Some Notes on the Iterative Operator Splitting

István Faragó*

Eötvös Loránd University, Institute of Mathematics and MTA-ELTE NumNet Research Group, Hungary

Abstract

Operator splitting is a powerful method for the numerical investigation of complex (physical) time-dependent models, where the stationary (elliptic) part consists of a sum of several simpler operators (processes). Some fields where different splitting methods play a crucial role include the air-pollution phenomena, Maxwell’s equations or the Hamiltonian systems. These tasks are usually very complicated, and therefore, the analytical solution is impossible to find, moreover, the numerical modelling with direct discretization is also hopeless from a practical point of view. To avoid this difficulty, the operator splitting is introduced and applied.

Algorithms of Operator Splitting

The basic idea of operator splitting is to split the original problem into a sequence of smaller (simpler) problems. The most popular method is the sequential splitting (sometimes referred to as the Lie-Trotter method). The general scheme of this approach can be formulated as follows:

We select a small positive time step h, and divide the whole time interval into subintervals of length h; On each subinterval we consecutively solve the time-dependent problems, each of which involves only one operator (physical process); We pass to the next time sub-interval.

We mention that the different problems are connected via the initial conditions. Another algorithm was defined in [1] as a global method to reaction diffusion equations and Fisher’s equation can also be done.

The proof can be found in [4].

Theorem 1. Suppose that A is a linear operator that generates a C0-semigroup and B is a bounded linear operator. Then the MIS generated split sub-problems have unique solutions, and the sequence of these solutions is consistent of order m.

The proof can be found in [4].

Generalizations to the non-linear cases and application of the method to reaction diffusion equations and Fisher’s equation can also be done.

References


*Corresponding author: István Faragó, Eötvös Loránd University, Institute of Mathematics and MTA-ELTE NumNet Research Group, Hungary, E-mail: faragois@cs.elte.hu

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