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Stability of Convergence Theorems of the Noor Iteration Method for an Enumerable Class of Continuous Hemi Contractive Mapping in Banach Spaces

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Abstract

The purpose of this is to study the Noor iteration process for the sequence $\{x_n\}$ converges to a common fix point for enumerable class of continuous hemi contractive mapping in Banach spaces.

Keywords: Stability; Noor iterations; Hemicontractive mapping; Convergence theorem; Continuous pseudocontractive mapping

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Introduction

Let E be a real Banach space and let J denote the normalized duality mapping from E to E^* defined by

$$J(x)=\{f\in E^*:\langle x,f\rangle=\|x\|\|f\|,\|x\|=\|f\|\ for\ all\ x\in E,$$

Where E^* denotes the dual space of E and $\langle .,.\rangle$ denotes the generalization duality pair.

It is well known that if E^* is strictly convex then J is single–valued. In the sequel, we shall denote the single–valued duality mapping by j. Let k be a nonempty closed convex subset of Banach space E and $T: K\to K$ be a self-mapping of K.

Definition 3.1: (i) A mapping T with domain D(T) and range R(T) in a Banach space is called pseudocontrative mapping, if for all $x, y\in D(T)$, there exists $j(x-y)\in J(x-y)$ such that **[1]**

$$\langle Tx-Ty, j(x-y)\rangle\le\|x-y\|^2 \qquad (1)$$

(ii) A mapping T with domain D(T) and range R(T) in E is called a hemicontractive mapping if $F(T)\neq\varnothing$ and for all $x\in D(T)x^*\in F(T)$ such that,

$$\langle Tx-x^*, j(x-x^*)\rangle\le\|x-x^*\|^2$$

(iii) A mapping $T: K\to K$ is called L-Lipschitizan there exists L>0 such that

$$\|Tx-Ty\|\le L\|x-y\| \text{ For all } x, y\in K$$

Definition 3.2: If $\{\propto_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are sequences of real numbers in [0, 1] **[2]**. For arbitrary $x_0\in$ E, Let $\{x_n\}_{n=0}^{\infty}$ be a Noor iteration defined by,

$$x_{n+1}=(1-\propto_n)x_n+\propto_n Tq_n$$

$$q_n=(1-\beta_n)x_n+\beta_n Tr_n$$

$$r_n=(1-\beta_n)x_n+\beta_n Tr_n$$

Lemma 3.3: Let E be a real uniformly convex Banach space **[3]**, K is nonempty closed convex subset of E and T a continuous pseudocontractive mapping of K, then *I-T* is demiclosed at zero, that is, for all sequences $\{x_n\}\subset K$ with $x_n\rightharpoonup p$ and $x_n-Tx_n\to 0$ it follows that $p=Tp$

Lemma 3.4: Let δ be a number satisfying $0\le\delta<1$ and $\{\epsilon_n\}$ a positive sequence satisfying $\lim_{n\to\infty}\epsilon_n=0$ [4,5]. Then, for any positive sequence $\{u_n\}$ satisfying: $u_{n+1}\le\delta u_n+\epsilon_n$, It follows that $\lim_{n\to\infty}u_n=0$.

Results

Theorem 4.1: Let $\{T_n\}_{n=1}^{\infty}$ be defined as above and $F:=\cap_{(i=1)}^{\infty}F(T_{(n)}\neq\phi)$ and let $(E,\|.\|)$ be a Banach space, $T:E\to E$ a self-map of E with a fixed point p, satisfying the contractive condition

$$\langle Tx-x^*, j(x-x^*)\rangle\le\|x-x^*\|^2 \text{ For } x_0\in E.$$

Let $\{x_n\}_{n=1}^{\infty}$ is converge to p and defined by the iteration (3.2) where $\{\propto_n\}_{n=1}^{\infty}$ is a real sequence in (0, 1) and define as $\epsilon_n=\|x_{n+1}-(1-\propto_n)x_n-\propto_n Tq_n\|$ Then

$\lim_{n\to\infty}\|x_n-p\|$ exists for all $p\in F$;

$\lim_{n\to\infty}d(x_n,F)=\{inf\|x_n-p\|: p\in F\}$;

$\{x_n\}$ converges strongly to a common fixed point of $\{T_n\}_{n=1}^{\infty}$ if and only if $\lim_{n\to\infty}d(x_n,F)=0$

Proof: Let $p\in F$ and $n\ge 1$ by 3.1 we choose $j(x_n-p)\in J(x_n-p)$ such that

$$\|x_{n+1}-p\|^2=\langle x_{n+1}-p, j(x_{n+1}-p)\rangle$$

$$\|x_{n+1}-p\|\le\|x_{n+1}-(1-\propto_n)x_n-\propto_n Tq_n\|+\|(1-\propto_n)x_n+\propto_n Tq_n-p\|$$

$$=\epsilon_n+\|(1-\propto_n)x_n+\propto_n Tq_n-((1-\propto_n)+\propto_n)p\|$$

$$=\epsilon_n+\|(1-\propto_n)\|x_n-p\|+\propto_n(Tq_n-p)\|$$

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$$\leq \epsilon_n + (1-\propto_n)\|x_n - p\| + \propto_n \|Tq_n - p\|$$

$$= \epsilon_n + (1-\propto_n)\|x_n - p\| + \propto_n \|p - Tq_n\|$$

$$\leq \epsilon_n + (1-\propto_n)\|x_n - p\| + \propto_n a\|p - q_n\|$$

$$= \epsilon_n + (1-\propto_n)\|x_n - p\| + \propto_n a\|q_n - p\| \quad (1)$$

For the estimate of in (1) we get

$$\|q_n - p\| = \|(1-\beta_n)x_n + \beta_n Tr_n - p\|$$

$$= \|(1-\beta_n)x_n + \beta_n Tr_n - ((1-\beta_n)+\beta_n)p\|$$

$$= \|(1-\beta_n)(x_n - p) + \beta_n(Tr_n - p)\|$$

$$\leq (1-\beta_n)\|x_n - p\| + \beta_n\|Tr_n - p\|$$

$$= (1-\beta_n)\|x_n - p\| + \beta_n\|p - Tr_n\|$$

$$\leq (1-\beta_n)\|x_n - p\| + \beta_n a\|p - r_n\|$$

$$= (1-\beta_n)\|x_n - p\| + \beta_n a\|r_n - p\| \quad (2)$$

Substituting (2) into (1) gives

$$\|x_{n+1} - p\| \leq \epsilon_n + (1-(1-a)\propto_n - \propto_n \beta_n a)\|x_n - p\| + \propto_n \beta_n a^2\|r_n - p\| \quad (3)$$

For $\|r_n - p\|$ in (3) we have,

$$\|r_n - p\| = \|(1-\gamma_n)x_n + \gamma_n Tx_n - p\|$$

$$= \|(1-\gamma_n)x_n + \gamma_n Tx_n \ ((1-\gamma_n)+\gamma_n) - p\|$$

$$= \|(1-\gamma_n)(x_n - p) + \gamma_n(Tx_n - p)\|$$

$$\leq (1-\gamma_n)\|x_n - p\| + \gamma_n\|Tx_n - p\|$$

$$= (1-\gamma_n)\|x_n - p\| + \gamma_n\|p - Tx_n\|$$

$$\leq (1-\gamma_n)\|x_n - p\| + \gamma_n a\|p - x_n\|$$

$$= (1-\gamma_n + \gamma_n a)\|x_n - p\| \quad (4)$$

Substituting (4) into (3) and using lemma 3.3

$$= \epsilon_n + (1-(1-a)\propto_n - \propto_n \beta_n a)\|x_n - p\| + \propto_n \beta_n a^2(1-\gamma_n + \gamma_n a)\|x_n - p\|$$

$$= \epsilon_n \ (1-(1-a)\propto_n - (1-a)\propto_n \beta_n a - (1-a)\propto_n \beta_n \gamma_n a^2)\|x_n - p\|$$

$$\leq (1-(1-a)\alpha - (1-a)\alpha\beta a - (1-a)\alpha\beta\gamma a^2)\|x_{n-1} - p\| + \epsilon_n$$

Observe that

$$0 \leq (1-(1-a)\alpha - (1-a)\alpha\beta a - (1-a)\alpha\beta\gamma a^2) < 1 \quad (5)$$

Therefore, taking the limit as $n \to \infty$ of both sides of the inequality (5) and using lemma 1.6 we get

$\lim_{n\to\infty}\|x_n - p\| = 0$, That is $\lim_{n\to\infty} x_{n=} p$

By theorem 3.2 $\|x_n - p\| \leq \|x_{n-1} - p\|$

Taking infimum over all $p \in F$, we have,

$$d(x_n, F) = \inf_{p\in F}\|x_n - p\| \leq \inf_{p\in F}\|x_{n-1} - p\| = d(x_{n-1}, F),$$

Thus $\lim_{n\to\infty} d(x_n, F)$ exist we finally prove (iii) suppose that $x_n \to p \in F$ from (ii) and

$d(x_n, F) \leq \|x_n - p\| \to 0$, We have $\lim_{n\to\infty} d(x_n, F) = 0$ for $n, m \in \mathbb{N}$ and $p \in F$, it follows

From (1.3) that

$$\|x_{n+m} - x_n\| \leq \|x_{n+m} - p\| + \|x_n - p\| \leq \ \|x_n - p\|$$

Consequently,

$$\|x_{n+m} - x_n\| \leq 2\|x_n - F\| \to 0$$

Therefore $\{x_n\}$ is a Cauchy sequence. Suppose $\lim_{n\to\infty} x_n = u$ for some $u \in E$. Then

$$d(u, F) = \lim_{n\to\infty} d(x_n, F) = 0$$

Since F is closed set, $u \in F$

So, Noor iteration process is T–stable.

Conclusion

Thus, the stability of Noor iteration considerable for finding fixed point for enumerable class of continuous hemi contractive mapping in Banach spaces.

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