

Some Issues on Hesitant Fuzzy Sets

Zeshui Xu* and Bin Zhu

School of Economics and Management, Southeast University, Nanjing, Jiangsu 211189, China

Hesitant fuzzy sets (HFSs) whose membership functions represented by a set of possible values, are a new effective tool to express human's hesitancy in daily life. Torra [1] originally introduced this concept as an extension of Zadeh's fuzzy sets (FSs [2]). HFSs have close relationships with the existing fuzzy sets, such as intuitionistic fuzzy sets (IFSs [3]), type-2 fuzzy sets (T2FSs [4]), fuzzy multisets (FMSs [5]). As discussed by Torra, the motivation to propose the HFSs is that when defining the membership of an element, the difficulty of establishing the membership is not a margin of error (as in IFSs), or some possibility distribution (as in T2FSs) on the possible values, but a set of possible values. With respect to the relationship among these fuzzy sets, HFSs were deemed IFSs when the HFS is a nonempty closed interval; HFSs can present FMSs in some cases, but the operations for FMSs do not apply correctly to HFSs. Then Torra defined the envelope of HFS, which can transform HFSs into IFSs. Moreover, some basic operations including "intersection", "union", "complement" have also been proposed.

In group decision making (GDM) problems, the decision makers (DMs) often have their unique characteristics with regard to knowledge, skills, experience, and personality, which imply that different DMs have different preferences when providing their preferences over alternatives. We consider that the DMs constituting a group hesitate about several possible values. In such a case, HFSs are convenient and effective to be used to collect and present DMs' preferences. For example, to the membership of x into the set A , assume the group of DMs provides some possible values as 0.3, 0.4 and 0.5 respectively, then we can use a HFE, h to collect them as $h = \{0.3, 0.4, 0.5\}$.

Due to the characteristics and advantages, HFSs have become a hot topic recently. Xia and Xu [6] further defined some operations of HFEs. Then on the basis of an extension principle introduced by Torra, they also developed some hesitant aggregation operators, such as the hesitant fuzzy weighted averaging (HFWA) operator, the hesitant fuzzy weighted geometric (HFWG) operator. The properties of the proposed aggregation operators have been studied extensively. As typical applications, these proposed operators have been applied to GDM. The Bonferroni mean (BM) can capture the interrelationships among arguments, which plays a crucial role in multi-criteria decision making problems. Zhu et al. [7] studied the BM under hesitant fuzzy environment, and developed the hesitant fuzzy geometric Bonferroni mean (HFGBM) and the hesitant fuzzy Choquet geometric Bonferroni mean (HFCGBM). Besides of aggregation operators, Xu and Xia [8,9] developed some distance, similarity and correlation measures for hesitant fuzzy information, which have also been used for decision making.

However, HFSs have some limitations when collecting DMs' preferences. For example, two DMs discuss the membership of x into the set A , if they both assign 0.5 to x , denoted by a HFE $h' = \{0.5\}$, we can only save one value, and loss the other one. In this situation, if the two DMs give their evaluation values anonymously, we can save one value reasonably; if the two DMs have different importance degrees, we have to loss some information. DMs are of vital importance in group decision making, we often need to consider their differences in practice.

In order to overcome information loss problem, Zhu [10] developed a new concept of generalized hesitant fuzzy sets (GHFSs) which are constructed by a Cartesian product of HFSs. For example, continued with the membership problem, if one DM assigns 0.5, another assigns 0.5 or 0.6, we collect one value provided by each DM as a group resulting in two possible groups of values (0.5, 0.5) and (0.5, 0.6). In such a case, all the values provided by the DMs are saved and distinguished clearly in the groups. Furthermore, GHFSs also have close relationships with some fuzzy sets, such as IFSs, T2FSs, FMSs and especially HFSs. GHFSs are an extension of HFSs, and HFSs can also be considered as a particular case of GHFSs. GHFSs increase the richness of numerical representation based on the groups of values, enhance the modeling abilities of HFSs, and can identify different DMs in decision making, which expand the applications of HFSs in practice.

Moreover, Zhu et al. [11], developed dual hesitant fuzzy sets (DHFSs) as a new extension of HFSs. The DHFS is a comprehensive set encompassing several existing fuzzy sets with certain conditions, whose membership and nonmembership are represented by a set of possible values respectively. In particular cases, DHFSs can reduce to some existing fuzzy sets, such as FSs, IFSs, HFSs and FMSs. From a mathematical point of view, DHFSs are also a particular case of T2FSs. DHFSs permit DMs consider as possible values for the membership a few different values, such as 0.1, 0.2 and 0.3 and for the nonmembership, such as 0.4, 0.5 and 0.6. So the uncertainty is somehow limited with all given memberships and nonmemberships. Under certain conditions, DHFSs encompass FSs, IFSs, HFSs and FMSs as special cases. Thus, DHFSs appear to be a more flexible method to be valued in multifold ways according to the practical demands than the existing fuzzy sets, taking much more information given by DMs into account.

Under linguistic environment, Rodriguez et al. [12], developed the hesitant fuzzy linguistic term sets (HFLTSS) to improve the modeling and computational abilities of linguistic approaches based on HFSs. The use of traditional fuzzy linguistic approaches are limited in a similar way to FSs, mainly because they are based on the elicitation of single and simple terms that encompass and express the linguistic information provided by the DMs regarding a linguistic variable. However, HFLTSS increase the flexibility of the elicitation of linguistic information that allows the DMs hesitate about several possible values to assess a linguistic variable. Compared to the fuzzy linguistic approaches which aim at statically assessing single linguistic terms for the linguistic variables, HFLTSS use several possible linguistic values or

*Corresponding author: Zeshui Xu, School of Economics and Management, Southeast University, Nanjing, Jiangsu 211189, China, Tel: +86 25 84483382; E-mail: xuzeshui@263.net

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richer expressions than a single term, which has particular advantages for collecting the DMs' preferences in decision making.

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