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Solving Nonlinear Integral Equations by using Adomian Decomposition Method

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Abstract

In this paper, we propose a numerical method to solve the nonlinear integral equation of the second kind. We intend to approximate the solution of this equation by Adomian decomposition method using He's polynomials. Several examples are given at the end of this paper with the exact solution is known. Also the error is estimated.

Keywords: Nonlinear Fredholm integral equation; Adomian decomposition method; Adomian polynomials; Approximate solutions; OriginPro 8 and MATHEMATICA v9 softwares

Introduction

Several scientific and engineering applications are usually described by integral equations. Integral equations arise in the potential theory more than any other field. Integral equations arise also in diffraction problems, conformal mapping, water waves, scattering in quantum mechanics, and population growth model. The electrostatic, electromagnetic scattering problems and propagation of acoustical and elastically waves are scientific fields where integral equations appear [1]. The Fredholm integral equation is of widespread use in many realms of engineering and applied mathematics [2].

Consider the general form non-linear Fredholm integral equation of the second kind

$$y(x) = f(x) + \lambda\int_a^b K(x,t)\mathrm{G}(y(t))dt,\ a \le x \le b$$

where $y(x)$ is the unknown solution, a and b are real constants. The kernel $K(x,t)$ and $f(x)$ are known smooth functions on R^2 and R, respectively. The parameter λ is a real (or complex) known as the eigenvalue when λ is a real parameter, and G is a nonlinear function of y.

Adomian Decomposition Method

Consider the following non-linear Fredholm integral equation of the second kind of the form

$$y(x) = f(x) + \lambda\int_a^b K(x,t)\mathrm{G}(y(t))dt, \quad a \le x \le b \tag{1}$$

We assume $G(y(t))$ is a nonlinear function of $y(x)$. That means that the nonlinear Fredholm integral equation (1) contains the nonlinear function presented by $G(y(t))$. Assume that the solution of equation (1) can be written in the form

$$y = \sum_{i=0}^{\infty} p^i y_i(x) = y_0 + p^1 y_1 + p^2 y_2 + p^3 y_3 + \ldots \tag{2}$$

The comparisons of like powers of p give solutions of various orders and the best approximation is

$$y = \lim_{p\to 1}\sum_{i=0}^{\infty} p^i y_i(x) = y_0 + y_1 + y_2 + y_3 + \ldots$$

The nonlinear term $G(y(t))$ can be expressed in Adomian polynomials [3-5] as

$$G(y) = \sum_{k=0}^{\infty} p^k H_k(y_0, y_1, y_2, \ldots, y_k) \tag{3}$$

$$= H_0(y_0) + p^1 H_1(y_0, y_1) + \ldots + p^k H_k(y_0, y_1, \ldots\ldots, y_k)$$

where H_k's are the so called Adomian polynomials which can be calculated by using the formula

$$H_k(y_0, y_1, \ldots\ldots, y_k) = \frac{1}{k!}\frac{d^k}{dp^k}\left[G\left(\sum_{i=0}^{k} p^i y_i\right)\right], k = 0,1,2\ldots\ldots \tag{4}$$

Using (2), (3) and (4) into (1), we have

$$\sum_{i=0}^{\infty} p^i y_i(x) = f(x) + \lambda\int_a^b K(x,t)\sum_{j=0}^{\infty}(p^j H_j)dt \tag{5}$$

Equating the term with identical power of p in equation (5),

p^0: $y_0(x) = f(x)$

$$p^1: y_1(x) = \lambda\int_a^b K(x,t)H_0(t)dt$$

and so on.

and in general form we have

$$\begin{cases} y_0(x) = f(x) \\ y_{k+1}(x) = \lambda\int_a^b K(x,t)H_k(t)dt, k = 0,1,2,\ldots \end{cases} \tag{6}$$

Using the recursive scheme (6), the n-term approximation series solution can be obtained as follows:

$$\varphi_n(x) = \sum_{j=0}^{n} y_j(x) \tag{7}$$

Numerical Implementations

In this section, we will apply the Adomian decomposition method

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to compute a numerical solution for non-linear integral equation of the Fredholm type. Then we will compare between the results which we obtain by the numerical solution technique and the results of the exact solution. To illustrate this, we consider the following example:

Example 1

Consider the following nonlinear Fredholm integral equation of the second kind

$$y(x)=\frac{7}{8}x+\frac{1}{2}\int_0^1 xty^2(t)\mathrm{d}t \qquad (8)$$

where the exact solution of the equation is $y(x)=x$. In the following, we will compute Adomian polynomials for the nonlinear terms $y^2(t)$ that arises in nonlinear integral equation.

For $k=0$, equation (4) becomes

$$H_0=\frac{1}{0!}\frac{d^0}{dp^0}\left[G\left(\sum_{i=0}^{\infty}p^i y_i\right)\right]_{p=0}$$

$$=\left[G\left(p^0y_0+p^1y_1+p^2y_2+\ldots\right)\right]_{p=0}$$

The Adomian polynomials for $G(y)=y^2$ are given by

$$=(y_0+py_1+p^2y_2+\ldots)^2\,|_{(p=0)}$$

$$\therefore H_0=y_0^{\,2}$$

By using the MATHEMATICA software, the next few terms, we have

$$H_1=2y_0y_1$$
$$H_2=2y_0y_2+y_1^{\,2}$$
$$H_3=2(y_0y_3+2y_1y_1)$$
$$H_4=2(y_1y_3+2y_0y_4)+y_2^{\,2}$$
$$H_5=2(y_2y_3+2y_1y_4+y_0y_5)$$
$$H_6=2(y_2y_4+y_1y_5+y_0y_6)+y_3^{\,2}$$
$$H_7=2(y_3y_4+y_2y_5+y_1y_6+y_0y_7)$$
$$H_8=2(y_3y_5+y_2y_6+y_1y_7+y_0y_8)+y_4^{\,2}$$
$$H_9=2(y_4y_5+y_3y_6+y_2y_7+y_1y_8)$$

and so on.

Applying the technique as stated above in equation (6), we have

$$p^0: y_0(x)=\frac{7}{8}x$$

$$p^1: y_1(x)=\frac{1}{2}\int_0^1 xt\,H_0(t)\,dt=\frac{x}{2}\int_0^1 t\,y_0^{\,2}(t)dt=\frac{x}{2}\int_0^1 t\left(\frac{7}{8}t\right)^2dt=\frac{49}{512}x=\frac{7^2}{8^3}x$$

$$p^2: y_2(x)=\frac{1}{2}\int_0^1 xt\,H_1(t)\,dt=\frac{x}{2}\int_0^1 2ty_0(t)y_1(t)dt=\frac{x}{2}\int_0^1 2t\left(\frac{7t}{8}\right)\left(\frac{49t}{512}\right)dt=\frac{7^3}{4\times 8^4}x$$

In a similar manner, we stop the iteration at the tenth step. Therefore we can write

$$y(x)=\left(\frac{7}{8}+\frac{7^2}{8^3}+\frac{7^3}{4\times 8^4}+\frac{5\times 7^4}{8^7}+\frac{7^6}{4\times 8^8}+\frac{3.7^7}{4\times 8^{10}}+\frac{1811\times 7^6}{2\times 8^{14}}\right.$$
$$\left.+\frac{5\times 283\times 7^8}{4\times 8^{15}}+\frac{5\times 3673\times 7^9}{2\times 8^{18}}+\frac{5\times 798101\times 7^{10}}{2\times 8^{22}}x\right)\approx 0.999947x$$

The table under shows the approximate solutions obtained by applying the Adomian Decomposition method giving to the value of x, which is in the interval [0-1] (Table 1 and Figure 1).

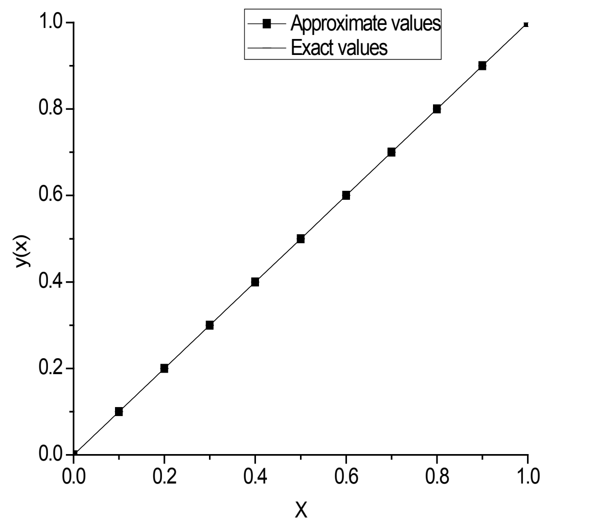

Figure 1: Numerical and exact solutions to the integral equation (8).

Nodes (x)	Exact solutions	Approximate solutions	Absolute error
0	0	0	0
0.10	0.100000	0.0999947	0.0000053
0.20	0.200000	0.1999890	0.0000110
0.30	0.300000	0.2999840	0.0000160
0.40	0.400000	0.3999790	0.0000210
0.50	0.500000	0.4999740	0.0000260
0.60	0.600000	0.5999680	0.0000320
0.70	0.700000	0.6999630	0.0000370
0.80	0.800000	0.7999580	0.0000420
0.90	0.900000	0.8999520	0.0000480
1.0	1.000000	0.9999470	0.0000530

Table 1: Numerical and exact solutions to the integral equation (8).

Example 2

Consider the following nonlinear Fredholm integral equation of the second kind

$$y(x)=3+0.6625x+\frac{x}{20}\int_0^1 ty^2(t)dt \qquad (9)$$

Applying above procedure, we have

$$p^0: y_0(x)=3+0.6625x$$

$$p^1: y_1(x)=\frac{x}{20}\int_0^1 ty_0^{\,2}(t)dt=0.296736x$$

In a similar manner, we stop the iteration at the tenth step. Therefore we can write

$$y(x)=3+(0.6625+0.296736+0.0345883+0.0356889+0.00441658+0.000794517+$$
$$0.000156235+0.0000438455+0.0000069959+0.000002199)\approx 3+1.03493x$$

The exact solution of the equation is $3+x$. The table below shows the approximate solutions obtained by applying the Adomian decomposition method according to the value of x, which is confined between zero and one. We compared these results with the results which were obtained by the exact solution (Table 2 and Figure 2).

Example 3

Consider the following nonlinear Fredholm integral equation

$$y(x)=\sin(\pi x)+\frac{1}{5}\int_0^1 \cos(\pi x)\sin(\pi t)\left(y(t)\right)^3 dt,\quad x\in[0,1] \qquad (10)$$

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Nodes (*x*)	Exact solutions	Approximate solutions	Absolute error
0	3	3	0
0.10	3.100000	3.10349	0.00349
0.20	3.200000	3.20699	0.00699
0.30	3.300000	3.31048	0.01048
0.40	3.400000	3.41397	0.01397
0.50	3.500000	3.51747	0.01747
0.60	3.600000	3.62096	0.02096
0.70	3.700000	3.72445	0.02445
0.80	3.800000	3.82794	0.02794
0.90	3.900000	3.93144	0.03144
1.0	4.000000	4.03493	0.03493

Table 2: Numerical and exact solutions to the integral equation (9).

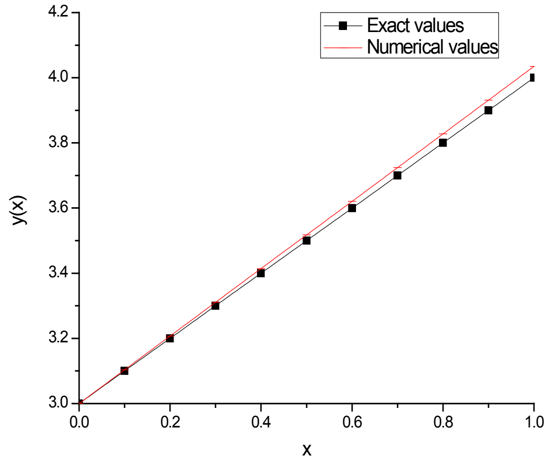

Figure 2: Numerical and exact solutions to the integral equation (9).

Nodes	Exact values	Approximate values	Absolute error
0.00	0.07542668890493687	0.07542668888687896	1.8057902×10^{-11}
0.05	0.23093252624133365	0.23093252622349808	1.7835566×10^{-11}
0.10	0.38075203836055493	0.3807520383433809	1.7174039×10^{-11}
0.15	0.5211961716517719	0.5211961716356822	1.6089685×10^{-11}
0.20	0.6488067254459994	0.6488067254313902	1.4609202×10^{-11}
0.25	0.7604415043936764	0.7604415043809076	1.2768786×10^{-11}
0.30	0.8533516897425216	0.8533516897319074	1.0614176×10^{-11}
0.35	0.9252495243780194	0.9252495243698213	8.198108×10^{-12}
0.40	0.9743646449962113	0.9743646449906311	5.580202×10^{-12}
0.45	0.9994876743237375	0.9994876743209126	2.824962×10^{-12}
0.50	1	1	0
0.55	0.9758890068665379	0.9758890068693629	2.824962×10^{-12}
0.60	0.9277483875940958	0.927748387599676	5.580202×10^{-12}
0.65	0.8567635239987162	0.8567635240069144	8.198108×10^{-12}
0.70	0.7646822990073733	0.7646822990179875	1.0614176×10^{-11}
0.75	0.6537720579794186	0.6537720579921874	1.2768786×10^{-11}
0.80	0.526763779138947	0.5267637791535562	1.4609202×10^{-11}
0.85	0.38678482782732176	0.38678482784341145	1.6089685×10^{-11}
0.90	0.23728195038933994	0.237281950406514	1.7174067×10^{-11}
0.95	0.08193640383912822	0.08193640385696378	1.7835566×10^{-11}
1.00	-0.07542668890493687	-0.07542668888687896	1.8057902×10^{-11}

Table 3: Numerical and exact solutions to the integral equation (10).

The exact solution of the equation (10) is $y(x)=\sin(\pi x)+\frac{1}{3}\left(20-\sqrt{391}\right)\cos(\pi x)$. In the following, we will calculate Adomian polynomials for the nonlinear terms $y^3(t)$ that arises in nonlinear integral equation.

For k=0, equation (4) becomes

$$H_0=\frac{1}{0!}\frac{d^0}{dp^0}\left[G\left(\sum_{i=0}^{\infty}p^i y_i\right)\right]_{p=0}$$

$$=\left[G\left(\sum_{i=0}^{\infty}p^i y_i\right)\right]_{p=0}$$

$$=\left[G\left(p^0 y_0+p^1 y_1+p^2 y_2+\ldots\right)\right]_{p=0}$$

The Adomian polynomials for $G(y)=y^3$ are given by

$$=\left(y_0+py_1+p^2y_2+\ldots\right)^3\Big|_{p=0}$$

$$\therefore H_0=y_0{}^3$$

By using the MATHEMATICA v9 software, the next few terms, we have

$$H_1=3y_0{}^2y_1$$

$$H_2=3(y_0y_1{}^2+y_0{}^2y_2)$$

$$H_3=y_1{}^3+6y_0y_1y_2+3y_0{}^2y_3)H_4=3(y_1{}^2y_2+y_0y_2{}^2+y_0{}^2y_4)+6y_0y_1y_3$$

$$H_5=3\left(y_1y_2{}^2+y_1{}^2y_3+y_0{}^2y_5\right)+6(y_0y_2y_3+y_0y_1y_4)$$

$$H_6=y_2{}^3+6\left(y_1y_2y_3+y_0y_2y_4+y_0y_1y_5\right)+3\left(y_0y_3{}^2+y_1{}^2y_4+y_0{}^2y_6\right)$$

$$H_7=3\left(y_2{}^2y_3+y_1y_3{}^2+y_1{}^2y_5+y_0{}^2y_7\right)+6(y_1y_2y_4+y_0y_3y_4+y_0y_2y_5+y_0y_1y_6)$$

$$H_8=3\left(y_2y_3{}^2+y_4y_2{}^2+y_4{}^2y_0+y_1{}^2y_6+y_0{}^2y_8\right)+6(y_1y_3y_4+y_1y_2y_5+y_0y_3y_5\quad+y_0y_2y_6+y_0y_1y_7$$

and so on.

Applying the procedure as stated above in equation (6), we have

$$p^0:y_0(x)=\sin(\pi x)$$

$$p^1:y_1(x)=\frac{\cos(\pi x)}{5}\int_0^1\sin(\pi t)H_0(t)\,dt=\frac{\cos(\pi x)}{5}\int_0^1\sin(\pi t)y_0{}^3(t)\,dt=\frac{3}{40}\cos(\pi x)$$

In a similar manner, we stop the iteration at the ninth step. Therefore we can write

$$y(x)=\sin(\pi x)+\left(\frac{3}{40}+0+\frac{27}{64000}+0+\frac{243}{51200000}+0+\frac{2187}{32768000000}+0+\frac{137781}{131072000000000}\right)\cos(\pi x)$$

$$=\sin(\pi x)+\frac{9886326965781}{131072000000000}\cos(\pi x)$$

The table under shows the approximate solutions obtained by applying the Adomian decomposition method according to the value of x, which is in the interval [0-1] (Table 3 and Figure 3).

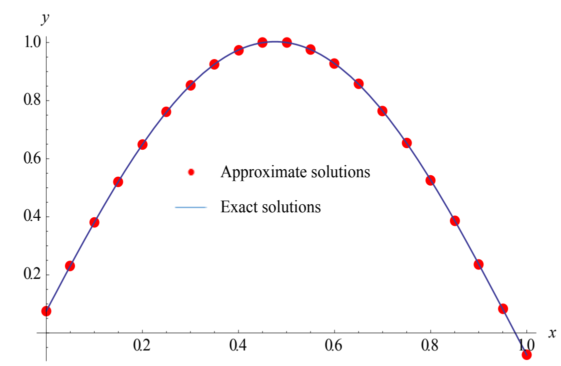

Figure 3: Numerical and exact solutions to the integral equation (10).

Conclusion

This paper presents a technique to find the result of a nonlinear Fredholm integral equation by Adomian decomposition method (ADM). The estimated solutions obtained by the ADM are compared with exact solutions. It can be concluded that the ADM is effective and accuracy of the numerical results demonstrations that the proposed method is well suited for the solution of such kind problems.

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