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Some Notes on the Iterative Operator Splitting

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Abstract

Operator splitting is a powerful method for the numerical investigation of complex (physical) time-dependent models, where the stationary (elliptic) part consists of a sum of several simpler operators (processes. Some fields where different splitting methods play a crucial role include the air-pollution phenomena, Maxwell's equations or the Hamiltonian systems. These tasks are usually very complicated, and therefore, the analytical solution is impossible to find, moreover, the numerical modelling with direct discretization is also hopeless from a practical point of view. To avoid this difficulty, the operator splitting is introduced and applied.

Algorithms of Operator Splitting

The basic idea of operator splitting is to split the original problem into a sequence of smaller (simpler) problems. The most popular method is the sequential splitting (sometimes referred to as the Lie-Trotter method). The general scheme of this approach can be formulated as follows:

We select a small positive time step h, and divide the whole time interval into subintervals of length h;

On each subinterval we consecutively solve the time-dependent problems, each of which involves only one operator (physical process);

We pass to the next time sub-interval.

We mention that the different problems are connected via the initial conditions. Another algorithm was defined in [1] as a global approximation method on the whole time interval [0,T]. Later, as a numerical method it was introduced in [2]. The method can be interpreted as the iterative improvement of the split result on some fixed split time interval $[(n\text{-}1), nh]$, where the first iteration is obtained by use of the continuous variant of the well-known ADI method. The algorithm for two operators A and B reads as follows:

$$\frac{dw_n^i}{dt}(t) = Aw_n^i(t) + Bw_n^{i-1}(t)$$

$$\frac{dw_n^{i+1}}{dt}(t) = Aw_n^i(t) + Bw_n^{i+1}(t)$$

for i=1, 3, 5, … , $2m^{-1}$, where w_n^o is a fixed starting function for the iteration. (The index i denotes the number of the iteration on the fixed n-th time sub-interval.) Then the split solution at the mesh-points is defined as $w_{sp}(nh)=w^2m(nh)$. This method can be considered as an operator splitting method because we decompose the original problem into a sequence of two simpler sub-problems, in which the first sub-problem should be solved for the first operator, while the second sub-problem for the second operator. This splitting is formally similar to the sequential splitting, but here each split sub-problem contains the other operator as well with some previously defined approximate solution. The convergence of the method is investigated in [3]. The results show that for linear problems with bounded operators the iterative splitting can provide accuracy of arbitrarily high order: increasing the number of iterations any required accuracy can be achieved.

In the following statement we modify this method by introducing the modified iterative splitting (MIS), which means, that in step we solve the same type of problem, with the same operator A. The algorithms read as follows:

$$\frac{dw_n^i}{dt}(t) = Aw_n^i(t) + Bw_n^{i-1}(t)$$

for i=1,2, 3, … ,m, and the split (approximate) numerical solution at the time point $t=tn=nh$ is defined as $w_{sp}(nh)=w^m(nh)$.

The main benefit of this new method is the following. Instead of alternating the operator in the inhomogeneous part of the sub-equations we keep the positions of the operators fixed, and hence A acts on the unknown function and B on the already known function obtained in the previous iteration. Thus the sub-equations become identical so the algorithm reduces to solving one equation repeatedly. A huge advantage of this method is that solving two problems is replaced by solving only one. Only the semigroup of A needs to be determined and in practice by its approximation an approximate solution can be generated. The implementation of this procedure is much simpler.

The following statement is true [4].

Theorem 1. Suppose that A is a linear operator that generates a C0-semigroup and B is a bounded linear operator. Then the MIS generated split sub-problems have unique solutions, and the sequence of these solutions is consistent of order m.

The proof can be found in [4].

Generalizations to the non-linear cases and application of the method to reaction diffusion equations and Fisher's equation can also be done.

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