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Slope Effects on the Pressure Head Profile Patterns of Sprinkler Irrigation Laterals, I. Theoretical Analysis

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Abstract

The pressure head profile of an irrigation lateral is a function of the hydraulic and geometric characteristics of the lateral and its slope. Slope effects on pressure variability along a lateral are typically limited. However, for a lateral with a spatially invariant parameter set well-defined relationships can be discerned between lateral slopes and the spatial patterns of the lateral pressure profiles. Thus, a comprehensive understanding of these relationships can be useful in the evaluation of lateral hydraulic simulation models and design recommendations. Existing studies have examined slope effects on the locations of pressure head extrema, along a lateral, in the context of hydraulic design of laterals. This paper presents a comprehensive analysis of slope effects on lateral pressure head profile patterns. The monotonic properties, with respect to distance from inlet, of lateral slope and friction slope profiles coupled with the pressure slope equation are used here to determine the full range of variation of lateral pressure head profile patterns as a function of lateral slope. Overall, the analysis show that the pressure profile patterns of a lateral, and possibly the locations of the pressure extreme along the lateral, can be determined by comparing the negative of the lateral slope with the corresponding friction slope at the upstream and/or downstream ends of the profile. The results also show that the full range of variation of the pressure profile patterns of a lateral, as affected by slope, consists of three distinct categories. These include a profile pattern that is increasing or decreasing, with distance from the inlet, over the entire length of the lateral and one that combines both trends within the length of the lateral. The relationships between lateral slopes and pressure profile patterns, deduced here through theoretical analysis, will be evaluated in a companion paper based on simulations.

Keywords: Continuous function; Step function; Friction slope; Lateral slope; Stationary point; Pressure slope

Notations

The following notations are used in this paper:

$h(x,\psi_i)$=Pressure head profile (i.e., function that relates pressure head with distance from lateral inlet) for a lateral slope of ψ_i;

$h'(x,\psi_i)$=Pressure slope profile (i.e., function that relates the slope of the pressure head profile with distance from lateral inlet) for a lateral slope of ψ_i;

$H_f'(x,\psi_i)$=Friction slope profile (i.e., function that relates the slope of the friction head loss profile with distance from lateral inlet) for a lateral slope of ψ_i;

h=Pressure head profile;

h'=Pressure slope profile;

H_f'=Friction slope profile;

x=Distance from lateral inlet;

ψ=Lateral slope.

Introduction

Hydraulic modeling of irrigation laterals has been the subject of various studies [1-6]. Earlier publications have also explored the application of mathematical models in the hydraulic design of sprinkler and drip irrigation laterals [7-12]. However, the focus of the current study is on the analysis of the spatial patterns of the pressure head profiles, along the lateral, of a solid-set or set-move sprinkler irrigation system.

The pressure head profile of an irrigation lateral is the main determinant of the spatial distribution of sprinkler discharge along the lateral. Lateral pressure head profile is a function of the hydraulic and geometric characteristics of the lateral and its slope. In most modern farming systems where lateral slopes are typically small, the effect of slope on pressure head variability along a lateral is in practice limited. However, for a lateral with a given hydraulic and geometric characteristic, it can be shown that there exists a well-defined and clearly discernible set of relationships between lateral slopes and the spatial patterns (i.e., the monotonic properties with respect to distance from inlet) of the corresponding pressure head profiles. Thus, a comprehensive understanding of these relationships can be useful in the verification of lateral hydraulic simulation models and design recommendations.

The effect of slope on pressure head extrema of irrigation laterals has generally been examined in the context of hydraulic design of laterals. Wu et al. [13] used the ratio of elevation differential and friction head loss along the lateral as criteria to define distinct categories of drip irrigation lateral pressure head profile patterns and to determine maximum pressure head differential along the lateral for design purposes. Keller and Bleisner [8] discussed slope effects on the locations of the minimum and maximum pressure heads along an irrigation lateral and on the average lateral pressure head. Scaloppi and

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Allen and Yildirim [3,12] proposed equations for calculating pressure head extrema and their locations along a lateral, assuming continuous and uniform and/or continuous and nonuniform spatial distributions of outflow discharge. Furthermore, Martin et al. [14] proposed a set of criteria, based on expressions that are functions of lateral slope and average friction slope, for determining the locations of pressure head extrema along the lateral and average pressure heads. In these studies, the friction head loss profile along a lateral is approximated with explicit functions that relate head loss directly with distance from lateral inlet. Furthermore, the effect of velocity head on pressure head profile is assumed either negligible or computed following a similar approach as that used to compute the friction head loss profile.

The objective of the study presented here is to conduct a comprehensive analysis of the effect of slope on the spatial patterns of lateral pressure head profiles, given the hydraulic and geometric characteristics of the lateral. Characterization of slope effects on pressure head profile patterns, and the locations of pressure head extrema along a lateral, is conducted using the pressure head profile equation of an irrigation lateral operated under steady flow condition. The monotonic properties, with respect to distance from inlet, of lateral slope and friction slope profiles coupled with the pressure slope equation are used here to determine the full range of variation of lateral pressure head profile patterns as a function of lateral slope. The analysis show that the spatial pattern of the pressure head profile of an irrigation lateral can be determined by comparing the negative of the lateral slope with the friction slopes at the upstream and/or distal ends of the profile. Furthermore, the results of the analysis show that the full range of variation of the spatial patterns of the pressure head profiles of a lateral, as affected by slope, consists of three distinct categories. These include a profile pattern that is increasing or decreasing with distance from the lateral inlet over the entire length of the lateral and one that combines both trends within the length of the lateral. The relationships between lateral slopes and pressure head profile patterns, derived here through theoretical analysis, will be evaluated in the companion paper [15] based on results of hydraulic simulations.

Lateral Pressure Head Profile and Related Equations

A schematic of the total head and piezometric head and their components along an irrigation lateral are depicted in Figure 1. The pressure head profile of a lateral is a function of the hydraulic (total head at the inlet, pipe surface roughness, and sprinkler hydraulic characteristics), geometric (pipe diameter, lateral length, and sprinkler spacing), and elevation profile of the lateral. Note that description and analysis of lateral pressure head profiles here assume that flow is steady; the hydraulic, geometric, and slope characteristics of a lateral are invariant with distance from lateral inlet; local head losses along the lateral are negligible; and the lateral parameter set constitutes a feasible hydraulic scenario. Note that in the current analysis a lateral hydraulic, geometric, and slope parameter combination is considered to constitute a feasible hydraulic scenario if flow takes place through a lateral defined as such.

For a lateral that satisfies the preceding requirements, it can be observed from Figure 1 that the energy balance equation between the lateral inlet and any point located at distance x from the lateral inlet can be expressed as

$$H_0 = H(x) + H_f(x) \tag{1}$$

In eqn. (1), H_0 is the total head at the inlet [L] and is considered constant; $H(x)$ is the total head at distance x from lateral inlet [L];

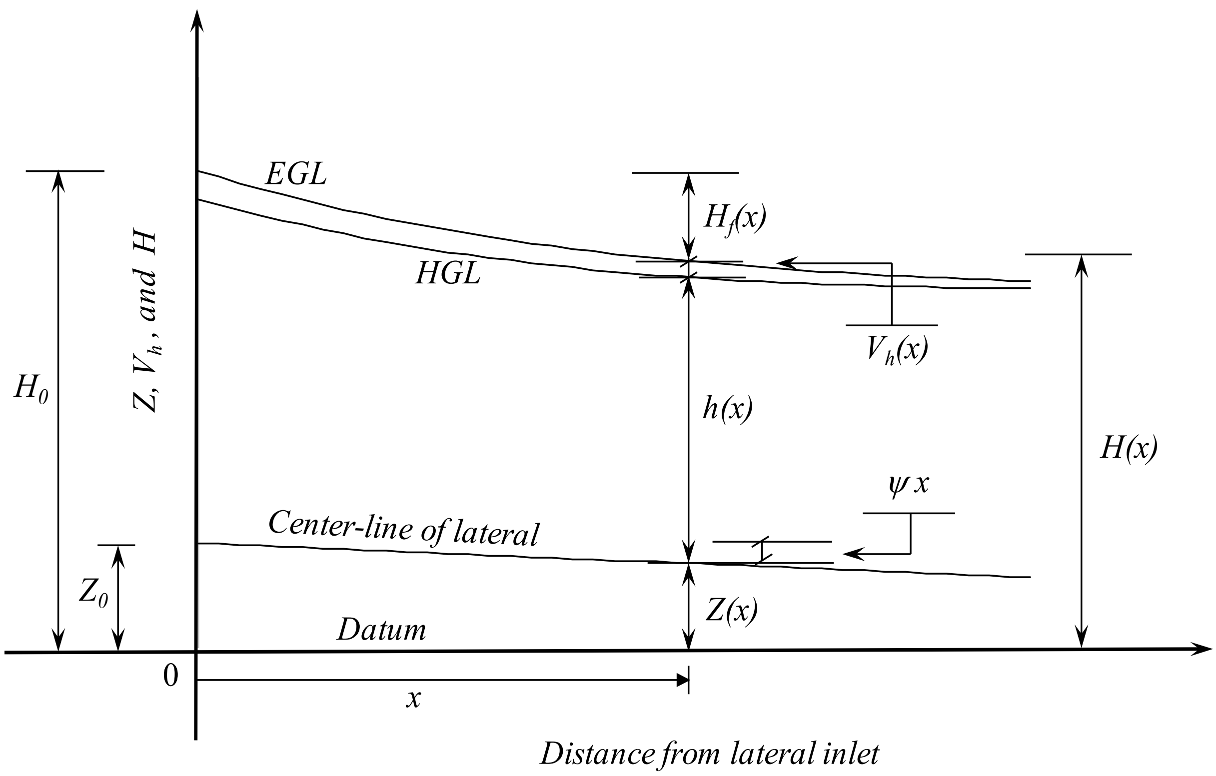

Figure 1: Schematic of components of specific energy of water for lateral pressure head profile calculation (EGL is energy grade line and HGL is hydraulic grade line).

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and $H_f(x)$ is the friction head loss between the lateral inlet and a point located at distance x from the inlet [L]. Expressing $H(x)$, in eqn. (1), in terms of its components yields

$$H_0 = Z(x) + h(x) + V_h(x) + H_f(x) \tag{2}$$

In eqn. (2), $Z(x)$ is lateral elevation at distance x from the lateral inlet [L]; $h(x)$ is lateral pressure head at distance x along the lateral [L]; and $V_h(x)$ is velocity head at distance x from the inlet [L].

Now, for a lateral with a constant slope, ψ_i, it can be readily shown that the pressure head profile along a lateral can be expressed as

$$h(x) = \left(H_0 - Z_0\right) - \left(H_f(x) + V_h(x) + \psi_i x\right) \tag{3}$$

where Z_0 is elevation of the lateral inlet [L], which is constant, and ψ_i is the constant lateral slope [-]. Eqn. (3) states that the pressure head at any given point along a lateral can be given as the difference between the sum of the velocity and pressure heads at the lateral inlet, which is a constant, and the sum of the friction head loss, the velocity head, and elevation differential, which varies with distance.

The interest here is to evaluate the effect of changes in the slope of a lateral on the spatial patterns of the corresponding pressure head profiles and hence the locations of pressure head extrema along the lateral. Thus, it is essential that the lateral slope is explicitly factored in the expression for lateral pressure head profile. Accordingly, a convenient expression for pressure head profile along a lateral, accounting specifically for lateral slope effects, can be obtained by recasting eqn. (3) in the following form

$$h(x,\psi_i) = H_0 - Z_0 - H_f(x,\psi_i) - V_h(x,\psi_i) - \psi_i x \tag{4}$$

Where ψ_i is the lateral slope used in computing lateral pressure head profile. Note that eqn. (4) is an exact expression for lateral pressure head profiles obtained based the application of the principle of energy conservation to steady flow condition in irrigation laterals. It can be used to define a range of pressure head curves, $h(x,\psi_i)$, as lateral slope, ψ_i, is varied within a feasible interval and to analyze slope effects on pressure head profile patterns. Note that it is implicitly assumed here that the friction head loss and velocity head terms in eqn. (4) are evaluated through hydraulic simulations in which continuity and energy balance requirements are satisfied.

For a given lateral slope, ψ_i, the spatial pattern of the lateral pressure head profile can be conveniently evaluated by studying the derivative of the pressure head profile with respect to distance from inlet. A close look at eqn. (4) reveals that, because of step changes in lateral discharge across sprinkler locations, $H_f(x,\psi_i)$ is a piece-wise linear function and $V_h(x,\psi_i)$ is a step function of distance from lateral inlet. By contrast, the lateral elevation differential, $\psi_i x$, is a (continuous) linear function of distance from lateral inlet. It then follows that eqn. (4) is a piece-wise differentiable function of distance and its derivative is a step function of distance from lateral inlet.

$$h'(x,\psi_i) = -H_f'(x,\psi_i) - \psi_i \tag{5}$$

Eqn. (5) states that at any given point along the lateral, excluding a sprinkler location, the slope of the pressure head profile, $h'(x,\psi_i)$, is equal to the negative of the algebraic sum of the slope of the friction head loss profile, $H_f'(x,\psi_i)$, and the lateral slope, ψ_i. In eqn. (5), both $H_f'(x,\psi_i)$ and $h'(x,\psi_i)$ are constant over a lateral pipe segment but vary from one lateral segment to another. A lateral segment is defined here as a pipe section spanning two consecutive nodes along the lateral and the term node is used to refer to the lateral inlet or a sprinkler location.

Note that eqn. (5) has the same form as an equation presented by Wu [2] for drip laterals. However, Wu's equation was developed based on the assumption that velocity head is negligible and with the presumption that friction head loss is a continuous function of distance from lateral inlet.

For simplicity, in subsequent discussion $H_f'(x,\psi_i)$ and $h'(x,\psi_i)$ will simply be referred to as the friction slope and pressure slope profiles, respectively. However, it needs to be observed here that the usage of the term friction slope in this paper is slightly different from the conventional use of the term, which refers to the slope of the energy line and hence equal to the negative of $H_f'(x,\psi_i)$.

Now, considering a lateral with a specified hydraulic, geometric, and slope parameter set, at any given point along the lateral, excluding sprinkler locations, the friction slope, $H_f'(x,\psi_i)$, can be calculated with

$$H_f'(x,\psi_i) = \frac{H_f(x_q + \Delta x,\psi_i) - H_f(x_q,\psi_i)}{\Delta x} \quad for \quad x_q < x < (x_q + \Delta x) \tag{6}$$

In eqn. (6), x_q is the distance of the qth node from the lateral inlet (q=0 at the lateral inlet and increases along the lateral), Δx is the length of the lateral pipe segment (sprinkler spacing), and $H_f(x_q,\psi_i)$ is the friction head loss in a section of the lateral that extends over a distance of x_q from the inlet.

The Spatial Properties of Friction Slope and Lateral Slope

As can be noted from eqn. (5), the pressure slope profile of a lateral is a function of the interactive effects of the lateral slope, ψ_i, and the corresponding friction slope profile, $H_f'(x,\psi_i)$. Thus, the monotonic properties of the ψ_i and $H_f'(x,\psi_i)$ profiles, with respect to distance from lateral inlet, are key inputs in developing simple criteria that can be used to characterize lateral pressure head profile patterns as a function of slope.

Friction head loss along an irrigation lateral is a monotonic increasing function of distance from the lateral inlet regardless of the lateral parameter set. Thus, it can be observed from eqn. (6) that H_f' is positive along the lateral (i.e., irrespective of the magnitude and algebraic sign of the lateral slope, $H_f'(x,\psi_i)>0$ over the entire length of a lateral). However, because of the decreasing discharge, the friction slope, $H_f'(x,\psi_i)$, of a lateral with a spatially invariant parameter set is a monotonic decreasing function of distance from the lateral inlet. Thus, the maximum and minimum of the friction slope profile occur at the inlet (x=0) and distal ends (x=l) of the lateral, respectively. In other words, for a lateral with a given ψ_i, $H_f'(0,\psi_i)$ and $H_f'(l,\psi_i)$ constitute the upper and lower limits, respectively, of the corresponding $H_f'(x,\psi_i)$ profile. By comparison, the slope of a lateral, ψ_i, is considered here invariant with distance from the lateral inlet and it can take a negative, $\psi_i<0$, or a positive, $\psi_i>0$, value. Thus, the slope profile of such a lateral, depicted in a graph of lateral slope versus distance from lateral inlet, would be a horizontal line. These observations will shortly be used to characterize lateral pressure head profile patterns as a function of slope. Before that, however, simulated results will be presented to graphically illustrate the spatial properties of these profiles.

Simulated examples of friction slope, H_f', and pressure slope, h', profiles for three different irrigation laterals covering a wide range of hydraulic, geometric, and slope characteristics are presented in Figure 2. Figures 2a-2c show that for each data set, the horizontal -ψ_i profiles are superimposed on the respective -H_f' and h' profiles. Note that to maintain consistency with eqn. (5) where h' is related to -H_f' and -ψ_i, the profiles of -H_f' and -ψ_i are depicted in Figure 2 instead of the H_f' and ψ_i profiles. Evidently, one should not have a problem in deducing

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the spatial patterns of the $H_f{}'$ and ψ_i profiles directly from those of the $-H_f{}'$ and $-\psi_i$ profiles. The h' and $H_f{}'$ profiles were calculated with eqns. (5) and (6), respectively, based on friction head loss profiles computed with a lateral hydraulic model developed by Zerihun and Sanchez [6]. The input data sets used in the simulation are presented in Table 1. The results depicted in Figures 2a-2c highlight the fact that h' and $H_f{}'$ are step functions of distance. Furthermore, it can be observed that the simulated $H_f{}'$ profiles are consistent with the preceding description of the general spatial pattern of an $H_f{}'$ profile (i.e., $H_f{}'$ is a monotonic decreasing function of distance from lateral inlet).

Note that for convenience, subsequent discussions would presume that the $H_f{}'$ and h' profiles are approximated by continuous functions. In the continuous approximation, the friction slope and pressure slope values for each lateral pipe segment are represented by a point at the upstream end node of the lateral segment. Evidently, with this representation of the $H_f{}'$ and h' profiles the conceptual distinction, noted above, between a sprinkler location and the other points along the lateral will be ignored in the interest of convenience and simplicity of presentation. Furthermore, with the continuous approximation the profile length would be shorter than the actual length of the lateral by sprinkler spacing. However, it should be noted that the $H_f{}'$ and h' values at the distal ends of each of the continuous profiles represent the conditions over the downstream end segment of the lateral.

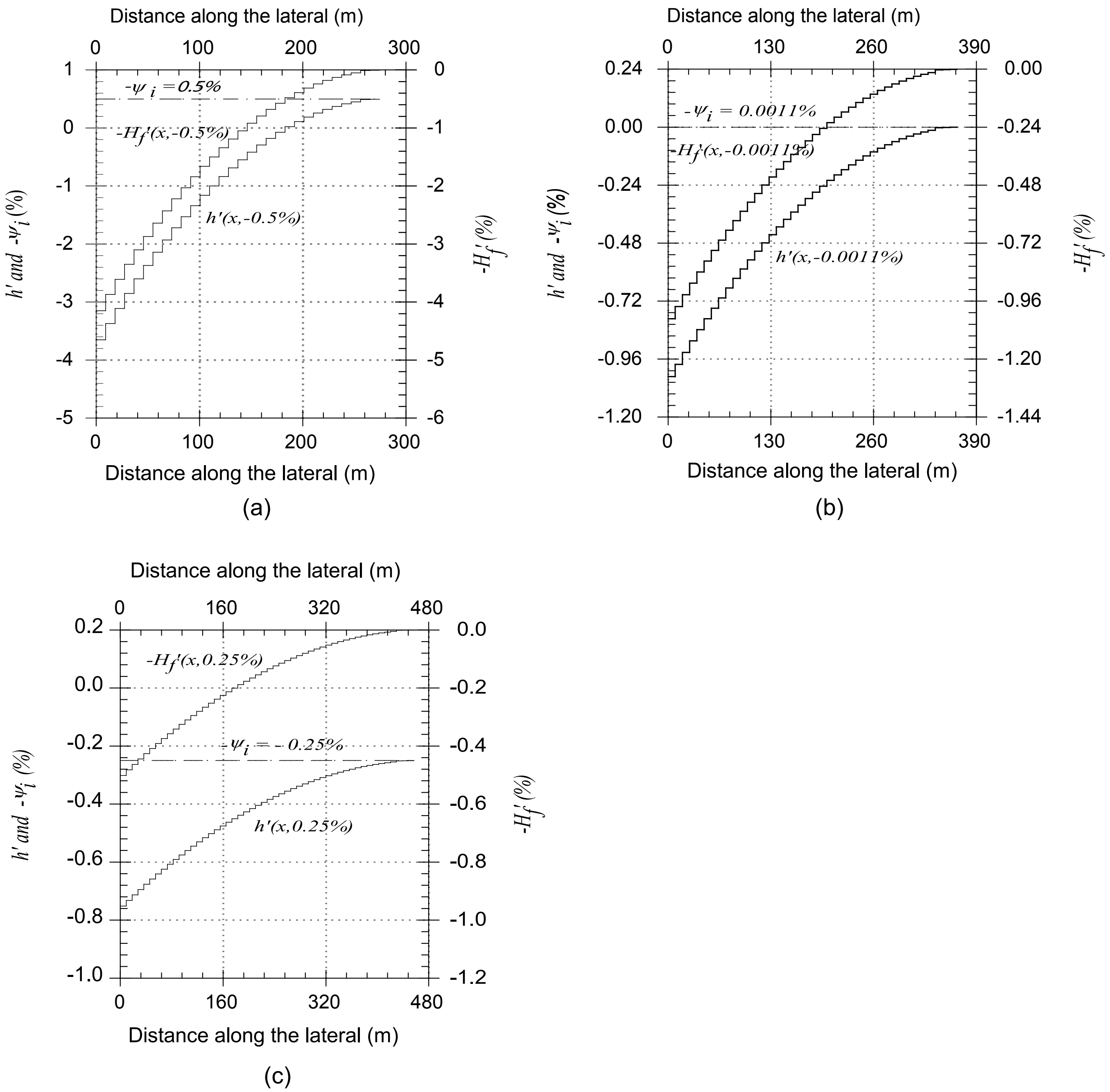

Figure 2: Lateral slope, ψ_i, and the corresponding simulated friction slope, $H_f{}'$, and pressure slope, h', profiles for (a) data set 1, (b) data set 2, and (c) data set 3.

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Lateral variables/parameters	Units	Data sets		
		1	2	3
Sprinkler spacing	m	9.14	9.14	9.14
Coefficient of sprinkler pressure head-discharge function, r,	(L/s) 1/m$^{\lambda}$	0.0125	0.0185	0.0258
Exponent of sprinkler pressure head-discharge function, l,	-	0.52	0.515	0.502
Lateral diameter	mm	50.8	76.2	101.6
Lateral length	m	274.2	365.6	457
Lateral slope	%	-0.5	-0.0011	0.25
Lateral pipe absolute roughness	mm	0.508	0.254	0.127
Constant total head at the lateral inlet	m	55	45	35
Elevations at lateral inlet	m	25	25	17

Table 1: Data used in lateral hydraulic simulations.

Characterization of the Spatial Patterns of Lateral Pressure Head Profiles as a Function of Slope

Eqn. (5) coupled with the monotonic properties, with respect to distance from inlet, of lateral slope, ψ_i, and friction slope, $H_f'(x,\psi_i)$, profiles will now be used to develop a set of criteria for characterizing the full range of variation of lateral pressure head profile patterns as affected by slope. Note that the criteria used here for the characterization of lateral pressure head profile patterns have some similarities with those presented by Martin et al. [14] for determination of pressure head extrema and their locations along the lateral.

It is to be recalled that the development here considers a lateral, with spatially invariant parameters and an arbitrarily set lateral slope (ψ_i), operating under a feasible hydraulic scenario. The discussion will be presented in two sections. First, a lateral with an elevation profile that is level or that increases at a constant rate with distance from the inlet ($\psi_i \geq 0$) will be considered. Followed by a lateral with an elevation profile that decreases at a constant rate along the lateral ($\psi_i < 0$).

Scenario i: A lateral with an elevation profile that is level or that increases at a constant rate with distance from the inlet ($\psi_i \geq 0$).

Noting that $H_f'(x,\psi_i) > 0$ regardless of the magnitude and algebraic sign of the lateral slope, it can be observed from eqn. (5) that for $\psi_i \geq 0$

$$h'(x,\psi_i) < 0 \quad for\ x \in [0,l] \tag{7}$$

Eqn. (7) states that for an irrigation lateral that is level or runs uphill on a constant slope, ψ_i, then the corresponding lateral pressure head profile, $h(x,\psi_i)$, should be a decreasing function of distance over the entire length of the lateral, regardless of the length of the lateral. Thus, the maximum and minimum pressure heads occur at the upstream and downstream ends of the lateral, respectively. Note that this is consistent with general physical reasoning.

Scenario ii: A lateral with an elevation profile that decreases at a constant rate with distance from the inlet ($\psi_i < 0$).

As noted earlier, given a spatially invariant ψ_i, the corresponding $H_f'(x,\psi_i)$ profile decreases monotonically with distance from the lateral inlet between a maximum of $H_f'(0,\psi_i)$ at the inlet and a minimum of $H_f'(l,\psi_i)$ at the distal end of the lateral. It can thus be observed from eqn. (5) that, in the interval ψ_i <0, the algebraic sign of h' (and hence the spatial pattern of h) depends on the magnitude of $-\psi_i$ relative to those of $H_f'(0,\psi_i)$ and $H_f'(l,\psi_i)$. The implication is that based on comparisons of $-\psi_i$ with the corresponding $H_f'(0,\psi_i)$ and/or $H_f'(l,\psi_i)$, a range of lateral pressure head profile patterns can be discerned.

ii.a: The lateral slope, ψ_i, is such that $-\psi_i > H_f'(0,\psi_i)$: For a lateral slope, ψ_i, that satisfies the inequality $-\psi_i > H_f'(0,\psi_i)$, it can be observed from eqn. (5) that

$$h'(x,\psi_i) > 0 \quad for\ x \in [0,l] \tag{8}$$

Eqn. (8) states that if a lateral runs downhill on a constant slope, ψ_i, and $-\psi_i$ exceeds the maximum of the friction slope profile, $H_f'(0,\psi_i)$, then the corresponding lateral pressure head profile, $h(x,\psi_i)$, should be an increasing function of distance over the entire length of the lateral, irrespective of the lateral length. Thus, the minimum and maximum pressure heads are at the inlet and distal ends of the lateral, respectively.

ii.b: The lateral slope, ψ_i, is such that $-\psi_i = H_f'(0,\psi_i)$: For a lateral slope, ψ_i, that satisfies the relationship $-\psi_i = H_f'(0,\psi_i)$, it can be observed from eqn. (5) that

$$\left.\begin{array}{l} h'(x,\psi_i) = 0 \quad at\ x = 0 \\ and \\ h'(x,\psi_i) > 0 \quad for\ x \in (0,l] \end{array}\right\} \tag{9}$$

Eqn. (9) shows that if an irrigation lateral runs downhill on a constant slope, ψ_i, and $-\psi_i$ equals the maximum of the friction slope profile, $H_f'(0,\psi_i)$, the corresponding lateral pressure head profile should be an increasing function of distance from the lateral inlet, regardless of the length of the lateral. Thus, the minimum and maximum pressure heads are located at the inlet and distal ends of the lateral, respectively.

Note that the pressure head profiles described by the current scenario, eqn. (9), and scenario ii.a, eqn. (8), have the same monotonic property with respect to distance. However, there is a difference in the pressure head profile described by eqn. (9) and those described by eqn. (8). For the pressure head profiles described by eqn. (8), $h'(0,\psi_i) > 0$. By comparison, for the $h(x,\psi_i)$ profile described by eqn. (9), $h'(0,\psi_i) = 0$. Evidently, this implies that under scenario iib the lateral inlet represents a stationary point on the corresponding pressure head profile. It will be shown in the companion paper that the lateral slope corresponding to the pressure profile, $h(x,\psi_i)$, described by eqn. (9) represents a special constant with some practical significance.

ii.c: The lateral slope, ψ_i, is such that $H_f'(l,\psi_i) < -\psi_i < H_f'(0,\psi_i)$: For a lateral slope, ψ_i, that satisfies the relationship $H_f'(l,\psi_i) < -\psi_i < H_f'(0,\psi_i)$, it can be observed from the monotonic properties of $H_f'(x,\psi_i)$ and ψ_i that the friction slope (which exceeds $-\psi_i$ at the lateral inlet) would steadily decrease as distance from lateral inlet increases and becomes equal to $-\psi_i$ at some point along the lateral between the inlet and distal ends and thereafter $-\psi_i$ will exceed the friction slope at all points downstream. In other words, for ψ_i that satisfies the inequality $H_f'(l,\psi_i) < -\psi_i < H_f'(0,\psi_i)$, the following holds

$$\left.\begin{array}{ll} -\psi_i < H_f'(x,\psi_i) & for\ [0,x_{\min}), \\ -\psi_i = H_f'(x,\psi_i) & for\ x = x_{\min},\quad and \\ -\psi_i > H_f'(x,\psi_i) & for\ (x_{\min},l] \end{array}\right\} \tag{10}$$

Thus, based on eqns. (5) and (10) it can be shown that

$$\left.\begin{array}{ll} h'(x,\psi_i) < 0 & for\ x \in [0,x_{\min}), \\ h'(x,\psi_i) = 0 & at\ x = x_{\min},\quad and \\ h'(x,\psi_i) > 0 & for\ x \in (x_{\min},l] \end{array}\right\} \tag{11}$$

In eqns. (10) and (11), $x_{\min}$ is the distance of the minimum pressure

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head point, from the lateral inlet, for a lateral slope of ψ_i. Eqn. (11) states that for a lateral slope, ψ_i, that satisfies the inequality $H_f'(l,\psi_i) < -\psi_i < H_f'(0,\psi_i)$, the lateral pressure head profile, $h(x,\psi_i)$, decreases with distance from lateral inlet, reaches a minimum at $x = x_{min}$, and then increases with distance from the inlet over the interval $x_{min} < x \leq l$. It needs to be pointed out here that in this and the companion manuscript x_{min} will be specifically used to refer only to the distance from the lateral inlet of a minimum pressure head point at which $h'(x_{min},\psi_i)= 0$. Thus, the point $(x_{min}, h(x_{min},\psi_i))$ represents a stationary point on the pressure head profile curve. Considering that $h(x,\psi_i)$ is function of the hydraulic, geometric, and slope characteristics of the lateral, it can be reasoned that x_{min} is a function of the lateral parameter set.

In practical terms, the preceding implies that if a lateral with a given hydraulic and geometric parameter set runs downhill on a constant slope of ψ_i and $-\psi_i$ satisfies the relationship $H_f'(l,\psi_i)<-\psi_i< H_f'(0,\psi_i)$, then the corresponding pressure head profile will have two segments: an upstream segment that is decreasing and a downstream segment that is increasing with distance from lateral inlet. The results also show that the minimum pressure head point is the point at which the rate of decrease in h due to friction head loss, $-H_f'$, equals the rate of increase in h due to elevation differential, $-\psi_i$. Thus, upstream of the minimum pressure head point the rate of decrease in h due to friction is larger than the rate of increase in h due to elevation differential. Observe that the converse is true over the lateral section downstream of the minimum pressure head point. Note that this result is consistent with the intuitive observation by Keller and Bliesner [8].

ii.d: Lateral slope, ψ_i, is such that $-\psi_i = H_f'(l,\psi_i)$: For a lateral slope, ψ_i, that satisfies the relationship $-\psi_i= H_f'(l, \psi_i)$, it can be observed from eqn. (5) that

$$\left.\begin{aligned} & h'(x,\psi_i)<0 \quad for \quad x\in[0,l) \\ & and \\ & h'(x,\psi_i)=0 \quad at \quad x=l \end{aligned}\right\} \qquad (12)$$

Thus, eqn. (12) shows that the pressure head profile for the current scenario is a decreasing function of distance over the entire length of the lateral. Accordingly, the maximum and minimum pressure heads are at the inlet and distal ends of the lateral, respectively.

Under the current scenario, where $-\psi_i=H_f'(l,\psi_i)$, it can be observed that $h'(l,\psi_i)=0$ and hence $x_{min}=l$. The implication is that the distal end of the pressure head profile is a stationary point. It will be shown in the companion paper that such a lateral slope, ψ_i, represents a constant that is of some practical significance.

Considering some arbitrarily set lateral slope range, $[\psi_{min},\psi_{max}]$, the preceding discussion suggests that in order for such an interval to cover the full range of variation of lateral pressure head profile patterns, the lower and upper limits of the slope range need to meet the following requirements: $\psi_{min}<0$ and $\psi_{max}>0$. Noting that $H_f'(x,\psi_i) > 0$ regardless of the values of ψ_i, a quick inventory of the lateral slope subintervals considered above shows that the preceding analysis did not cover the subinterval in which lateral slopes satisfy the inequality $0<-\psi_i < H_f'(l,\psi_i)$. The following describes the spatial pattern of the corresponding pressure head profiles.

ii.e: The lateral slope, ψ_i, is such that $0 <-\psi_i < H_f'(l,\psi_i)$: It can be shown based on eqn. (5) that in the interval $0 <-\psi_i < H_f'(l,\psi_i)$ the pressure head profile is a decreasing function of distance from inlet over the entire length of the lateral [i.e., $h'(x,\psi_i) < 0$ for $0 \leq x \leq l$]. Thus, the pressure head profile, $h(x,\psi_i)$, for the current scenario, has the same monotonic property as that described under scenario i, eqn. (7). It can then be observed that the slope intervals for scenarios i and ii.e can be merged to form a continuous interval: $-\psi_i < H_f'(l,\psi_i)$. Note that the pressure head profiles corresponding to scenario ii.d are also decreasing functions of distance from the lateral inlet over the entire length of the lateral. However, under scenario ii.d, $h'(l,\psi_i) = 0$, but in scenarios i and ii.e, $h'(l,\psi_i)<0$. Thus, for scenario ii.d the distal end of the lateral pressure head profile is a stationary point, which is not the case for scenarios i and iie.

Pressure head profile pattern categories

The preceding theoretical analysis developed a set of criteria that can be used to determine the spatial pattern of the pressure head profile of a lateral, provided the hydraulic, geometric, and slope parameter set of the lateral is spatially invariant and constitutes a feasible hydraulic scenario. Overall the results show that the pressure head profile pattern of a lateral, and possibly the locations of the pressure head extreme, along the lateral can be determined by comparing the negative of the lateral slope, $-\psi_i$, with the friction slopes at the upstream end, $H_f'(0,\psi_i)$, and/or distal end, $H_f'(l,\psi_i)$, of the profile. Based on these relationships six scenarios, labeled here as scenarios i to iie, have been identified. As noted above scenarios i and iie can be merged into a single continuous slope subinterval, reducing the number of scenarios to five. A summary of the scenarios defined in the preceding analysis, the corresponding profile patterns, and the locations along the lateral of the respective minimum and maximum pressure heads is presented in Table 2. Furthermore, a closer look at the results show that the full range of variation of the spatial patterns of the pressure head profiles of a lateral consists of three distinct categories. These include a profile pattern that is increasing or decreasing with distance from lateral inlet over the entire length of the lateral or one that combines both spatial trends within the length of the lateral. A description of the pressure head profile categories is presented as follows:

Category I: Lateral slopes that satisfy the criterion $-\psi_i \geq H_f'(0,\psi_i)$: Lateral pressure head profiles are monotonic increasing functions of distance from the lateral inlet over the entire length of the lateral. Thus, the minimum and maximum pressure heads are located at the upstream and downstream ends of the profiles, respectively. Note that this category of pressure head profile pattern corresponds to scenarios ii.a and ii.b.

Category II: Lateral slopes that satisfy the criterion $H_f'(l,\psi_i) < -\psi_i < H_f'(0,\psi_i)$: Lateral pressure head profiles have two segments: an upstream segment that decreases with distance and a downstream segment that increases with distance from inlet, with a minimum pressure head point somewhere along the lateral in between the upstream and distal ends of the profiles. At the minimum pressure head point $h'(x_{min},\psi_i)=0$, where x_{min}=the distance of the minimum pressure head point from the lateral inlet. The maximum pressure head can be at the upstream and/or distal end. Note that this category of pressure head profile pattern corresponds to scenario ii.c.

Category III: Lateral slopes that satisfy the criterion $-\psi_i \leq H_f'(l,\psi_i)$: Lateral pressure head profiles are decreasing functions of distance from the inlet over the entire length of the lateral. Thus, the maximum and minimum lateral pressure heads are located at the upstream and distal ends of the profiles, respectively. Note that this pressure head profile pattern category corresponds to scenarios i, ii.d, and ii.e.

The preceding analysis presumes that the feasible range of variation of the slope of a lateral is sufficiently wide to cover the full range of

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	Lateral slope ranges					
	$\psi_i<0$					$\psi_i\geq0$
Relationships between ψ_i and H_f' at the inlet and/or distal ends of the profile	$-\psi_i>H_f'(0,\psi_i)$	$-\psi_i=H_f'(0,\psi_i)$	$H_f'(l,\psi_i)<-\psi_i<H_f'(0,\psi_i)$	$-\psi_i=H_f'(l,\psi_i)$	$0<-\psi_i<H_f'(l,_i)$	$-\psi_i<H_f'(l,\psi_i)$
Scenarios[(a)]	*Scenario iia*	*Scenario iib*	*Scenario iic*	*Scenario iid*	*Scenario iie*	*Scenario i*
Pressure head, *h*, profile pattern[(b)]	*Increasing*, ($h'(x,\psi_i)>0$ for $x\in[0,l]$)	*Increasing*, ($h'(x,\psi_i)=0$, for $x=0$ and $h'(x,\psi_i)>0$, for $x\in(0,l]$)	*Decreasing/Increasing*[(c)], ($h'(x,\psi_i)<0$, for $x\in[0,x_{min})$, $h'(x,\psi_i)=0$, for $x=x_{min}$, and $h'(x,\psi_i)>0$, for $x\in(x_{min}, l]$)	*Decreasing*, ($h'(x,\psi_i)<0$, for $x\in[0,l)$ and $h'(x,\psi_i)=0$, for $x=l$)	*Decreasing*, ($h'(x,\psi_i)<0$ for $x\in[0,l]$)	
Location of minimum *h*	*Inlet end*	*Inlet end*	$(0<x_{min}<l)$[(d)]	*Distal end*	*Distal end*	
Location of maximum *h*	*Distal end*	*Distal end*	*Inlet/distal end*[(e)]	*Inlet end*	*Inlet end*	
Categories [(f)]	*I*		*II*	*III*		

[(a)]Scenarios defined based on relationships between lateral and friction slopes;
[(b)]The spatial patterns of lateral pressure head profiles are described as *Increasing* and *Decreasing*. The term *Increasing* implies that the lateral pressure head profile increases with distance from inlet and the converse is true for those profile categories labeled as *Decreasing*;
[(c)]The lateral pressure head profile consists of a decreasing upstream segment and an increasing downstream segment;
[(d)]The minimum pressure head occurs somewhere in between the inlet and distal ends of the lateral, its exact location varies depending on the lateral slope, specific discussion on this is provided in the companion paper;
[(e)]The maximum pressure head occurs at the inlet and/or distal end of the lateral;
[(f)]Categories of pressure head profile patterns;

Table 2: Lateral pressure head profile pattern categories.

variation of the spatial patterns of lateral pressure head profiles defined here. It ought to be pointed out here that even if the feasible range of variation of the slope of a given lateral is not sufficiently wide, the results presented earlier would still be applicable. However, the range of variation of the pressure head profile patterns of such a lateral would be only a subset of the complete set described above.

Note that Wu et al. [13] had defined a similar set of pressure head profile categories based on the ratio of elevation differential to friction head loss along a drip irrigation lateral. However, the criteria used are intuitive and approximate compared to those derived in the analysis presented here. The relationships between lateral slopes and pressure head profile patterns, derived in the preceding analysis, will be evaluated in the companion paper [15] based on outputs of hydraulic simulations.

Summary and Conclusions

The paper presents a comprehensive analysis of slope effects on the spatial patterns of lateral pressure head profiles and the locations of pressure head extreme along a lateral. The analysis assumes that the hydraulic and geometric characteristics and the slope of a lateral are invariant with distance from lateral inlet and local head losses along the lateral are negligible. The lateral specifically considered here is that of solid-set or set-move sprinkler irrigation system, although the results obtained can be readily applied to drip irrigation laterals as well. The effect of slope on the magnitude of pressure head variability along a lateral is generally limited. Nonetheless, for a lateral with a given hydraulic and geometric characteristic, there exists a well-defined and readily discernible relationship between lateral slopes and the spatial patterns of the corresponding pressure head profiles. The practical significance of this is that these relationships can be used as criteria for verification of hydraulic simulation models and design recommendations.

Analysis of slope effects on the spatial patterns of lateral pressure head profiles is conducted here using the lateral pressure head profile equation for steady flow condition. The monotonic properties, with respect to distance from inlet, of lateral slope and friction slope profiles coupled with the pressure slope equation are used here to determine the full range of variation of lateral pressure head profile patterns as a function of lateral slope. Overall, the significant results of the theoretical study are:

(1) Considering a lateral with a parameter set that constitutes a feasible hydraulic scenario, the spatial pattern of the corresponding pressure head profile, and possibly the locations of the pressure head extreme, along the lateral can be determined by comparing the negative of the lateral slope with the friction slopes at the upstream and/or downstream ends of the profile and

(2) Based on these relationships it is deduced here that the full range of variation of the spatial patterns of the pressure head profiles of a lateral consist of three distinct categories, defined as category I, II, and III, provided the lateral slope can be varied over a sufficiently wide range.

The important characteristic features of the pressure head profile pattern categories are:

(1) if the negative of the lateral slope equals or exceeds the maximum of the corresponding friction slope profile (which is located at the lateral inlet), then the results show the pressure head profile is a monotonic increasing function of distance from inlet and such a spatial pattern is labeled here as category I;

(2) if the negative of a lateral slope is within the range of variation of the corresponding friction slope profile, the results of the analysis show that the pressure head profile should have two segments, consisting of a decreasing upstream and an increasing downstream section with a minimum pressure head somewhere in between the inlet and distal ends, the spatial pattern of such a profile is described here as category II; and

(3) if the minimum of the friction slope profile of a lateral (which occurs at the distal end) equals or exceeds the negative of the corresponding lateral slope, then the pressure head profile is a decreasing function of distance from inlet and is defined here as category III pattern.

The relationships between lateral slopes and pressure head profile patterns, derived here through theoretical analysis, will be evaluated in the companion paper based on results of hydraulic simulations.

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