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Some Boundary Value Problems for the Hyperbolic: Hyperbolic type Equation with Two Line of Degeneration in Special Domain

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Abstract

In the present paper we study unique solvability of the analogues of problem Bitsadze for the degenerating hyperbolic-hyperbolic type equation. Uniqueness and existence theorem for solution of this problems are proven with principle extremum and by the method of integral equations.

Keywords: Boundary value problem; Existence and uniqueness of solution; Degenerating equation; Hyperbolic-hyperbolic type; A principle an extremum; Method of integral equations

Introduction

Last years the increasing attention of mathematicians is involved with problems of correctness of the boundary value problems (BVP) for the degenerating equations of the mixed parabolic-hyperbolic, elliptic-hyperbolic and hyperbolic-hyperbolic types. It is closely connected with appendices of such problems to the decision of problems of mechanics, gas dynamics, biology and in other material sciences. The first basic researches under the theory of the degenerating equations of the mixed and mixed-compound type are Trikomi's [1], Gellerstedt's et al, [2], Bitsadze's [3] and Salakhitdinov's [1,4,5] works. The degenerating and singularity equations possess that nature, that for them the correctness of some classical problems not always takes place. This fact rather for the degenerating equations of elliptic type, in the first has been noticed of MKeldych [6], and concerning the degenerating equations of hyperbolic type of Gellerstedt. In this cases Bitsadze has suggested to study modify problems Cauchy for the degenerating equation of hyperbolic type because the problems Cauchy for such equations it is put incorrectly. Since Bitsadze's [2] works, in the theory partial differential equations there was a new direction, in which the analogue of problem Tricomi for the first time is formulated and investigated in double connected domain for the modeling equations of the mixed type. After this work various problems for the equation of the mixed type on the second order in multiply and doubly connected domains are investigated, in works as Bers [3] and Salahitdinov, Urinov [1,5]. However, BVPs in double-connected domain are studied for the not degenerated modeling equations of the third order of elliptic-hyperbolic type [7], and uniqueness of solution of the problem for the degenerated hyperbolic-hyperbolic type equation in double-connected domain was proved by Islomov et al. [4].

The Statement of Problems

In the present work the analogues of problem A.V. Bitsadze [2] is formulated and investigated for the hyperbolic-hyperbolic type equation with two degenerating lines of the following kind:

$$(-y)^n u_{xx} - |x|^n u_{yy} = 0, n = const > 0 \tag{1}$$

in the special domain Ω, bounded at y<0 with characteristics

$$A_j C_1 : \left((-1)^{j-1} x\right)^{\frac{n+2}{2}} + (-y)^{\frac{n+2}{2}} = q^{\frac{n+2}{2}}; (j = 1,2)$$

$$B_j C_2 : \left((-1)^{j-1} x\right)^{\frac{n+2}{2}} + (-y)^{\frac{n+2}{2}} = 1; (j = 1,2), (0 < q < 1)$$

of the equation (1), and at y = 0 with segments A_jB_j where $A_j\left((-1)^{j-1} q; 0\right)$ $B_j\left((-1)^{j-1}; 0\right), (j = 1,2)$.

We introduce the following notations:

Through Ω_{1j} and Ω_{3j} we will designate characteristic triangles $A_jB_jE_j$ and $C_1F_jC_2$ (j =1, 2), accordingly, and through Ω_{2j} we will designate characteristic quadrangles $A_jE_jF_jC_1$ (j=1, 2). In the section 2 we have formulated and proved unique solvability of a problems I(I*) and II (II*) in the domain of , which, consist of four characteristic triangles and from two quadrangles. The result, which is obtained in this section shows that when we will investigate problems I(I*) and II(II*), in each sub domains, we find the solution of equation (1)in an explicit form In the section 3 we studying uniqueness and existence of solution of a problem III (III*). Uniqueness of solution of problem III (III*) are proven with principle an extremum. Existence of the solution of problem III (III*) we have proved, by method integral equations. The main result of this section shows that when we will studying existence of the solution of problem III(III*), we have singularity integral equation, which regularities by the method of Karleman's-Vekua [8], to the integral equation of Fredgolm of the second kind.

Unique solvability of the problems I(I*) and II(II*)

Problem I: Find a function u(x, y) in the domain Ω with following properties:

1) $u(x, y) \in C(\overline{\Omega}) \cap C^2(\Omega)$;

2) u (x, y) satisfies the equation (1) in domains Ω_{1j}, Ω_{2j}, Ω_{3j} (j = 1, 2);

3) u (x, y) satisfies the following conditions

$$u(x,0) = \tau_j(x), x \in \overline{A_jB_j} \tag{2j}$$

$$u(x,y)\big|_{B_1C_2} = \varphi(x), x \in [0,1] \tag{3}$$

$$u(x,y)\big|_{A_2C_1} = \psi(x), x \in [-q,0] \tag{4}$$

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Where $T_j(x)$, $\varphi(x)$, $\Psi(x)$ - given functions, and $\tau_j(x) \in C(\overline{A_jB_j}) \cap C^3(A_jB_j)$,

$$\tau_1(1) = \varphi(1), \tau_2(-q) = \psi(-q) \qquad (5_1)$$

$$\varphi(x) \in C[0,1] \cap C^3(0,1) \psi(x) \in C[-q,0] \cap C^3(-q,0) \qquad (5_2)$$

Problem I*: Find a function u(x, y) in domain Ω, satisfies to all conditions problem I, except (2_j) (j=1, 2) which are replaced with conditions

$$u_y(x,0) = v_j(x), x \in A_jB_j \qquad (6_j)$$

where $v_j(x)$ - given functions, and

$$v_j(x) \in C^2(A_jB_j), (j=1,2) \qquad (7_j)$$

Problem II (II*): Find a function u(x, y) in domain Ω, satisfies to all conditions problem I(I*), except (3), (4) which are replaced with conditions:

$$u\big|B_jE_j = \mu_j(x), x \in \overline{B_jE_j} \qquad (8_j)$$

And

$$u\big|A_jC_1 = \psi_j(x), x \in \overline{A_jC_1} \qquad (9_j)$$

where $\mu_j(x)$, $\Psi_j(x)$- given functions, and:

$$u_j(x) \in C(\overline{B_jE_j}) \cap C^3(B_jE_j), \psi_j(x) \cap C(\overline{A_jC_1}) \cap C^3(\overline{A_jC_1}) \qquad (10)$$

Theorem 1: If conditions (5_j) $((5_2)$ and $(7_j))$ are satisfied that the problem I(I*) is unique solvability.

Proof: Is known, that the solution of problem Cauchy in domain Ω_{11} for the equation (1) satisfying to conditions (2_1) and (6_1) looks like [8,9]:

$$u(x,y) = \gamma_1 \int_q^1 \tau 1(z_1^{\frac{1}{n+2}})\left[(t-q)(1-t)\right]^{\beta-1} dt$$

$$-\gamma_2 xy \int_q^1 \upsilon(z_1^{\frac{1}{n+2}}) z_1^{-\frac{1}{n+2}} \left[(t-q)(1-t)\right]^{-\beta} dt \qquad (11)$$

Where

$$z_1 = x^{n+2} + (-y)^{n+2} + \frac{2(2t-q-1)}{1-q}(-xy)\frac{n+2}{2}, \gamma_1 = \frac{\Gamma(2\beta)(1-q)^{1-2\beta}}{\Gamma^2(\beta)} \gamma_2 = \frac{\Gamma(2\beta)(1-q)^{1-2\beta}}{\Gamma^2(1-\beta)}$$

From here, by virtue condition (3), considering (51) ((71)) it is easily possible to define unknown function $v_1(x)$ $(T_1(x))$, hence, owing to uniqueness of the solution of problem Cauchy, the solution of the problem I(I*) in domain Ω_{11} is uniquely defined.

Further, designating through $h_1(x)$ a trace of the solution of problem Cauchy- Gaursat 1 (Cauchy-Gaursat 2) from domain Ω_{11} on the characteristic A_1E_1 and considering the condition (3) taking into account (5^2), by the method of Riemann, we restore the solution of the problem I(I*) in domain the of Ω_{21} as the solution of problem Gaursat and this solution is given by the formula:

$$u(\xi,\eta) = -2\beta \int_\xi^{q^{\frac{n+2}{2}}} \varphi^*\left((\frac{t+1}{2})^{1-2\beta}\right) \frac{t(t^2-1)^{1-2\beta}}{(t^2-n^2)^\beta(\xi^2-1)^\beta} F(\beta,\beta,1;\sigma)dt$$

$$-\int_\eta^1 d\left[\left(h_1\left(\frac{q^{(n+2)/2}+t}{2}\right)^{1-2\beta}\right)\right] \frac{(q^{n+2}-t^2)^{2\beta}}{(\xi^2-t^2)^\beta(q^{n+2}-\eta^2)^\beta} F(\beta,\beta,1;\sigma)dt +$$

$$+2\beta \int_\eta^1 \left(h_1\left(\frac{q^{(n+2)/2}+t}{2}\right)^{1-2\beta}\right) \frac{t(q^{n+2}-t^2)^{2\beta-1}}{(\xi^2-t^2)^\beta(q^{n+2}-\eta^2)^\beta} F.(\beta,\beta,1;\sigma)dt$$

$$-\int_\eta^{q^{(n+2)/2}} d\left[\varphi^*\left(\left(\frac{t+1}{2}\right)^{1-2\beta}\right)\right] \frac{(t^2-1)^{2\beta}}{(\xi^2-1)^\beta(t^2-\eta^2)^\beta} F(\beta,\beta,1;\sigma)dt$$

$$+h_1\left(\left(\frac{q^{(n+2)/2}+t}{2}\right)^{1-2\beta}\right) \frac{(q^{n+2}-1)^{2\beta-1}}{(\xi^2-1)^\beta(q^{n+2}-\eta^2)^\beta} F\left(\beta,\beta,1;\frac{(\xi^2-q^{n+2})(1-\eta^2)}{(\xi^2-1)^\beta(q^{n+2}-\eta^2)}\right)$$

Where

$$\sigma = \frac{(\xi^2-t^2)(1-\eta^2)}{(\xi^2-1)(t^2-\eta^2)}, \sigma_1 = \frac{(\xi^2-q^{n+2})(t^2-\eta^2)}{(\xi^2-t^2)(q^{n+2}-\eta^2)}$$

Similarly, we find the unique solution of problem Gaursat for the equation (1) in domain Ω_{31} in an explicit form. As the solution u (x, y) is found in domain Ω_{31} an obvious kind, can write out the solution of problem Cauchy for the equation (1) in domain Ω_{32} too. Hence, in domains Ω_{22} and Ω_{12} the solution of problem I(I*) is restored as the solution of problem Gaursat and Cauchy-Gaursat-1 (Cauchy-Gaursat-2) accordingly. The theorem is proved. Theorem 2. If conditions (51) and (10) $((7_j)$ and (10)) are satisfied that the problem II (II*) is unique solvability. The theorem 2 is proved similarly as the theorem 1.

Uniqueness and Existence of Solutions of the Problem III (III*)

Problem III (III*): Find a function u(x, y) in domain Ω satisfies to all conditions problem I(I*),except (3) which are replaced with conditions:

$$u\big|B_1E_1 = \varphi^*(x), x \in \left[\frac{1-q}{2}, 1\right], \qquad (13)$$

$$u\big|F_2C_2 = \psi^*(x), x \in \left[\frac{q-1}{2}, 0\right], \qquad (14)$$

where $\varphi^*(x)$, $\Psi^*(x)$ - given functions, and

$$\varphi^*(x) \in C\left[\frac{1-q}{2}, 1\right] \cap C^3\left(\frac{1-q}{2}, 1\right), \psi^*(x) \in C\left[\frac{q-1}{2}, 0\right] \cap C^3\left(\frac{q-1}{2}, 0\right) \qquad (15)$$

Theorem 3: If conditions (5_1) and (15) $((7_j)$ are satisfied and (15)) that solution of a problem III (III*) exists and is unique.

Proof: Is known, that the solution of problem Cauchy in Ω_{31} (Ω_{32}) for the equation (1), satisfying to conditions $u(0,y) = \tau_3(y), y \in \overline{C_1C_2}$ *and* $u_x(0,y) = \nu_3(y)$, $y \in C_1C_2$ looks like:

$$u(x,y) = \gamma_1 \int_{-1}^{-q} \tau_3(z_1^{\frac{1}{n+2}})\left[(-t-q)(1+t)\right]^{\beta-1} dt -$$

$$-\gamma_2 xy \int_{-1}^{-q} \upsilon_3(z_1^{\frac{1}{n+2}}) z_1^{-\frac{1}{n+2}} \left[(-t-q)(1+t)\right]^{\beta-1} dt, \qquad (16)$$

$$\left(u(x,y)\gamma_1 \int_{-1}^{-q} \tau_3(z_3^{\frac{1}{n+2}})\left[(-t-q)(1+t)\right]^{\beta-1} dt - \gamma_2 xy \int_{-1}^{-q} \upsilon_3(z_3^{\frac{1}{n+2}})\left[(-t-q)(1+t)\right]^{\beta-1} dt,\right) \qquad (17)$$

$$z_3 = (-x)^{n+2} + (-y)^{n+2} + \frac{2(2t-q-1)}{1-q}(xy)^{\frac{n+2}{2}}$$

Where

$$\gamma_1 = \frac{\Gamma(2\beta)(1-q)^{1-2\beta}}{\Gamma^2(\beta)}, \gamma_2 = \frac{\Gamma(2-2\beta)(1-q)^{2\beta-1}}{\Gamma^2(1-\beta)}$$

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From here, owing to condition $u(x, y)|_{C_1F_1} = h_2(y)$ and (14) taking into account properties of integro-differential operators of fractional order [8,9] accordingly we will receive:

$$\nu_3^*(y) = \frac{\gamma^*}{\gamma_1\tau(\beta)}(y^*)^{\frac{1}{n+2}} D^{1-2\beta}_{q^{n+2}y^*}(y^* - q^{n+2})^{1-\beta}\, D^{\beta}_{q^{n+2}y^*}\, h_2(y^*)(Y^* - q^{n+2})^{2\beta-1} - (y^*)^{\frac{1}{n+2}}\gamma^* D^{1-2\beta}_{q^{n+2}y^*}\tau_3^*(y), \quad (18)$$

And

$$\nu_3^*(y) = \frac{\gamma^*}{\gamma_1\tau(\beta)}(y^*)^{\frac{1}{n+2}} D^{1-2\beta}_{q^{n+2}y^*}(1 - y^*)^{1-\beta}\, D^{\beta}_{y^*1}\, \psi^*(y^*)(1 - y^*)^{2\beta-1} + (y^*)^{\frac{1}{n+2}}\gamma^* D^{1-2\beta}_{y^*1}\tau_3^*(y), \quad (19)$$

where $h_2(y)$- a trace of solution of the Gaursat in domain Ω_{21}, satisfying condition (13) and

$$u(x, y)|_{A_1E_1} = h_1(x),\ and\ \gamma^* = \frac{\gamma_1\Gamma(\beta)}{\gamma_2\Gamma(1-\beta)4^{2\beta-1}},\ \tau_3^*(y) = \tau\left((y^*)^{\frac{1}{n+2}}\right),$$

$$\nu_3^*(y) = \nu\left((y^*)^{\frac{1}{n+2}}\right)\ here\ y^* = \left(2(-y)\frac{n+2}{2} - 1\right)^2$$

At the proof of the theorem 3 takes place

Lemma: The solution u(x, y) of the problem III (III*) at

$$\tau j(x) \equiv \varphi^*(x) \equiv \psi^*(x) \equiv 0\, \nu_j(x) \equiv \varphi^*(x) \equiv \psi^*(x) \equiv 0, (j = 1, 2) \quad (20j)$$

in the domain the positive maximum and negative minimum reaches only in points C_1 and C_2.

Proof: By virtue (20_j) and considering solutions of problem Cauchy-Gaursat (in the domain Ω_{11}) and Gaursat (in the domain 21) for the equation (1) we will receive, that u(x, y)=0 on the characteristic C_1F_1. From here, owing to a principle of extremum for the hyperbolic equations [4,9,10] function u(x, y) reaches the positive maximum and the negative minimum in the domain 31 only on the piece C1C2. Similarly, owing to the principle of an extremum for the hyperbolic equations [10] with the account $\Psi^*(x) \equiv 0$, we have, that the function u(x, y) reaches the positive maximum and the negative minimum in domain Ω_{32} only on the piece C_1C_2.

Let function u(x, y) reaches the positive maximum (the negative minimum) in some point y_0 of the interval C_1C_2 (i.e. y_0 € (−1,−q)) then owing to (18) and (19) taking into account a principle of extremum for the integro-differential operators of fractional order [9], accordingly we will receive $u_x(+0, y_0) <0$, y_0 € (−1,−q), and $u_x(-0, y_0) >0$, y_0 € (−1,−q). From here owing to continuity of solution u(x, y) have received the contradiction, i.e., the function u(x, y) does not reach the positive maximum (the negative minimum) in the interval C_1C_2. Hence, function u(x, y) can reach the positive maximum (the negative minimum) only on points C_1 and C_2. The lemma is proved. As, u(x, y)=0 on the characteristic C_1F_1 and C_2F_2, we have that, u(x, y)=0 on the points C_1 and C_2. From here, owing to continuity of solution u(x, y) in the domain of, the problem III (III*) with zeroes dates, has only trivial solution, i.e. uniqueness of the solution of problem III (III*) is roved. Existence of the solution of the problem III (III*) is proved, by method integral equations.

From functional relation

$$\tau_3^*(y) = \frac{1}{\gamma_1\tau(\beta)}(y^* - q^{n+2})^{1-\beta} D^{\beta}_{q^{n+2}y^*} h_2(y^*)(y^* - q^{n+2})^{2\beta-1} - \frac{\gamma_2 4^{2\beta-1}(1-\beta)}{\gamma_1\tau(\beta)} D^{1-2\beta}_{q^{n+2}y^*}\nu_3^*(y)(y^*)^{-\frac{1}{n+2}} \quad (21)$$

And

$$\tau_3^*(y) = -\frac{1}{\gamma_1\tau(\beta)}(1 - y^*)^{1-\beta} D^{\beta}_{y^*1}\psi^*(y^*)(1 - y^*)^{2\beta-1} + \frac{\gamma_2 4^{2\beta-1}\tau(1-\beta)}{\gamma_1\tau(\beta)} D^{2\beta-1}_{y^*1}\nu_3^*(y)(y^*)^{-\frac{1}{n+2}} \quad (22)$$

excluding $\tau_3^*(y)$ and considering properties of the integro-differential operators, we will receive singular integral equation.

$$\nu(y^*)(1 + \cos\pi(1-2\beta)) - \frac{\sin\pi(1-2\beta)}{\pi}\int_{q^{n+2}}^{1}\left(\frac{1-t}{1-y^*}\right)^{1-2\beta}\left(\frac{y^*}{t}\right)^{\frac{1}{n+2}}\frac{\nu(t)}{t - y^*}dt = \phi(y^*) \quad (23)$$

where

$$\phi(y^*) = \frac{y^{*1-2\beta}}{\gamma^*} D^{1-2\beta}_{y^*1}(y^* - q^{n+2}) D^{1-2\beta}_{q^{n+2}y^*}\frac{(y^* - q^{n+2})^{2\beta-1}h_2^*(y^*)}{\gamma_1\tau(\beta)} + \frac{y^{*1-2\beta}}{\gamma^*} D^{1-2\beta}_{y^*1}(1 - y^*)^{1-\beta} D^{1-2\beta}_{q^{n+2}y^*}\frac{(1 - y^*)^{2\beta-1}\psi^*(y^*)}{\gamma_1\tau(\beta)} \quad (24)$$

Entering designations, $a(y^*)=1 + \cos \pi(1-2\beta)$, $b(y^*)=-i \sin \pi(1 - 2\beta)$ and

$$K(t, y^*) = \left(\frac{y^*}{t}\right)^{1-2\beta}\left(\frac{1-t}{1-y^*}\right)^{1-2\beta} - 1, \quad (25)$$

we will copy the equation (23) in the form of integral Cauchy [11,12]:

$$a(y^*)\nu(y^*) + \frac{b(y^*)}{\pi i}\int_{q^{n+2}}^{1}\frac{\nu(t)}{t - y^*}dt = \phi(y) - \frac{b(y^*)}{\pi i}\int_{q^{n+2}}^{1}K(t, y^*)\frac{\nu(t)}{t - y^*}dt. \quad (26)$$

We will estimate the function $\Phi(y^*)$, for this considering properties integro-differential operators, we have from (24)

$$\phi(y^*) = \frac{4^{1-2\beta}y^{*1-2\beta}}{\gamma_2\tau(2\beta)\tau^2(1-\beta)}\frac{d}{dy^*}\int_{y^*}^{1}\frac{\left(t - q^{n+2}\right)^{1-\beta}}{\left(t - y^*\right)^{1-2\beta}}dt\,\frac{d}{dt}\int_{q^{n+2}}^{1}\frac{\left(s - q^{n+2}\right)^{2\beta-1}}{\left(t - s\right)^{\beta}}h2(s)\,ds +$$
$$\frac{4^{1-2\beta}y^{*1-2\beta}}{\gamma_2\tau(2\beta)\tau^2(1-\beta)}\frac{d}{dy^*}\int_{y^*}^{1}\frac{\left(1-t\right)^{1-\beta}}{\left(t - y^*\right)^{1-2\beta}}dt\,\frac{d}{dt}\int_{t}^{1}\frac{\left(1-s\right)^{2\beta-1}}{\left(s - t\right)^{\beta}}\psi^* ds$$

From here, having executed replacements $s=(t - q^{n+2})z + q^{n+2}$ in first inner integral and $s=(1 - t)z + t$ in second inner integral, we will receive:

$$\phi(y^*) = \frac{\beta.4^{1-2\beta}y^{*1-2\beta}}{\gamma_2\tau(2\beta)\tau^2(1-\beta)}\frac{d}{dy^*}\int_{y^*}^{1}\left(t - y^*\right)^{2\beta-1}\int_{0}^{1}(1-z)^{-\beta}z^{2\beta-1}h_2^*(t, z)\,dzdt +$$
$$\frac{4^{1-2\beta}y^{*1-2\beta}}{\gamma_2\tau(2\beta)\tau^2(1-\beta)}\frac{d}{dy^*}\int_{y^*}^{1}\frac{t - q^{n+2}}{\left(t - y^*\right)^{1-2\beta}}\int_{0}^{1}(1-z)^{-\beta}z^{2\beta}h_2^{*'}(t, z)\,dzdt +$$
$$\frac{\beta.4^{1-2\beta}y^{*1-2\beta}}{\gamma_2\tau(2\beta)\tau^2(1-\beta)}\frac{d}{dy^*}\int_{y^*}^{1}\left(t - y^*\right)^{2\beta-1}\int_{0}^{1}(1-z)^{2\beta-1}z^{1-\beta}\widetilde{\psi}^{*'}(t, z)\,dzdt +$$
$$\frac{4^{1-2\beta}y^{*1-2\beta}}{\gamma_2\tau(2\beta)\tau^2(1-\beta)}\frac{d}{dy^*}\int_{y^*}^{1}\frac{t - t}{\left(t - y^*\right)^{1-2\beta}}\int_{0}^{1}(1-z)^{2\beta-1}z^{1-\beta}h_2^{*'}(t, z)\,dzdt$$

where $h_2^*(t,z) = h_2\left[(t - q^{n+2})z + q^{n+2}\right]$ and $\widetilde{\psi}^*(t,z) = \psi^*\left[(t - q^{n+2})z + q^{n+2}\right]$. Hence, owing to properties B(a,b) functions and taking into account a continuity of functions $h_2^*(t,z)$, $h_2^{*'}(t,z)$ and $\widetilde{\psi}^*(t,z)$, $\psi^{*'}(t,z)$ we will receive the estimate for function $\Phi(y^*)$:

$$\left|\Phi(y^*)\right| \le const.(1 - y^*)^{2\beta-1}$$

As, $a^2(y^*)-b^2(y^*) \neq 0$ the integral equation (23) is singular integral equation of the normal type, and by virtue (27) and $|K(t, y^*)| \le const \cdot t^{1-2\beta}(1-y^*)^{2\beta-1}$ we have that, index of integral equation (26), is equal to zero. Hence, by virtue of the theory singular integral equations and by the method regularities of Karlemens-Vekua [11], the integral equation (26) will be reduced to the integral equation of Fredgolm of the second kind with weak singularity. Thus, by virtue uniqueness of solution of the problem III(III*) the function $v_3(y)$ € C(−1;−q] C^2(−1;−q) will be unequivocally find from the equation (26), and this function can have singularity of an order less than 1 − 2β at the y→ −1 and continuous at y→ −q. After it is found $v_3(y)$, from (21) and (22) we will found $v_3(y)$ accordingly in domains Ω_{31} and Ω_{32}, hence in the domains of Ω_{31} and Ω_{32} the solution of problem III (III*) is restored as the solution of problem Cauchy, and in the domains of Ω_{21} and Ω_{22} as the solution of problem Cauchy-Gaursat. The theorem 3 is proved.

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