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Some Important Properties of Beta Functions

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Abstract

In this paper, we discuss some important properties of Beta functions. These properties make the study of these functions more meaningful. Beta function is also known as the Eulerian integral of the first kind.

Introduction

Beta function

The definite integral [1-3]

$\int_0^1 x^{m-1}(1-x)^{n-1}dx$, for $m > 0, n > 0$

is called the Beta function and is denoted by B(m,n)[read as "Beta m,n] . Thus

B$(m,n)\int_0^1 x^{m-1}(1-x)^{n-1}dx,$

Where m, n are positive numbers integral or fractional. Beta function is also called the Eulerian integral of the first kind [1-3].

Properties

Some Simple Properties of Beta Functions (i) Symmetry of Beta function i.e. B (m,n)B(m,n) We have B(m,n)= $\int_0^1 x^{m-1}(1-x)^{n-1}dx, dy$ the def. of the Beta function

=$\int_0^1(1-x)^{m-1}\{1-(1-x)\}^{n-1}dx, [\because \int_0^a (x)dx = \int_0^a f(a-x)dx]$

=$\int_0^1(1-x)^{m-1}x^{n-1}dx = \int_o^1 x^{n-1}(1-x)^{m-1}dx$

= B (n, m), by the def. of Beta function. Hence B (m,n) = B(n, m)

(ii) If m or n is a positive integral, B (m,n) can be evaluated in an application form. Case I: When n is a positive integer. If n = 1, the result is obvious because B

$(m,1)$=$\int_0^1 x^{m-1}(1-x)^{1-1} = \int_0^1 x^{m-1}dx$

$= [\frac{x^m}{m}]_0^1 = \frac{1}{m}$

So let us take n > 1. We have

B $(m,n)\int_0^1 x^{m-1}(1-x)^{n-1} = \int_0^1 1 - x^{n-1}x^{m-1}dx$

=$\left[(1-x)^{n-1}.\frac{x^m}{m}\right] - \int_0^1 (n-1)(1-x)^{n-2}(-1).\frac{x^m}{d}dx$

integrating by parts taking x^{m-1} as the second function

=0+$\frac{n-1}{m}\int_0^1 x^m(1-x)^{n-2}dx$

$$= \frac{n-1}{m}\int_0^1 x^{(m+1)-1}(1-x)^{(n-1)-1}dx$$

$$= \frac{n^m - 1}{m}B(m+1, n-1$$

By the repeated application of this process, we get

$$B(m,n) = \frac{n-1}{m}.\frac{n-2}{m+1}.\frac{n-3}{m+2}\cdots\frac{1}{m+n-2}B(m+n-1,1)$$

$$= \frac{n-1}{m}.\frac{n-2}{m+1}.\frac{n-3}{m+2}\cdots\frac{1}{m+n-2}\int_0^1 x^{m+n-2}(1-x)^0 dx$$

$$= \frac{n-1}{m}.\frac{n-2}{m+1}.\frac{n-3}{m+2}\cdots\frac{1}{m+n-2}\int_0^1 x^{m+n-2}\, dx$$

$$\frac{(n-1)!}{m(m+1)(m+2)\ldots(m+n-2)}.\left[\frac{x^{m+n-1}}{m+n-1}\right]$$

$$\therefore B(m,n) = \frac{(n-1)!}{m(m+1)(m+2)\ldots(m+n-2)(m+n-2)}$$

Case II: When m is a positive integer. Since the Beta function is symmetrical in m and n i.e., **B**(m, n) = **B**(n, m), therefore by case I, we conclude that

$$B(m,n) = \frac{(n-1)!}{n(n+1)(n+2)\ldots(n+m-2)(n+m-2)}$$

(iii) If both m and n are positive integers, then

B(m, n) =$\frac{(m-1)!(n-1)!}{(m+n-1)!}$

From (ii), we have

$$B(m,n) = \frac{(n-1)!(m-1)!}{m(m+1)(m+2)\ldots(m+n-2)(m+n-2)}$$

$$= \frac{(n-1)!}{(m+n-1)(m+n-2)\ldots(m+1)m(m-1)!}$$

writing the denominator in the reversed order and multiplying the Nr and Dr by $(m-1)!$

$$= \frac{(m-1)!\,(n-1)!}{(m+n-1)!}$$

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