a nonlinear coupler with gain and loss is less studied. In this paper, at first we obtain an analytical solution for dynamical equations of *PT*-symmetric temporal dark solitons in nonlinear *PT*-symmetric couplers and discuss the stability condition. Then, the numerical results in the form of diagrams for *PT*-symmetric temporal dark solitons are illustrated.

Model

The *PT*-symmetric coupler with gain in one waveguide and loss

***Corresponding author:** Lida Safaei, Faculty of Science, Department of Physics, Yazd University, Yazd, Iran, Tel: +98 35 3123 2222; E-mail: Lidasafaei2015@gmail.com

Received May 01, 2016; **Accepted** November 07, 2016; **Published** November 23, 2016

Citation: Safaei L, Hatami M, Zarndi MB (2016) Stability of Temporal Dark Soliton in *PT*-Symmetric Nonlinear Fiber Couplers in Normal Dispersion Regime. J Laser Opt Photonics 3: 141. doi: 10.4172/2469-410X.1000141

in the other one has been studied theoretically and experimentally [17,19,28]. In general, the model that describes the propagation of light in an optical waveguide is based on Non-Linear Schrodinger Equation (NLS) equations coupled with the linear terms which represent the tunneling of light. To describe beams and pulses in such a system including the nonlinear *PT*-symmetric coupler and existence of dark solitons, we have the following dimensionless dynamical.

Equation:

$$iu_z \mp u_{TT} + 2|u|^2 u = -\upsilon + i\gamma u \qquad (1)$$
$$iu_z \mp u_{TT} + 2|u|^2 u = -\upsilon - i\gamma u$$

Here, u and v are the normalized amplitude variables at the top and bottom fiber waveguides, z and τ indicate the length of fiber and normalized time, respectively. The plus and minus signs are standing for anomalous and normal dispersion, respectively. Two kinds of solitons have been discovered depending on the dispersion sign:

Bright solitons correspond to the positive sign and in general have the solution as $U(z, \tau)=A\operatorname{sech}(z, \tau) \exp(iA2z)$ in an anomalous regime.

Similar to the case of bright solitons, *dark solitons* corresponding to negative sign of dispersion are generally in the form of $u(z, \tau)=A\tanh(z, \tau) \exp(iA2z)$.

Figure 1a and 1b display the bright soliton and the dark soliton in general. The main difference between the dark and bright solitons is that the domains of dark solitons approach a constant as $\tau \to \pm\infty$ while for bright solitons they vanish to zero [2,3].

The *PT*-symmetric coupler with gain in one fiber and loss in the other one has recently been studied theoretically and experimentally [1,25-27]. To investigate the solution in a *PT*-symmetric nonlinear coupler, a simple object composed of a pair of nonlinear coupled optical fibers is considered. We illustrate a schematic of *PT*-symmetric coupler in Figure 2.

The coupler consists of four ports for input and output pulses. To satisfy the *PT*-symmetric condition, we assume the group velocities and the second-order dispersions in fibers are matched, and we normalize the coefficients to $u\tau\tau$ and $v\tau\tau$ to unity, and hence, the two waveguides have the same Kerr nonlinearity coefficients. Malomed et al. show that it is necessary for the existence of solitons [13,17].

In Eq.(1), the first term in the right hand side is related to coupling between the modes propagating in the two fiber waveguides and γ terms stand for the gain in one fiber and loss in the other. Without loss of generality, γ can be considered to be positive which means that the gain is supposed to be in the top fiber waveguide and loss in the bottom one. To confirm the *PT*-symmetric condition, the gain and loss coefficients must be equal [17].

The associated powers are:

$$p_u = \int |u|^2 dT \qquad (2)$$
$$p_\upsilon = \int |\upsilon|^2 d_T$$

In this system, neither the individual powers associated with the two modes nor their sum are conserved if $\gamma/=0$.

For the total power:

$$\frac{d}{dt}(p_u + p_\upsilon) = 2\gamma(p_u - p_\upsilon) \qquad (3)$$

In order to analyze Eq (1), it is convenient to represent the variables in the following form:

$$u(z,T) = \exp^{i(\Omega z-\theta)}U(z,T),\ u(z,T) = \exp^{i(\Omega z)}V(z,T) \qquad (4)$$

Where θ is a constant angle satisfying:

$$\sin\theta = \gamma \qquad (5)$$

and Ω is a real parameter.

The substitution of Eq. (4) into Eq.1 leads to:

$$iU_z - U_{TT} - \Omega U + 2|U|^2 U = -\cos\theta V + i\gamma(U - V), \qquad (6)$$
$$iV_z - V_{TT} - \Omega V + 2|V|^2 V = -\cos\theta U + i\gamma(U - V)$$

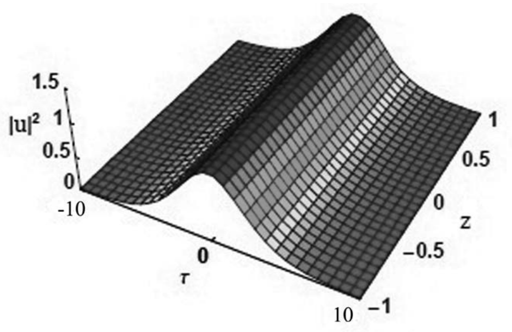
By applying $U=V \equiv \varphi$ Eq.6 reduce into:

$$i\varphi z - \varphi\tau\tau - a2\varphi + 2 \mid \varphi \mid 2\ \varphi=0 \qquad (7)$$

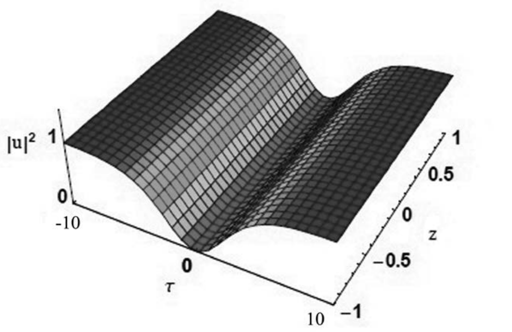
Here $a^2 = \Omega - \cos\theta$

By solving equation (7), it is obvious that we arrive at familiar dark soliton solution:

$$\phi(T,z) = a\tanh(aT)\exp(ia^2 z) \qquad (8)$$



(a)



(b)

Figure 1: Scheme of (a) Bright soliton (b) Dark soliton in general form.

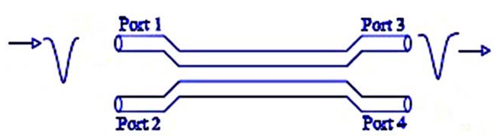


Figure 2: Scheme of nonlinear *PT*-symmetric coupler with gain in top waveguide and loss in bottom waveguide.

Solition Stability

Perturbation

To investigate the stability of these solitons, we use the same perturbation method as used by Alexeeva et al. [17].

Let

$$U(T,z)=\phi(T)+\delta U(T,z),\ V(T,z)=\phi(T)+\delta V(T,z) \tag{9}$$

By using symmetric and antisymmetric combination

$$p=\frac{\delta U+\delta V}{\sqrt{2}},\ q=\frac{\delta U-\delta V}{\sqrt{2}} \tag{10}$$

We linearized Eqs.(8) in δU and δV. Consider a separable solution for the linearized equation in the form of:

$$p=\exp(\nu t)\left[(p_1'+ip_2')\cos\omega t+(p_1''+ip_2'')\sin\omega t\right] \tag{11}$$

$$q=\exp(\nu t)\left[(q_1'+iq_2')\cos\omega t+(q_1''+iq_2'')\sin\omega t\right]$$

Where

$$p_1=p_1'+ip_1''\ p_2=p_2'+ip_2''$$

$$q_1=q_1'+iq_1''\ q_2=q_2'+iq_2''$$

By introducing the operator

$$L=\begin{pmatrix}\frac{d^2}{dT^2}+\Omega-6\phi^2 & 0\\ 0 & \frac{d^2}{dT^2}+\Omega-2\phi^2\end{pmatrix} \tag{12}$$

and the component vectors :

$$\vec{p}=\begin{pmatrix}p_1\\ p_2\end{pmatrix},\vec{q}=\begin{pmatrix}q_1\\ q_2\end{pmatrix} \tag{13}$$

We arrive at two eigenvalue problems

$$\left(L\text{ - }cos\theta\right)\vec{p}+2\gamma J\vec{q}=\mu J\vec{p} \tag{14}$$

$$\left(L\text{ - }cos\theta\right)\vec{q}+2\gamma J\vec{p}=\mu J\vec{q} \tag{15}$$

We assume ν and ω are real, $\mu=\nu-i\omega$ and also the skew-symmetric matrix, J is:

$$J=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} \tag{16}$$

We can obtain two eigenvectors for the Eqs. (14) and (15)

$$\begin{pmatrix}\vec{p}\\ 0\end{pmatrix},\begin{pmatrix}\vec{p}\\ \vec{q}\end{pmatrix} \tag{17}$$

In the first eigenvector, both eigenvalues are zero for q. This component leads Eq.12 to be a linearized eigenvalue equation which can be integrated. Therefore, it is not a perturbed equation and it will be stable.

The second eigenvector has q/=0, and using this eigenvector for Eq (12), it becomes a non-homogeneous equation and with no nonzero eigenvalue. This vector is an eigenfunction for Eq (13) and hence, investigating the stability is reduced to solving Eq. (13). These results are the same as bright soliton solutions.

The eigenvalue problem (15) can be written as:

$$\begin{pmatrix}L_{1+n} & 0\\ 0 & L_{0+n}\end{pmatrix}\begin{pmatrix}q_1\\ q_2\end{pmatrix}=\lambda J\begin{pmatrix}q_1\\ q_2\end{pmatrix} \tag{18}$$

Here $T=a\tau$, $\lambda=\mu/a2$, $\eta=2(\cos\theta/a2)$ and $L0,1$ are Sturm-Liouville operators

$$L_0=d^2/dT^2+1-2\tanh^2 T, \tag{19}$$

$$L_1=d^2/dT^2+1-6\tanh^2 T, \tag{20}$$

According to equation $L_0y_0=\lambda y_0$, the eigenvalue of L_0 is $\lambda=-1$, and it's corresponding eigenfunction is $y0=\tanh X$.

For eigenfunction equation $L_1y_1=\lambda y_1$, there exists two eigenvalues $\lambda=-1$ and $\lambda=0$, the corresponding eigenfunctions are respectively $y_1=(1-\tanh2\ T)$, and $y1=sechT\ tanhT$. The second one resembles the bright solitons' perturbation eigenfunction.

As we can see in propagating a dark soliton in a *PT*-symmetric coupler, the perturbed equation has two kinds of solutions, namely the bright and dark solitary solutions.

The lowest eigenvalue of the operator $L0+\eta$ equals η. For the scalar function of $q1(T)$, the eigenvalue problem in Eq.(16) can be written in the form of:

$$\left(L_1+\eta\right)q_1=-\lambda^2\left(L_0+\eta\right)^{-1}q_1 \tag{21}$$

As the operator on the left side is symmetric, so the right side is symmetric and positive. The lowest eigenvalue is defined by:

$$-\lambda^2=\min\frac{<q_1\left|L_1+\eta\right|q_1>}{<q_1\left|(L_0+\eta)^{-1}\right|q_1>} \tag{22}$$

The minimum is positive if the numerator is positive, and as mentioned before, the lowest eigenvalue of the operator $L1$ is equal to (-1) and the numerator, $(-1+\eta)$, is positive if $\eta>-1$.

Using $\eta=2\frac{\cos\theta}{a^2}$, the stability condition is calculated as $a\le a_c, a_c^2=2\sqrt{1-\gamma^2}$.

Numerical results

According to the *PT*-equations, for evaluating a dark soliton inside a *PT*-symmetric nonlinear coupler, the numerical methods might be applied to investigate the stability of these perturbed equations by means of the eigenvalues introduced in solutions of Eqs.19 and 20 in the last section.

For the numerical simulation and verification of the propagation of the solitons, we attempted to construct such solitons by adding perturbed eigenfunctions and corresponding eigenvalues where the perturbation method to the initial pulses needs to be applied. The initial condition is taken such that the dark soliton pulse is launched into the coupler as follows:

$$\begin{aligned}u_1(0,\tau)&=\tanh(a\tau)\\ u_2(0,\tau)&=\tanh(a\tau)\end{aligned} \tag{23}$$

Then the perturbed eigenfunctions is added to these pulses.

As it was mentioned in solution of Eq (14) and (15), three different eigenfunctions with their real eigenvalues yield. The evolution of dark soliton inside a coupler to obtain the expected gain in top waveguide and loss in bottom by calculating the eigenfunctions and their eigenvalues with the perturbed method is presented. Figure 3a and 3b show the evolution of dark soliton with $y0=tanh(T)$ and perturbed eigenfunction where it propagates through the *PT*-symmetric nonlinear coupler for $|u|2$ versus τ and z, and $|v|2$ versus τ and z.

Also in Figure 3c, the evaluation of total powers, $Pu+Pv$, is calculated and illustrated according to Eq. (2).

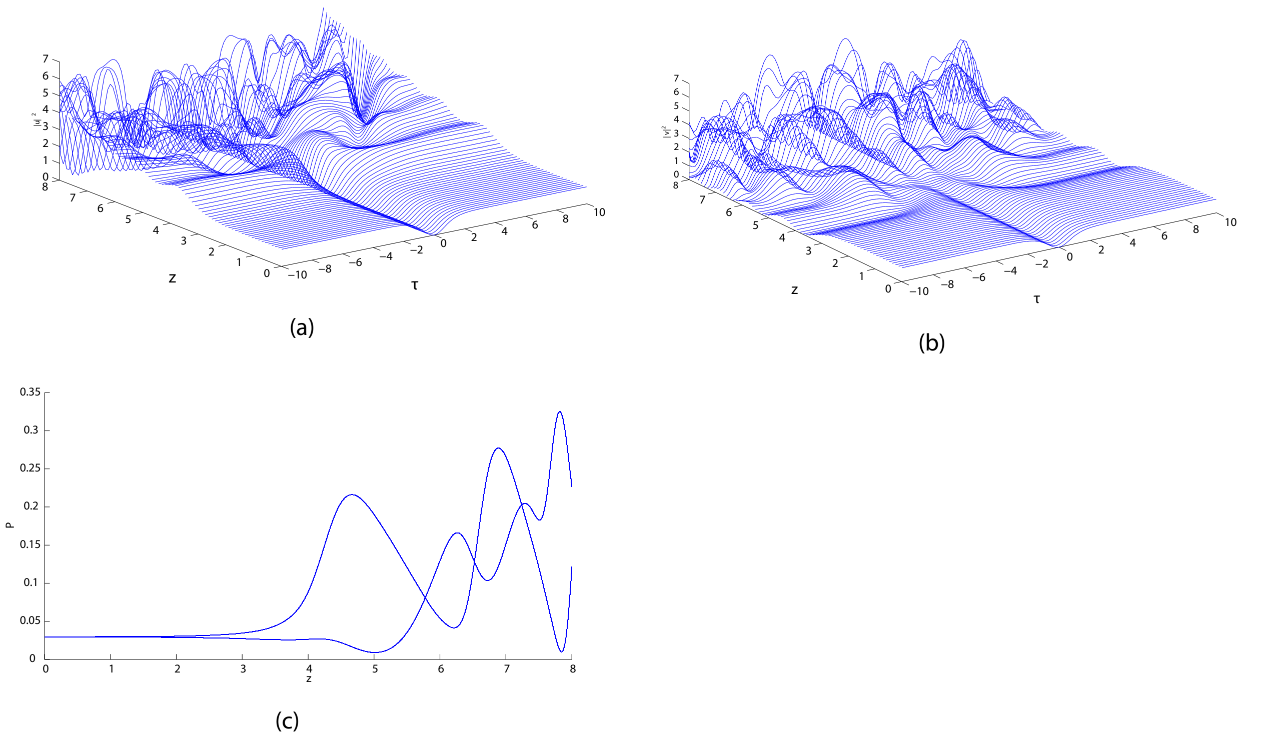


Figure 3: The evolution of dark soliton with the perturbed eigenfunction y0=tanh(T) (a) The evolution of j u j2 versus _ and z , (b) The evolution of j v j2 versus _ and z, (c) The evaluation of total power.

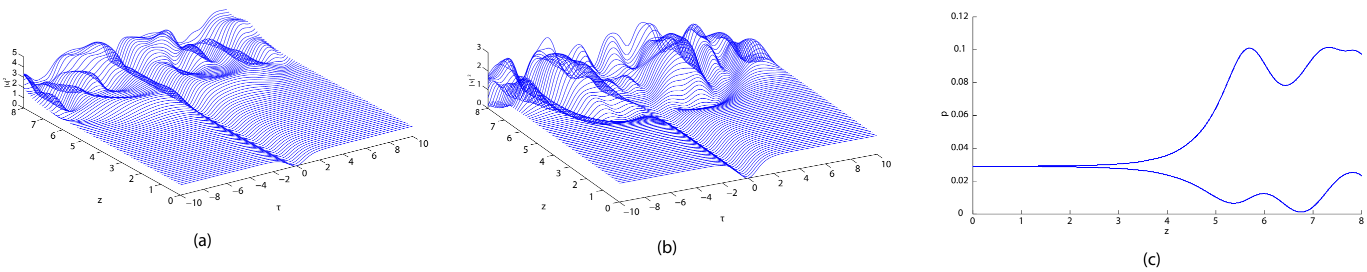


Figure 4: The numerical results according to the analytical solution for the perturbed eigenfunction y_1=sech(T)tanh(T) added to the initial pulse (a) The evolution of j u j_2 (b) The evolution of j v j_2 (c) Total power.

The domain and phase is changed in this situation and according to Figure 3a-3c and their equations, this soliton with the added perturbed eigenvalue and corresponding eigenfunction is unstable.

In Figure 4a and 4b the numerical results are presented in terms of the analytical solution for the perturbed eigenfunction $y_1 = sech(T)\, tanh(T)$ added to the initial pulse for $|u|2$ and $|v|2$. Moreover, according to Figure 4c, it can be concluded that although some fluctuation in this solution is observable, it is stable.

In Figure 5a and 5b the evolution of the dark soliton with the second perturbed eigenfunction, $y_1=(1 - tanhT\ 2)$, and corresponding eigenvalues which are calculated by perturbation method according to Eqs (13) and 16 for $|\ u\ |2$ and $|v|2$ are depicted.

The evaluation of the associated total power, $Pu + Pv$, of this result is also obtained and presented in Figure 5c.

Given these results, it can be safely assumed that propagating dark soliton in a *PT*-symmetric nonlinear coupler in this case is unstable.

Conclusion

In this work, we obtained the analytical and numerical solutions of propagating a pulse in a nonlinear coupler with nonlinearity in *PT*-symmetric potentials with gain in one fiber waveguide and loss in the other one. Analytical investigations showed that the coupled equations which propagate in the fiber waveguide had a solution in the form of temporal dark soliton. The stability of these solutions was obtained by analytical and numerical analyses via perturbation method. Evaluating perturbed analytically showed that perturbations which guaranty the stability were in two forms of bright and dark solitons. However, the usual solutions of non-coupled equations were merely dark solitons and for a bright solution propagating in the nonlinear coupled fiber, only a bright soliton perturbation satisfied the stability. According to the numerical results and total observable power perturbed in the form of bright solutions, we conclude that although this soliton has some fluctuations, it is stable. In addition, in the perturbed dark form, the temporal dark soliton is proved to be unstable.

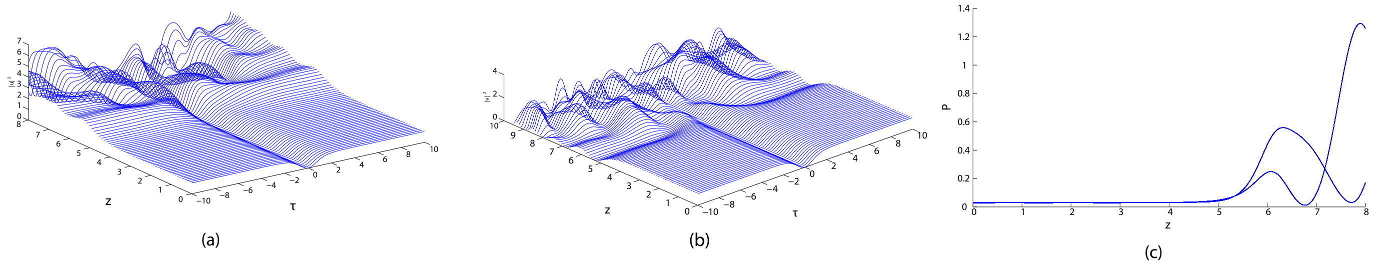


Figure 5: The evaluation of perturbed eigenfuntion, "1tanh (T)" added to the initial pulse is presented for (a) j u j$_2$ and (b)j v j$_2$ and in (c) the corresponding total power.

References

1. Kivshar YS, Agrawal G (2003) Optical Solitons from fibers to photonic crystals. (1stedn) Academic Press pp: 4-20.
2. Agrawal G (2007) Nonlinear Fiber optics. (4thedn) Academic Press pp: 135-180.
3. Boyed R (2009) Nonlinear Optics. (3rdedn) Academic Press pp: 336-383.
4. Kivshar YS, Barry LD (1998) Dark optical solitons. physics and applications Elsevier pp: 81-197.
5. Fernandes DE, Silveirinha MG (2014) Bright and dark spatial solitons in metallic nanowire arrays. Photonics and Nanostructures Fundamentals and Applications 12: 340-349.
6. Hasegawa A, Tappert F (1973) Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. II. Normal dispersion. ppl Phys Lett 23: 171.
7. Simovski CR, Belov PA, Atrashchenko AV, Kivshar YS (2012) Wire metamaterials: physics and applications. Adv Mater 24: 4229-4248.
8. Linzon Y, Malomed BA, Zaezjev M, Morandotti R, Volatier-Ravat M, et al. (2009) Wave interactions and wave-structure scattering in nonlinear patterned waveguide. Mathematics and Computers in simulation 80: 666-673.
9. Diebel1 F, Leykam D, Boguslawski M, Rose P, Denz C, et al. (2014) All-optical switching in optically induced nonlinear waveguide couplers. Appl Phys Lett 104: 261111.
10. Biswas A, Milovic D, Edwards M (2010) Mathematical Theory of Dispersion-Managed Optical Solitons. Springer Science and Business Media.
11. Burlak G, Malomed BA (2013) Stability boundaries and collisions of two dimensional solitons in PT-symmetric couplers with the cubic-quintic nonlinearity. Phys Rev E 88: 062904.
12. Bender CM (2007) Introduction to PT-symmetric quantum theory. Rep Prog Phys 46: 277-292.
13. Bender CA, Fring A, Gunther U, Jones H (2012) Quantum physics with non-Hermitian operators. J Phys A: Math Theor 45: 440301.
14. Markis KG, El-Gnainy R, Christodoulides DN, Musslimani ZH (2008) Optical Solitons in PT periodic potentials. Phys Rev Lett 100: 030402.
15. Sukhorukov AA, Dimitriev SV, Kivshar YS (2012) Nonlocality in PT-symmetric waveguide array with gain and loss. Opt Lett 37: 2148-2150.
16. Barashenkov IV, Suchkov SV, Sukhorukov AA, Dimitriev SV, Kivshar YS (2012) Breathers in PT-symmetric nonlinear couplers. Phys Rev A 86: 053809.
17. Alexeeva NV, Barashenkov IV, Sukhorukov AA, Kivshar YS (2012) Optical solitons in PT-symmetric nonlinear couplers with gain and loss. Phys Rev A 85: 063837.
18. Guo A, Salamo GJ, Duchensne D, Morandotti R, Volatier-Ravat M, et al. (2009) Observation of PT-Symmetry Breaking in Complex Optical Potentials. Phys Rev Lett 103: 093902.
19. Ruter CE, Markis K, G,El-Gnainy R, Christodoulides DN, Segev M, (2010) Observation of parity-time symmetry in optics. Nat Phys 6: 192.
20. Sukhorukov AA, Kivshar YS (2010) Nonlinear suppression of time-reversals in PT-symmetric optical couplers. Phys Rev A 82: 1-5.
21. Duanmu M, Li K (2013) Linear and nonlinear parity-time-symmetric oligomers: A dynamical system analysis. Phil Tra Soc A 371: 20120171.
22. Agrawal GP (2001) Applications of nonlinear fiber optics. Accademic press.
23. Ramezani H, Kottos T, El-Gnainy R, Christodoulides DN (2010) Unidirectinal nonlinear PT-symmetric optical structures. Phys Rev A 82: 043803.
24. Driben R, Malomed BA (2011) Stability of solitons in parity-time-symmetric couplers. Opt Lett 36: 4323-4325.
25. Suchkov SV, Molomed BA, Dimitriev SV, Kivshar YS (2011) Solitons in a chain of parity-time invariant dimers. Phys Rev E 84: 046609.
26. Zakeri A, Hatami M (2007) Design of an ultra-fast all-optical dark soliton switch in a nonlinear directional coupler (NLDC) made of chalcogenide glasses. Applied Physicd D 40: 12.
27. Tahmasebi Z, Hatami M (2010) Study of the gain saturation effect on the propagation of dark solitonin Er+3-doped, Ga5Ge20Sb10S65 chalcogenide fiber amplifier. Optics Communication 284: 656-659.
28. Zakeri A, Hatami M (2005) Nonlinear optical properties of pulsed-laser-deposited GeAsSe films and simulation of a nonlinear directional coupler switch. Journal of Optical Society of American B 22: 591-597.
29. Shakeri S, Hatami M (2010) Self-tunable chalcogenide Raman laser. Journal of Optical society of American B 27: 679-684.
30. Ruschhaupt A, Delgado F, Muga JG (2005) Physical realization of PT-symmetric potential scattering in a planner slab waveguide. J Phys A: Math Gen 38: 02.