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Solving Traveling Salesmen Problem using Ant Colony Optimization Algorithm

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Abstract

Ant Colony Optimization is a new meta-heuristic technique used for solving different combinatorial optimization problems. ACO is based on the behaviors of ant colony and this method has strong robustness as well as good distributed calculative mechanism. ACO has very good search capability for optimization problems. Travelling salesman problem is one of the most famous combinatorial optimization problems. In this paper we applied the ant colony optimization technique for symmetric travelling salesperson problem. Analysis are shown that the ant select the rich pheromone distribution edge for finding out the best path.

Keywords: Ant colony optimization; Travelling salesman problems; Algorithm models

Introduction

Ant Colony Optimization (ACO) is a relatively new meta-heuristic and successful technique in the field of swarm intelligence. This technique was first introduced by Dorigo and his colleagues [1,2]. This technique is used for many applications especially problems that belong to the combinatorial optimization. ACO algorithm models represent the behavior of real ant colonies in establishing the shortest path between food sources and nests. The ants release pheromone on the ground while walking from their nest to food and then go back to the nest. The ants move according to the richer amount of pheromones on their path and other ants would be followed and will tend to choose a shorter path which would have a higher amount of pheromone. Artificial ants imitate the behavior of real ants, but can solve much more complicated problem than real ants can.

ACO has been widely applied to solving various combinatorial optimization problems such as traveling salesman problem (TSP), job-shop scheduling problem (JSP), vehicle routing problem (VRP), quadratic assignment problem (QAP), etc. [3]. Although ACO has a powerful capacity to find out solutions to combinatorial optimization problems, it has the problems of stagnation, premature convergence and the convergence speed of ACO is always slow. These problems will be more obvious when the problem size increases (Figure 1).

The traveling salesman problem (TSP) is the problem of finding a shortest closed tour which visits all the cities in a given set. In a symmetric TSP the distance between two cities is the same regardless of the direction of travel whereas in the asymmetric TSP the distance is different with regards to the direction of travel [4]. This paper restricts attention to symmetric TSPs in which cities are on a plane and a path (edge) exists between each pair of cities. The definition of a TSP is: given N cities, if a salesman starting from his home city is to visit each city exactly once and then return home, find the order of a tour such that the total distances (cost/time/money/energy etc) traveled should be minimum. A complete weighted graph G= (N, E) can be used to represent a TSP, where N is the set of n cities and E is the set of edges (paths) fully connecting all cities. Each edge $(i,j)\in E$ is assigned a cost d_{ij}, which is the distance between cities i and j [5-7].

Methodology of Ant Colon Optimization Model

The Ant System was first introduced and applied to TSP by Marcodorigo et al. Initially, each ant is placed on some randomly chosen city. An ant k currently at city i choose to move to city j by applying the following probabilistic transition rule:

$$p_{ij}^{k}(t)=\begin{cases}\dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{k\in allowed_k}[\tau_{ik}(t)]^{\alpha}[\eta_{ik}]^{\beta}} & \text{if } j\in \text{allowed}_k\\ 0 & \text{otherwise}\end{cases}$$

where n_{ij} is the heuristic visibility of edge (i, j), generally it is a value of $1/d_{ij}$, where d_{ij} is the distance between city i and city j. City J is a set of cities which remain to be visited when the ant is at city i. α and β are two adjustable positive parameters that control the relative weights of the pheromone trail and of the heuristic visibility. If α=0, the closed vertex i more likely to be selected. This is responding to a classical stochastic greedy algorithm. If β=0, only pheromone amplification is at work: This method will lead the system to a stagnation situation, i.e. a situation in which all the ants generate a sub-optimal tour. So the trade-off between edge length and pheromone intensity is necessary. After each ant completes its tour, the pheromone amount on each path will be adjusted according to equation $(1-\rho)$ is the pheromone decay parameter $(0<\rho<1)$ where it represents the trail evaporation when the ant chooses a city and decides to move. Number of ants represented by m, L_k is the length of the tour performed by ant k and Q is an arbitrary constant [8,9]. After all the ants complete their tour the pheromone is required to be updated using

$$\tau_{ij}(t+1)=(1-\rho)\tau_{ij}(t)+\Delta\tau_{ij}(t)$$

Where

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$$\Delta\tau_{ij}(t) = \sum_{k=1}^{m} \Delta\tau_{ij}^{\ k}(t)$$

$$\text{Tour Length: } \Delta\tau_{i,j}^{k} = \begin{cases} \frac{Q}{L_k} & if\,(i,j) \in \text{tour}_k \\ 0 & \text{otherwise} \end{cases}$$

Methodology

In this paper a symmetric travelling salesman problem is presented for five cities. The distance of each city is given in the Table 1 and their visibility for each edge is shown in Table 2 and 3.

Parameter selection

No of Ants=5

α =0.7, β=0.7, Q=1, Number of iteration=5

Initially pheromone (τ) distribution for each edge=1

Results and Discussion

Ant colony optimization methodology is presented for symmetric cities of travelling salesmen problem in this paper. A detailed procedure for city selection for respective ant is presented in the Tables 4-8 for iteration 1 to 5. The selection of next city is based on the maximum probability within the set of possible selection of the cities for next move. This is marked as '*' in the respective iteration. In the Table 4, ant 2, 3 and 5 gives the minimum distance with the initially assumed pheromone which is one for each edge. The minimum distance remain same for the ant 2, 3 and 5 for iteration 2, however the pheromone (τ) distribution is different because of updating the pheromone in the iteration 2 for each edge based on the updating rule. In the iteration 4, the ant 1, 2, 3 and 5 are selected the best path with minimum distance. However in the iteration 5, all the ants are converged to the minimum distance. The pheromone (τ) distribution for iteration, 2, 3, 4 and 5 are updated for each iteration. The distribution of the pheromone (τ) for each edge is shown in the beginning of the respective table.

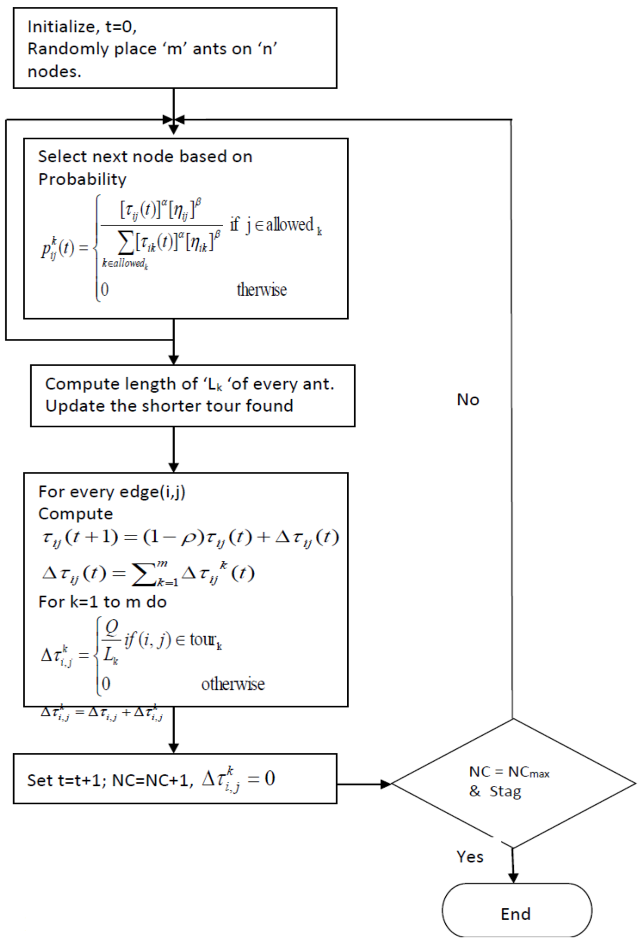

Figure 1: Flow Chart for ACO.

Cities/ Cities	A	B	C	D	E
A	∞	3	6	2	3
B	3	∞	5	2	3
C	6	5	∞	6	4
D	2	2	6	∞	6
E	3	3	4	6	∞

Table 1: Distance (d_{ij}) of cities.

Cities/ Cities	A	B	C	D	E
A	∞	0.33	0.16	0.5	0.33
B	0.33	∞	0.2	0.5	0.33
C	0.16	0.2	∞	0.16	0.25
D	0.5	0.5	0.16	∞	0.16
E	0.33	0.33	0.25	0.16	∞

Table 2: Visibility of edge = 1/dij.

Ant	1	2	3	4	5
City	A	B	C	D	E

Table 3: Randomly select the first City by each ant.

Pheromone Distribution

City	A	B	C	D	E
A	0	1	1	1	1
B	1	0	1	1	1
C	1	1	0	1	1
D	1	1	1	0	1
E	1	1	1	1	0

Selection of Cities for respective ant

City	τ	τ^{α}	Cost(n)	1/n	$(1/n)^{\beta}$	$\tau^{\alpha}(1/n)^{\beta}$	$\sum\tau^{\alpha}(1/n)^{\beta}$	Probability of next city
Ant 1 Path								
Selection of Second City								
AB	1	1.000	3	0.3333	0.4635	0.4635	1.8278	0.2536
AC	1	1.000	6	0.1667	0.2853	0.2853	1.8278	0.1561
*AD	1	1.000	2	0.5000	0.6156	0.6156	1.8278	0.3368
AE	1	1.000	3	0.3333	0.4635	0.4635	1.8278	0.2536
Selection of third City								
*DB	1	1.000	2	0.5000	0.6156	0.6156	1.1862	0.5190
DC	1	1.000	6	0.1667	0.2853	0.2853	1.1862	0.2405
DE	1	1.000	6	0.1667	0.2853	0.2853	1.1862	0.2405
Selection of fourth City								
BC	1	1.000	5	0.2000	0.3241	0.3241	0.7876	0.4115
*BE	1	1.000	3	0.3333	0.4635	0.4635	0.7876	0.5884
Selection of last City								
*EC	1	1.000	4	0.2500	0.3789	0.3789	0.3789	1.0000
Path for ant 1: A			**D**	**B**	**E**	**C**	**A**	
Distance			**2**	**2**	**3**	**4**	**6**	**Total= 17**
Ant 2 Path								
Selection of Second City								
BA	1	1.000	3	0.3333	0.4635	0.4635	1.8666	0.2483
BC	1	1.000	5	0.2000	0.3241	0.3241	1.8666	0.1736
*BD	1	1.000	2	0.5000	0.6156	0.6156	1.8666	0.3298
BE	1	1.000	3	0.3333	0.4635	0.4635	1.8666	0.2483
Selection of third City								
*DA	1	1.000	2	0.5000	0.6156	0.6156	1.1862	0.5190
DC	1	1.000	6	0.1667	0.2853	0.2853	1.1862	0.2405
DE	1	1.000	6	0.1667	0.2853	0.2853	1.1862	0.2405
Selection of fourth City								
AC	1	1.000	6	0.1667	0.2853	0.2853	0.7488	0.3810
*AE	1	1.000	3	0.3333	0.4635	0.4635	0.7488	0.6189
Selection of last City								
*EC	1	1.000	6	0.1667	0.2853	0.2853	0.2853	1.0000
Path for ant 2: B			**D**	**A**	**E**	**C**	**B**	
Distance			**2**	**2**	**3**	**4**	**5**	**Total= 16**
Ant 3 Path								
Selection of Second City								
CA	1	1.000	6	0.1667	0.2853	0.2853	1.2737	0.2240
CB	1	1.000	5	0.2000	0.3241	0.3241	1.2737	0.2545
CD	1	1.000	6	0.1667	0.2853	0.2853	1.2737	0.2240
*CE	1	1.000	4	0.2500	0.3789	0.3789	1.2737	0.2975
City	τ	τ^{α}	Cost(n)	1/n	$(1/n)^{\beta}$	$\tau^{\alpha}(1/n)^{\beta}$	$\sum\tau^{\alpha}(1/n)^{\beta}$	Probability of next city
Selection of third City								
*EA	1	1.000	3	0.3333	0.4635	0.4635	1.2122	0.3823
EB	1	1.000	3	0.3333	0.4635	0.4635	1.2122	0.3823
ED	1	1.000	6	0.1667	0.2853	0.2853	1.2122	0.2354
Selection of fourth City								
AB	1	1.000	3	0.3333	0.4635	0.4635	1.0790	0.4295
*AD	1	1.000	2	0.5000	0.6156	0.6156	1.0790	0.5705
Selection of last City								
*DB	1	1.000	2	0.5000	0.6156	0.6156	0.6156	1.0000
Path for ant 3: C			**E**	**A**	**D**	**B**	**C**	
Distance			**4**	**3**	**2**	**2**	**5**	**Total=16**
Ant 4 Path								
Selection of Second City								
*DA	1	1.000	2	0.5000	0.6156	0.6156	1.8017	0.3417
DB	1	1.000	2	0.5000	0.6156	0.6156	1.8017	0.3417
DC	1	1.000	6	0.1667	0.2853	0.2853	1.8017	0.1583
DE	1	1.000	6	0.1667	0.2853	0.2853	1.8017	0.1583
Selection of third City								
*AB	1	1.000	3	0.3333	0.4635	0.4635	1.2122	0.3823
AC	1	1.000	6	0.1667	0.2853	0.2853	1.2122	0.2354
AE	1	1.000	3	0.3333	0.4635	0.4635	1.2122	0.3823
Selection of fourth City								
BC	1	1.000	5	0.2000	0.3241	0.3241	0.7876	0.4115
*BE	1	1.000	3	0.3333	0.4635	0.4635	0.7876	0.5884
Selection of last City								
*EC	1	1.000	4	0.2500	0.3789	0.3789	0.3789	1.0000
Path for ant 4: D			**A**	**B**	**E**	**C**	**D**	
Distance			**2**	**3**	**3**	**4**	**6**	Total=18
Ant 5 Path								
Selection of Second City								
*EA	1	1.000	3	0.3333	0.4635	0.4635	1.5912	0.2913
EB	1	1.000	3	0.3333	0.4635	0.4635	1.5912	0.2913
EC	1	1.000	4	0.2500	0.3789	0.3789	1.5912	0.2381
ED	1	1.000	6	0.1667	0.2853	0.2853	1.5912	0.1793
Selection of third City								
AB	1	1.000	3	0.3333	0.4635	0.4635	1.3643	0.3397
AC	1	1.000	6	0.1667	0.2853	0.2853	1.3643	0.2091
*AD	1	1.000	2	0.5000	0.6156	0.6156	1.3643	0.4512
Selection of fourth City								
*DB	1	1.000	2	0.5000	0.6156	0.6156	0.9009	0.6833
DC	1	1.000	6	0.1667	0.2853	0.2853	0.9009	0.3167
Selection of last City								
*BC	1	1.000	5	0.2000	0.3241	0.3241	0.3241	1.0000
Path for ant 5: E			**A**	**D**	**B**	**C**	**E**	
Distance			**3**	**2**	**2**	**5**	**4**	**Total =1 6**

* Selected Edge

Table 4: Iteration-1.

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Pheromone Distribution

	A	B	C	D	E
A	0	0.5	0.5	0.625	0.625
B	0.5	0	0.5625	0.625	0.5
C	0.5	0.625	0	0.5	0.6875
D	0.5625	0.5625	0.5	0	0.5
E	0.5625	0.5	0.5	0.5	0

Selection of Cities for respective ant

City	τ	τ^{α}	Cost(n)	1/n	$(1/n)^{\beta}$	$\tau^{\alpha}(1/n)^{\beta}$	$\sum\tau^{\alpha}(1/n)^{\beta}$	Probability of next city
Ant 1 Path								
Selection of Second City								
AB	0.500	0.6156	3	0.3333	0.4635	0.2853	1.2374	0.2306
AC	0.500	0.6156	6	0.1667	0.2853	0.1756	1.2374	0.1419
*AD	0.625	0.7196	2	0.5000	0.6156	0.4430	1.2374	0.3580
AE	0.625	0.7196	3	0.3333	0.4635	0.3335	1.2374	0.2695
Selection of third City								
*DB	0.563	0.6685	2	0.5000	0.6156	0.4115	0.7627	0.5395
DC	0.500	0.6156	6	0.1667	0.2853	0.1756	0.7627	0.2303
DE	0.500	0.6156	6	0.1667	0.2853	0.1756	0.7627	0.2303
Selection of fourth City								
BC	0.563	0.6685	5	0.2000	0.3241	0.2167	0.5020	0.4316
*BE	0.500	0.6156	3	0.3333	0.4635	0.2853	0.502	0.5683
Selection of last City								
*EC	0.500	0.6156	4	0.2500	0.3789	0.2333	0.2333	1.0000
Path for ant 1: A			**D**	**B**	**E**	**C**	**A**	
Distance			**2**	**2**	**3**	**4**	**6**	Total = 17
Ant 2 Path								
Selection of Second City								
BA	0.5	0.6156	3	0.3333	0.4635	0.2853	1.2303	0.2319
BC	0.5625	0.6685	5	0.2000	0.3241	0.2167	1.2303	0.1761
*BD	0.625	0.7196	2	0.5000	0.6156	0.4430	1.2303	0.3601
BE	0.5	0.6156	3	0.3333	0.4635	0.2853	1.2303	0.2319
Selection of third City								
*DA	0.5625	0.6685	2	0.5000	0.6156	0.4115	0.7627	0.5395
DC	0.5	0.6156	6	0.1667	0.2853	0.1756	0.7627	0.2303
DE	0.5	0.6156	6	0.1667	0.2853	0.1756	0.7627	0.2303
Selection of fourth City								
AC	0.5	0.6156	6	0.1667	0.2853	0.1756	0.5091	0.3449
*AE	0.625	0.7196	3	0.3333	0.4635	0.3335	0.5091	0.6551
Selection of last City								
*EC	0.5	0.6156	4	0.2500	0.3789	0.2333	0.2333	1.0000
Path for ant 2: B			**D**	**A**	**E**	**C**	**B**	
Distance			**2**	**2**	**3**	**4**	**5**	Total = 16
Ant 3 Path								
Selection of Second City								
CA	0.5	0.6156	6	0.1667	0.2853	0.1756	0.8572	0.2049
CB	0.625	0.7196	5	0.2000	0.3241	0.2333	0.8572	0.2721
CD	0.5	0.6156	6	0.1667	0.2853	0.1756	0.8572	0.2049
*CE	0.625	0.7196	4	0.2500	0.3789	0.2727	0.8572	0.3181
Selection of third City								
*EA	0.5625	0.6685	3	0.3333	0.4635	0.3098	0.7707	0.4020
EB	0.5	0.6156	3	0.3333	0.4635	0.2853	0.7707	0.3702
ED	0.5	0.6156	6	0.1667	0.2853	0.1756	0.7707	0.2279
Selection of fourth City								
AB	0.5	0.6156	3	0.3333	0.4635	0.2853	0.7283	0.3917
*AD	0.625	0.7196	2	0.5000	0.6156	0.4430	0.7283	0.6083
Selection of last City								
*DB	0.5625	0.6685	2	0.5000	0.6156	0.4115	0.4115	1.0000
Path for ant 3: C			**E**	**A**	**D**	**B**	**C**	
Distance			**4**	**3**	**2**	**2**	**5**	Total = 16
Ant 4 Path								
Selection of Second City								
*DA	0.5625	0.6685	2	0.5000	0.6156	0.4115	1.1742	0.3504
DB	0.5625	0.6685	2	0.5000	0.6156	0.4115	1.1742	0.3504
DC	0.5	0.6156	6	0.1667	0.2853	0.1756	1.1742	0.1496
DE	0.5	0.6156	6	0.1667	0.2853	0.1756	1.1742	0.1496
Selection of third City								
AB	0.5	0.6156	3	0.3333	0.4635	0.2853	0.7944	0.3591
AC	0.5	0.6156	6	0.1667	0.2853	0.1756	0.7944	0.2211
*AE	0.625	0.7196	3	0.3333	0.4635	0.3335	0.7944	0.4198
Selection of four City								
EC	0.5	0.6156	5	0.2000	0.3241	0.1995	0.4848	0.4115
*EB	0.5	0.6156	3	0.3333	0.4635	0.2853	0.4848	0.5885
Selection of last City								
*BC	0.5625	0.6685	4	0.2500	0.3789	0.2533	0.2533	1.0000
Path for ant 4: D			**A**	**E**	**B**	**C**	**D**	
Distance			**2**	**3**	**3**	**5**	**6**	Total = 19
Ant 5 Path								
Selection of Second City								
*EA	0.5625	0.6685	3	0.3333	0.4635	0.3098	1.0040	0.3086
EB	0.5	0.6156	3	0.3333	0.4635	0.2853	1.0040	0.2842
EC	0.5	0.6156	4	0.2500	0.3789	0.2333	1.0040	0.2323
ED	0.5	0.6156	6	0.1667	0.2853	0.1756	1.0040	0.1749
Selection of third City								
AB	0.5	0.6156	3	0.3333	0.4635	0.2853	0.9039	0.3156
AC	0.5	0.6156	6	0.1667	0.2853	0.1756	0.9039	0.1943
*AD	0.625	0.7196	2	0.5000	0.6156	0.4430	0.9039	0.4901
Selection of fourth City								
*DB	0.5625	0.6685	2	0.5000	0.6156	0.4115	0.5871	0.7009
DC	0.5	0.6156	6	0.1667	0.2853	0.1756	0.5871	0.2991
Selection of last City								
*BC	0.5625	0.6685	5	0.2000	0.3241	0.2167	0.2167	1.0000
Path for ant 5: E			**A**	**D**	**B**	**C**	**E**	
Distance			**3**	**2**	**2**	**5**	**4**	Total = 16

* Selected Edge

Table 5: Iteration-2.

Pheromone distribution

	A	B	C	D	E
A	0	0.25	0.25	0.4375	0.4375
B	0.25	0	0.34375	0.4375	0.25
C	0.25	0.4375	0	0.25	0.53125
D	0.34375	0.34375	0.25	0	0.25
E	0.34375	0.25	0.25	0.25	0

Selection of Cities for respective ant

City	τ	τ^α	Cost(n)	1/n	$(1/n)^\beta$	$\tau^\alpha (1/n)^\beta$	$\sum\tau^\alpha (1/n)^\beta$	Probability of next city
Ant 1 Path								
Selection of Second City								
AB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.8887	0.1976
AC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.8887	0.1216
*AD	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.8887	0.3883
AE	0.4375	0.5606	3	0.3333	0.4635	0.2598	0.8887	0.2924
Selection of third City								
*DB	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.5613	0.6148
DC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5613	0.1926
DE	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5613	0.1926
Selection of fourth City								
BC	0.3438	0.4736	5	0.2000	0.3241	0.1535	0.3291	0.4664
*BE	0.25	0.3789	3	0.3333	0.4635	0.1756	0.3291	0.5336
Selection of last City								
*EC	0.5313	0.6423	4	0.2500	0.3789	0.2434	0.2434	1.0000
Path for ant 1:	A		B	E	C	A		
Distance			2	2	3	4	6	Total = 17
Ant 2 Path								
Selection of Second City								
BA	0.25	0.3789	3	0.3333	0.4635	0.1756	0.8499	0.2066
BC	0.3438	0.4736	5	0.2000	0.3241	0.1535	0.8499	0.1806
*BD	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.8499	0.4061
BE	0.25	0.3789	3	0.3333	0.4635	0.1756	0.8499	0.2066
Selection of third City								
*DA	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.5613	0.6148
DC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5613	0.1926
DE	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5613	0.1926
Selection of fourth City								
AC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.3679	0.2938
*AE	0.4375	0.5606	3	0.3333	0.4635	0.2598	0.3679	0.7063
Selection of last City								
*EC	0.5313	0.6423	4	0.2500	0.3789	0.2434	0.2434	1.0000
Path for ant 2:	B		D	A	E	C	B	
Distance			2	2	3	4	5	Total = 16
Ant 3 Path								
Selection of Second City								
CA	0.25	0.3789	6	0.1667	0.2853	0.1081	0.6413	0.1686
CB	0.4375	0.5606	5	0.2000	0.3241	0.1817	0.6413	0.2834
CD	0.25	0.3789	6	0.1667	0.2853	0.1081	0.6413	0.1686
*CE	0.5313	0.6423	4	0.2500	0.3789	0.2434	0.6413	0.3795

City	τ	τ^α	Cost(n)	1/n	$(1/n)^\beta$	$\tau^\alpha (1/n)^\beta$	$\sum\tau^\alpha (1/n)^\beta$	Probability of next city
Selection of third City								
*EA	0.4375	0.5606	3	0.3333	0.4635	0.2598	0.5436	0.4780
EB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.5436	0.3231
ED	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5436	0.1989
Selection of fourth City								
AB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.5207	0.3373
*AD	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.5207	0.6628
Selection of last City								
*DB	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.3451	1.0000
Path for ant 3:	C		E	A	D	B	C	
Distance			4	3	2	2	5	Total = 16
Ant 4 Path								
Selection of Second City								
*DA	0.3438	0.4736	2	0.5000	0.6156	0.2915	0.7993	0.3647
DB	0.3438	0.4736	2	0.5000	0.6156	0.2915	0.7993	0.3647
DC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.7993	0.1353
DE	0.25	0.3789	6	0.1667	0.2853	0.1081	0.7993	0.1353
Selection of third City								
AB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.5032	0.3490
AC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5032	0.2148
*AE	0.3438	0.4736	3	0.3333	0.4635	0.2195	0.5032	0.4362
Selection of fourth City								
EC	0.25	0.3789	5	0.2000	0.3241	0.1228	0.2984	0.4115
*EB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.2984	0.5885
Selection of last City								
*BC	0.4375	0.5606	4	0.2500	0.3789	0.2124	0.2124	1.0000
Path for ant 4:	D		A	E	B	C	D	
Distance			2	3	3	5	6	Total = 19
Ant 5 Path								
Selection of Second City								
*EA	0.3438	0.4736	3	0.3333	0.4635	0.2195	0.7365	0.2980
EB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.7365	0.2385
EC	0.5	0.6156	4	0.2500	0.3789	0.2333	0.7365	0.3167
ED	0.25	0.3789	6	0.1667	0.2853	0.1081	0.7365	0.1468
Selection of third City								
AB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.5753	0.3053
AC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5753	0.1879
*AD	0.3438	0.4736	2	0.5000	0.6156	0.2915	0.5753	0.5068
Selection of fourth City								
*DB	0.3438	0.4736	2	0.5000	0.6156	0.2915	0.3996	0.7295
DC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.3996	0.2705
Selection of last City								
*BC	0.4375	0.5606	5	0.2000	0.3241	0.1817	0.1817	1.0000
Path for ant 5:	E		A	D	B	C	E	
Distance			3	2	2	5	4	Total = 16

* Selectged Edge

Table 6: Iteration-3.

Citation: Raghavendra BV (2015) Solving Traveling Salesmen Problem using Ant Colony Optimization Algorithm. J Appl Computat Math 4: 260. doi:10.4172/2168-9679.1000260

Pheromone distribution

	A	B	C	D	E
A	0	0.25	0.25	0.4375	0.4375
B	0.25	0	0.34375	0.4375	0.25
C	0.25	0.4375	0	0.25	0.53125
D	0.34375	0.34375	0.25	0	0.25
E	0.34375	0.25	0.25	0.25	0

Selection of Cities for respective ant

City	τ	$τ^α$	Cost(n)	1/n	$(1/n)^β$	$τ^α(1/n)^β$	$Στ^α(1/n)^β$	Probability of next city
Ant 1 Path								
Selection of Second City								
AB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.8887	0.1976
AC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.8887	0.1216
*AD	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.8887	0.3883
AE	0.4375	0.5606	3	0.3333	0.4635	0.2598	0.8887	0.2924
Selection of third City								
*DB	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.5613	0.6148
DC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5613	0.1926
DE	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5613	0.1926
Selection of fourth City								
BC	0.3438	0.4736	5	0.2000	0.3241	0.1535	0.3291	0.4664
*BE	0.25	0.3789	3	0.3333	0.4635	0.1756	0.3291	0.5336
Selection of last City								
*EC	0.5313	0.6423	4	0.2500	0.3789	0.2434	0.2434	1.0000
Path for ant 1:	A		B	E	C	A		
Distance			2	2	3	4	6	Total = 17
Ant 2 Path								
Selection of Second City								
BA	0.25	0.3789	3	0.3333	0.4635	0.1756	0.8499	0.2066
BC	0.3438	0.4736	5	0.2000	0.3241	0.1535	0.8499	0.1806
*BD	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.8499	0.4061
BE	0.25	0.3789	3	0.3333	0.4635	0.1756	0.8499	0.2066
Selection of third City								
*DA	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.5613	0.6148
DC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5613	0.1926
DE	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5613	0.1926
Selection of fourth City								
AC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.3679	0.2938
*AE	0.4375	0.5606	3	0.3333	0.4635	0.2598	0.3679	0.7063
Selection of last City								
*EC	0.5313	0.6423	4	0.2500	0.3789	0.2434	0.2434	1.0000
Path for ant 2:	B		D	A	E	C	B	
Distance			2	2	3	4	5	Total = 16
Ant 3 Path								
Selection of Second City								
CA	0.25	0.3789	6	0.1667	0.2853	0.1081	0.6413	0.1686
CB	0.4375	0.5606	5	0.2000	0.3241	0.1817	0.6413	0.2834
CD	0.25	0.3789	6	0.1667	0.2853	0.1081	0.6413	0.1686
*CE	0.5313	0.6423	4	0.2500	0.3789	0.2434	0.6413	0.3795
Selection of third City								
*EA	0.4375	0.5606	3	0.3333	0.4635	0.2598	0.5436	0.4780
EB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.5436	0.3231
ED	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5436	0.1989
Selection of fourth City								
AB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.5207	0.3373
*AD	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.5207	0.6628
Selection of last City								
*DB	0.4375	0.5606	2	0.5000	0.6156	0.3451	0.3451	1.0000
Path for ant 3:	C		E	A	D	B	C	
Distance			4	3	2	2	5	Total = 16
Ant 4 Path								
Selection of Second City								
*DA	0.3438	0.4736	2	0.5000	0.6156	0.2915	0.7993	0.3647
DB	0.3438	0.4736	2	0.5000	0.6156	0.2915	0.7993	0.3647
DC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.7993	0.1353
DE	0.25	0.3789	6	0.1667	0.2853	0.1081	0.7993	0.1353
Selection of third City								
AB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.5032	0.3490
AC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5032	0.2148
*AE	0.3438	0.4736	3	0.3333	0.4635	0.2195	0.5032	0.4362
Selection of fourth City								
EC	0.25	0.3789	5	0.2000	0.3241	0.1228	0.2984	0.4115
*EB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.2984	0.5885
Selection of last City								
*BC	0.4375	0.5606	4	0.2500	0.3789	0.2124	0.2124	1.0000
Path for ant 4:	D		A	E	B	C	D	
Distance			2	3	3	5	6	Total = 19
Ant 5 Path								
Selection of Second City								
*EA	0.3438	0.4736	3	0.3333	0.4635	0.2195	0.7365	0.2980
EB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.7365	0.2385
EC	0.5	0.6156	4	0.2500	0.3789	0.2333	0.7365	0.3167
ED	0.25	0.3789	6	0.1667	0.2853	0.1081	0.7365	0.1468
Selection of third City								
AB	0.25	0.3789	3	0.3333	0.4635	0.1756	0.5753	0.3053
AC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.5753	0.1879
*AD	0.3438	0.4736	2	0.5000	0.6156	0.2915	0.5753	0.5068
Selection of fourth City								
*DB	0.3438	0.4736	2	0.5000	0.6156	0.2915	0.3996	0.7295
DC	0.25	0.3789	6	0.1667	0.2853	0.1081	0.3996	0.2705
Selection of last City								
*BC	0.4375	0.5606	5	0.2000	0.3241	0.1817	0.1817	1.0000
Path for ant 5:	E		A	D	B	C	E	
Distance			3	2	2	5	4	Total = 16

* Selectged Edge

Table 7: Iteration-4.

	A	B	C	D	E
A	0	0.0625	0.0625	0.3594	0.2969
B	0.0625	0	0.2422	0.3594	0.0625
C	0.0625	0.2969	0	0.0625	0.4141
D	0.1797	0.1797	0.0625	0	0.0625
E	0.2422	0.0625	0.125	0.0625	0

Selection of Cities for respective ant

	City	τ	τ^α	Cost(n)	1/n	$(1/n)^\beta$	$\tau^\alpha (1/n)^\beta$	$\sum\tau^\alpha (1/n)^\beta$	Probability of next city
Ant 1 Path									
Selection of Second City									
	AB	0.0625	0.1436	3	0.3333	0.4635	0.0665	0.6063	0.1098
	AC	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.6063	0.0676
	*AD	0.3594	0.4885	2	0.5000	0.6156	0.3007	0.6063	0.4960
	AE	0.2969	0.4274	3	0.3333	0.4635	0.1981	0.6063	0.3267
Selection of third City									
	*DB	0.3594	0.4885	2	0.5000	0.6156	0.3007	0.3826	0.7859
	DC	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.3826	0.1071
	DE	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.3826	0.1071
Selection of fourth City									
	*BC	0.2342	0.3620	5	0.2000	0.3241	0.1173	0.1839	0.6381
	BE	0.0625	0.1436	3	0.3333	0.4635	0.0665	0.1839	0.3619
	*CE	0.1250	0.2333	4	0.2500	0.3789	0.0884	0.0884	1.0000
Path for ant 1:			A	**D**	**B**	**C**	**E**	**A**	
Distance				**2**	**2**	**5**	**4**	**3**	**Total = 16**
Ant 2 Path									
Selection of Second City									
	BA	0.0625	0.1436	3	0.3333	0.4635	0.0665	0.5539	0.1201
	BC	0.2422	0.3706	5	0.2000	0.3241	0.1201	0.5539	0.2169
	*BD	0.3594	0.4885	2	0.5000	0.6156	0.3007	0.5539	0.5429
	BE	0.0625	0.1436	3	0.3333	0.4635	0.0665	0.5539	0.1201
Selection of third City									
	*DA	0.3594	0.4885	2	0.5000	0.6156	0.3007	0.3826	0.7859
	DC	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.3826	0.1071
	DE	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.3826	0.1071
Selection of fourth City									
	AC	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.2390	0.1714
	*AE	0.2969	0.4274	3	0.3333	0.4635	0.1981	0.2390	0.8287
Selection of last City									
	*EC	0.4141	0.5394	4	0.2500	0.3789	0.2044	0.2044	1.0000
Path for ant 2:			B	**D**	**A**	**E**	**C**	**B**	
Distance				**2**	**2**	**3**	**4**	**5**	**Total = 16**
Ant 3 Path									
Selection of Second City									
	CA	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.4249	0.0964
	CB	0.2969	0.4274	5	0.2000	0.3241	0.1385	0.4249	0.3260
	CD	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.4249	0.0964
	*CE	0.4141	0.5394	4	0.2500	0.3789	0.2044	0.4249	0.4811
Selection of third City									
	*EA	0.2969	0.4274	3	0.3333	0.4635	0.1981	0.3056	0.6482
	EB	0.0625	0.1436	3	0.3333	0.4635	0.0665	0.3056	0.2178
	ED	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.3056	0.1340
Selection of fourth City									
	AB	0.0625	0.1436	3	0.3333	0.4635	0.0665	0.3673	0.1812
	*AD	0.3594	0.4885	2	0.5000	0.6156	0.3007	0.3673	0.8187
Selection of last City									
	*DB	0.3594	0.4885	2	0.5000	0.6156	0.3007	0.3007	1.0000
Path for ant 3:			C	**E**	**A**	**D**	**B**	**C**	
Distance				**4**	**3**	**2**	**2**	**5**	**Total = 16**
Ant 4 Path									
Selection of Second City									
	*DA	0.1797	0.3007	2	0.5000	0.6156	0.1851	0.4522	0.4094
	DB	0.1797	0.3007	2	0.5000	0.6156	0.1851	0.4522	0.4094
	DC	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.4522	0.0906
	DE	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.4522	0.0906
Selection of third City									
	AB	0.0625	0.1436	3	0.3333	0.4635	0.0665	0.2793	0.2383
	AC	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.2793	0.1467
	*AE	0.2422	0.3706	3	0.3333	0.4635	0.1718	0.2793	0.6150
Selection of fourth City									
	*EC	0.1250	0.2333	5	0.2000	0.3241	0.0756	0.1422	0.5319
	EB	0.0625	0.1436	3	0.3333	0.4635	0.0665	0.1422	0.4680
Selection of last City									
	*CB	0.2969	0.4274	4	0.2500	0.3789	0.1619	0.1619	1.0000
Path for ant 4:			D	**A**	**E**	**C**	**B**	**D**	
Distance				**2**	**3**	**4**	**5**	**2**	**Total = 16**
Ant 5 Path									
Selection of Second City									
	*EA	0.2422	0.3706	3	0.3333	0.4635	0.1718	0.3677	0.4672
	EB	0.0625	0.1436	3	0.3333	0.4635	0.0665	0.3677	0.1810
	EC	0.1250	0.2333	4	0.2500	0.3789	0.0884	0.3677	0.2404
	ED	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.3677	0.1114
Selection of third City									
	AB	0.0625	0.1436	3	0.3333	0.4635	0.0665	0.2926	0.2274
	AC	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.2926	0.1400
	*AD	0.1797	0.3007	2	0.5000	0.6156	0.1851	0.2926	0.6327
Selection of fourth City									
	*DB	0.1797	0.3007	2	0.5000	0.6156	0.1851	0.2261	0.8188
	DC	0.0625	0.1436	6	0.1667	0.2853	0.0410	0.2261	0.1812
Selection of last City									
	*BC	0.2969	0.4274	5	0.2000	0.3241	0.1385	0.1385	1.0000
Path for ant 5:			E	**A**	**D**	**B**	**C**	**E**	
Distance				**3**	**2**	**2**	**5**	**4**	**Total = 16**
* Selected Edge									

Table 8: Iteration-5.

Conclusion

It is shown in the iteration number 5 that all the ants converge to the best path which gives minimum distance. The pheromone distribution for iteration and the next city selection based on maximum probability is determined in the iteration. It is evident from the analysis that the rich pheromone edge is converges the best path for the travelling salesmen problems.

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