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Solitons in Mode-Locked Lasers

Theodoros P. Horikis*

Department of Mathematics, University of Ioannina, Ioannina 45110, Greece

Following the discovery of the Kerr-lens mode-locking, in recent years femtosecond (10–15second) solidstate lasers, such as those based on the Ti:Sapphire gain medium, have had a major impact in the field of ultrafast science. The enhanced performance of these lasers has led to their widespread use. Increasingly compact, user-friendly, reliable, and economical instruments have allowed femtosecond lasers to expand beyond research environments and find new applications such as in multi-user biology facilities, medical clinics, manufacturing environments, and even in mobile facilities and aircraft. A mode-locked (ML) fiber laser cavity with a passive polarizer is an example of a technologically and commercially promising ultrafast device. It can be expected that improved mathematical modeling of ultra-short pulse passively mode-locked laser devices will help speed their development. Mode-locked laser pulses can be generated actively by use of an external element or passively via the Kerr-lens mechanism. The latter produces shorter pulses and will be the focus here.

A *mode-locked* laser refers to the frequency domain description of how ultra-short pulses are generated by a laser system. The requirement that the electromagnetic field be unchanged after one round trip in the laser means that lasing only occurs for frequencies such that the cavity length is an integer number of wavelengths. If multiple modes lase at the same time, then a short pulse can be formed, but only if the modes are locked in phase, i.e., the laser is mode-locked. Passive mode-locking can be described in terms of a saturable absorber, i.e., higher intensity is less attenuated than lower intensity. Nonlinearity is important in ML lasers because of the large intensities produced. *Mode-locked* lasers and their ultra stable frequency combs have many potential uses in basic physics and technology and are becoming extremely important in many other areas of science ranging from enhanced material processing and measuring of the fundamental constants of nature to high-harmonic generation and optical clock technology.

Pulse propagation in a laser cavity is governed by the interplay of chromatic dispersion, self-phase modulation, saturable gain and filtering and intensity discrimination. In the anomalous regime many models have been used to describe these phenomena, including Ginzburg-Landau (GL) types and the so-called master-equation. The master-equation is a generalization of the classical nonlinear Schrodinger equation modified to contain gain, filtering and loss terms. Gain and filtering are saturated by energy (i.e. the time integral of the pulse power), while loss is represented by a cubic nonlinearity. If the pulse energy is taken to be constant the master-equation reduces to a GL type system. These equations have limited mode-locking capabilities and have been shown to exhibit a variety of solutions ranging from unstable, chaotic, to quasi-periodic and blow up.

We are interested in very particular solutions of the above equations termed solitons. A soliton is a form of solitary wave with unique properties: (a) it maintains its shape while it travels at constant speed and (b) it does not interact with other solitons while propagating. Two types of solitons exist: bright, namely solutions that decay at infinity and dark, solitons that tend to form on a constant background (tend to a complex constant at infinity).

In our research [1,2] we have been studying a power-energy saturation (PES) model, which, we find, naturally describes the locking and evolution of pulses in ML lasers that are operating in the soliton regime. For a pulse with amplitude $u(z,t)$, power $P(z,t)=|u|^2$, and energy $E(z)=\int_{-\infty}^{+\infty}|u|^2\mathrm{d}t$, which is propagating in the z direction, our normalized equation takes the form

$$i\frac{\partial u}{\partial z}+\frac{d(z)}{2}\frac{\partial^2 u}{\partial t^2}+n(z)|u|^2u=\frac{ig}{1+E/E_{sat}}u+\frac{iT}{1+E/E_{sat}}\frac{\partial^2 u}{\partial t}-\frac{il}{1+P/P_{sat}}u$$

where the constant parameters g, T, l, E_{sat}, P_{sat} are positive, while $d(z)$ is the dispersion and $n(z)$ the nonlinear (Kerr) coefficient. Usually one considers a two step dispersion map where $d(z)$ is taken to be a constant changing across different parts of the laser. Also, $n(z)=1$ inside the Ti:Sapphire crystal and $n(z)=0$ outside the crystal. The first term on the right hand side represents saturable gain, the second is nonlinear filtering and the third saturable loss.

Power saturation models also arise in other problems [3] in nonlinear optics and are important in the underlying theory. For example, in the study of the dynamics of localized lattice modes (solitons, vortices, etc) propagating in photorefractive nonlinear crystals. If the nonlinear term in these equations was simply a cubic nonlinearity, without saturation, two dimensional fundamental lattice solitons would be vulnerable to blow up singularity formation, which is not observed. Thus saturable terms are crucial in these problems.

In the *anomalous dispersive* ($d(z)$ is a positive constant) regime and if there is insufficient gain in the PES, pulses dissipate to zero [1,2]. On the other hand, remarkably, a distinguishing feature of this model is that even under large gain, pulses do not blow-up nor do they exhibit instabilities. On the contrary: when the gain is greater than some threshold value $g=g^*$, during the evolution, the pulse readjusts itself as it mode-locks into a stable localized mode, or soliton solution. Complicated evolution (chaotic, radiation, or strong growth) is not observed even when the perturbations cannot be considered small at the initial instant. The saturated (energy) gain and filtering, and saturated (power) loss, though crucial to the mode-locking mechanism, after evolution they are only found to be perturbative effects. The resulting modes are essentially the modes of the unperturbed system, i.e., hyperbolic secants.

Soliton pulses in *normally dispersive* ($d(z)$ is a negative constant)

***Corresponding author:** Theodoros P. Horikis, Department of Mathematics, University of Ioannina, Ioannina 45110, Greece, Tel: +30 26510 08268, Fax: +30 26510 08201, E-mail: horikis@uoi.gr

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mode-locked lasers can also be found [4] using the same PES equation. In this regime, pulses are wide and strongly chirped; unlike solitons their anomalous regime counterparts they are not well approximated by the unperturbed equations without gain, filtering and loss and thus perturbation theory is used to derive a set of uncoupled equations for the amplitude and the phase of the soliton pulse.

The latest development is that dark solitons can also been found [5,6] under the same model appropriately modified to take into account the fact that these soliton do not have decaying tails at infinity. It was found that general initial conditions evolve (mode-lock) into dark solitons under appropriate requirements also met in experimental observations.

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