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Size and Power Properties of Some Test Statistics for Testing the Population Correlation Coefficient ρ

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Abstract

Correlation measures the strength of association between two variables, which plays an important role in various fields, such as Health Science, Economics, Finance, Engineering, Environmental science among others. Several tests for testing the population correlation coefficient are proposed in a literature by various researchers at different time points. This paper evaluates the performance of some of the prominent test statistics for testing the population correlation coefficient air proposed in a literature by various researchers at different time points. This paper evaluates the performance of some of the prominent test statistics for testing the population correlation coefficient based on empirical size and power of the tests. Some bivariate distributions, such as normal, lognormal, gamma and chi-square are considered to compare the performance of the test statistics. We believe that the findings of this paper will make an important contribution to select some good test statistics to find the relationship between two variables.

Keywords: Bivariate distribution; Bootstrapping; Correlation coefficients; Hypothesis tests; Monte Carlo simulation; Power; Size

Introduction

One of the most useful statistical tools for quantifying the relationship between two continuous variables is the coefficient of correlation that developed by Pearson [1] from a related idea introduced by Galton [2]. In statistics, Pearson's correlation coefficient is used to find the linear relationship between two quantitative variables (say) X and Y. It gives a value between -1 and +1 inclusive, where -1 indicates a perfect negative correlation, 0 is no correlation and +1 indicates a perfect positive correlation between X and Y. Since the population correlation coefficient, ρ is usually unknown, it is necessary to estimate it by estimator, r from the observed data or sample information. Even the sample correlation coefficient (r) is a biased estimator of population correlation coefficient ρ, the biasness disappears with the increase of sample size. When there is a question of estimation, its estimation accuracy and thus the validity through the hypothesis testing is essential. Several researchers considered several confidence intervals for estimating the population correlation coefficient ρ [3]. However, a comparison of several test statistics for testing the population correlation coefficient is limited in literature. In this paper, we have made an attempt to consider several test statistics for testing the population correlation coefficient. Since, a theoretical comparison among the test procedures is not possible, a simulation study will be conducted to compare the performance of the test statistics based on empirical size and power of the test. We believe that the findings of this study will make an important contribution to literature to choose appropriate test statistics for testing the population correlation coefficient for practitioners.

The paper is organized as follows: One proposed and some existing methods for testing the population correlation coefficient are described in section 2. A Monte Carlo simulation study along with results is discussed in section 3. Finally, some concluding remarks are given in section 4.

Methods for Testing the Population Correlation Coefficient

Suppose we are interested to find the linear relationship between two variables X and Y. Then the population correlation coefficient between two variables X and Y is denoted by ρ and is defined by

$$\tilde{n} = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}} = \frac{{}^{\sigma}XY}{\sqrt{{}^{\sigma}X}{}^{\sigma}Y}}.$$

The corresponding sample correlation coefficient is defined by

$$r = \frac{\hat{\sigma} xy}{\sqrt{\hat{\sigma} x \hat{\sigma} y^2}}.$$

It can be shown that $-1 \le \rho \le 1$. A value of 1 implies that a linear equation describes the relationship between X and Y perfectly, with all data points lying on a line for which Y increases as X increases. A value of -1 implies that all data points lie on a line for which Y decreases as X increases. A value of 0 implies that there is no linear association between X and Y. Several methods for testing for population correlation coefficient, $H_0: \rho=0$ vs. $H_1: \rho \neq 0$ are given as follows.

The classical test statistic

Suppose, the sample correlation coefficient, r is a point estimator of ρ . The distribution of r when ρ is zero was for the first time studied by a student [4]. Thus, a common test is that of whether or not a linear relationship exists between two variables X and Y. The test statistic is defined as follows:

$$f_{r} = (\sqrt{(n-2)}) \sqrt{\frac{r^{2}}{1-r^{2}}}$$
(1)

where *n* is the sample size and (n-2) is the degrees of freedom(df). Thus the critical value for this test statistic can be obtained from t-distribution with (n-2) degrees of freedom.

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Fisher's large sample test statistic

Since the sampling distribution of Pearson's r is not normally distributed, Pearson's r is converted to Fisher's z and the test statistic for testing H_0 : $\rho=0$ vs. H_1 : $\rho \neq 0$ is computed using Fisher's [5] transformation and is given as follows:

$$t_z = \sqrt{n-3}(z-z_0) \tag{2}$$

Where $z = \frac{1}{2} \log \left(\frac{1+r}{1-r} \right)$ and z_0 is the value of z under the null hypothesis where $z_0 = \frac{1}{2} \log \left(\frac{1+\rho_0}{1-\rho_0} \right)$

The distribution of t_{x} has a standard normal distribution.

Gorsuch and Lehmann test statistics

To improve the performances of the classical statistic, Gorsuch and Lehmann [6] modified the classical statistic and the Fisher statistic and proposed the following four statistics for testing H_0 : $\rho=0$ vs. H_1 : $\rho \neq 0$ based on different standard errors of *r*:

Modified classical statistics:

$$t_{GL1} = (\sqrt{(n-1)} \left(\frac{r}{1-r^2}\right)$$
(3)

$$t_{GL2} = (\sqrt{(n-1)} \left(\frac{r}{1-r^2}\right)$$
(4)

Where
$$r = \frac{e^{2\Psi} - 1}{e^{2\Psi} + 1}$$
, $\hat{\Psi} = \frac{1}{2} \log\left(\frac{1+r}{1-r}\right)$ the critical value of t_{GL}

is assumed to be 2 (details see Gorsuch and Lehmann [6]) and the distribution of t_{GL2} follows t distribution with (*n*-1) df.

Modified Fisher statistics:

$$t_{GL3} = \sqrt{n-3}(z-z_0)$$
(5)
$$t_{GL4} = \sqrt{n-3}(z-z_0)$$
(6)

where the critical value of $t_{_{GL3}}$ is assumed to be 2 (details see Gorsuch and Lehmann [6]) and $t_{_{GL4}}$ has a t-distribution with (n-1) degrees of freedom.

Proposed test statistic

We know that $r = b_1 \times \frac{s_x}{s_y}$, where b1 is estimator of β_1 for the model $y_t = \beta_0 + \beta_1 x_t + e_t$, β_0 is the constant, β_1 is the regression coefficient of y on

 $y_t=p_0+p_1x_t+e_r$, p_0 is the constant, p_1 is the regression coefficient of y on x and $e_t \sim N(0, \sigma^2)$. The test statistic for testing H_0 : $\rho=0$ vs. H_1 : $\rho \neq 0$ is given by

$$t_{SK} = \frac{r}{\sqrt{\frac{MSE}{\sum_{i=1}^{n} (\mathbf{y}_i - \mathbf{\bar{y}})^2}}}$$
(7)

The distribution of t_{SK} has t-distribution with (n-2) df.

Parametric bootstrap test statistic

Let $\mathbf{x}^{(*)} = x_1^{(*)}, x_2^{(*)}, ..., x_n^{(*)}$ and $y^{(*)} = y_1^{(*)}, y_2^{(*)}, ..., y_n^{(*)}$ where ith random samples are denoted by $\mathbf{x}^{(i)}$ and $y^{(i)}$ for i = 1, 2, ..., B and B is the number of bootstrap samples [7]. The test statistic for testing \mathbf{H}_0 : $\rho = 0$ *vs*. \mathbf{H}_1 : $\rho \neq 0$ is given by

$$t_{pboot} = (\sqrt{(n-2)}\sqrt{\frac{r^2}{1-r^2}}$$
(8)

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Where critical values of the above statistic is the $t^*_{(n-2)}, \frac{\alpha}{2}$ which is the $(\alpha/2)^{\text{th}}$ sample quintiles of t_{pboot} .

Parametric bootstrap Fisher z test statistic

The test statistic for testing H_0 : $\rho=0$ vs. H_1 : $\rho \neq 0$ is computed using Fisher's z [8] transformation and is given as follows:

$$t_{Fboot} = \sqrt{n - 3} (z^* - z_0)$$
(9)

Where critical values of the above statistic is the $z^*_{\alpha/2}$, which is the $(\alpha/2)^{\text{th}}$ sample quintiles of t_{Fboot} .

Parametric bootstrap version of proposed test statistic

The test statistic for testing H_0 : $\rho=0$ vs. H_1 : $\rho \neq 0$ is computed follows:

$$t_{SKbool} = \frac{r}{\sqrt{\frac{MSE}{\sum_{i=1}^{n} (\mathbf{y}_{i-} \overline{\mathbf{y}})^2}}}$$
(10)

Where critical values of the above statistic is the $t^*_{(n-2)}, \frac{\alpha}{2}$, which is the $(\alpha/2)^{\text{th}}$ sample quintiles of $t_{_{SKboot}}$.

Bootstrap bias corrected acceleration test statistic

This method is introduced by Efron and Tibshirani [9]. The test statistic for testing H₀: $\rho = 0 vs$. H₁: $\rho \neq 0$ is the t-statistic defined in eqn. (1) and the critical value is calculated by $t_{BCA}^{cri} = r^{(\alpha)}$, where r⁰ is the (100 α)% percentile of the distribution of $r, \alpha = \phi \left[\hat{z}_0 + \frac{\hat{z}_0 + z^{(\alpha)}}{1 - \hat{a} \{ \hat{z}_0 + z^{(\alpha)} \}} \right], \phi(.)$ is

the standard normal cumulative distribution function, bias correction

 $\hat{z}_{0} = \phi^{-1} \left(\frac{\#(\mathbf{r}_{i}^{*} < \mathbf{r})}{B} \right), \phi^{-1}(.) \text{ is the inverse function of cumulative distribution function of the Z distribution, acceleration factor <math>\hat{a} = \sum_{i=1}^{n} (\mathbf{r} - \mathbf{r}_{-i})^{3} / 6 \left[\sum_{i=1}^{n} (\mathbf{r} - \mathbf{r}_{-i})^{2} \right]^{1.5}$, **r** is the correlation between **x** and **y** and **r**_i is the correlation between **x** and **y** of (n-1) observations without the ith observation.

Simulation Study

The main goal of this paper is to evaluate the performance of test statistics for testing population correlation coefficient based on size and power properties, discussed in section 2. Since a theoretical comparison among the tests is not possible, a simulation study has been conducted in this section.

Simulation design

MATLAB (2015) programming language was used to run simulations and to make necessary tables. The most common level of significance α =0.05 is considered and assumed random sample sizes n=10, 30, 50, 80 and 100 and ρ_1 =-0.5, -0.9, 0.3, 0.8 and 0.99. We have considered 2500 replications for our simulation experiments and 1500 bootstrap samples for each selected random samples sizes. Random samples produced from the following population distributions:

(a) Bivariate normal with $\mu_1 = 15$, $\mu_2 = 20$ and $\sigma_1 = 10$, $\sigma_2 = 4$

(b) Bivariate log normal with $\mu_1 = 15$, $\mu_2 = 20$ and $\sigma_1 = 10$, $\sigma_2 = 4$

(c) Bivariate gamma with shape parameters 1 and mean parameters 2 $\ensuremath{\mathsf{2}}$

(d) Bivariate chi-square with dfs 1 and 3

Results Discussion

Table 1 presents estimated sizes of the selected test statistics for selected values of n and ρ , when random samples are generated from the bivariate normal distribution. For a visual expression, simulation results are presented graphically in Figure 1.

We can see from Figure 1 and Table 1 is that for all sample sizes, all proposed test statistics except GL1 and SK Boot have empirical sizes close to the 5% nominal level.

We have presented estimated sizes when data are generated from the bivariate lognormal distribution in Table 2 and depicted results for visual inspection in Figure 2. From Table 2 and Figure 2, we observe that for moderate to large sample sizes, tBoot, FBoot, SKBoot and BCABoot have sizes close to the nominal level, while rest of the tests achieve nominal level only when sample sizes are large.

In Tables 3 and 4, we have reported estimated sizes when data generated from the bivariate gamma and bivariate chi-square distribution respectively. We find that all tests have correct sizes expect GL1 and SKboot. GL1 test has small sizes than the nominal level and SKboot has higher sizes than the nominal level (Figures 3 and 4).

In Table 5, we have presented the estimated powers when data are generated from the bivariate normal distribution for various sample sizes and various values of ρ . We observed that for small sample size n=10 (Figure 5), Fisher, G0L3, GL4, FBoot and SKBoot have good

Tests	n=10	n=30	n=50	n=80	n=100	
t	0.0476	0.0520	0.0480	0.0588	0.0560	
Fisher	0.0488	0.0548	0.0484	0.0604	0.0560	
GL1	0.0920	0.0608	0.0528	0.0592	0.0552	
GL2	0.0592	0.0552	0.0512	0.0608	0.0576	
GL3	0.0440	0.0512	0.0452	0.0560	0.0520	
GL4	0.0248	0.0476	0.0436	0.0564	0.0532	
SK	0.0460	0.0360	0.0356	0.0496	0.0468	
tBoot	tBoot 0.0300		0.0336	0.0400	0.0392	
FBoot	FBoot 0.0424		0.0552	0.0560	0.0500	
SKBoot	SKBoot 0.1328		0.1020 0.0860		0.0568	
BCABoot	BCABoot 0.0540		0.0520	0.0480	0.0412	

Table 1: Estimated sizes at the 5% level of significance for bivariate normal data.



■ n=10 ■ n=30 ■ n=50 ■ n=80 ■ n=100

Figure 1: Estimated sizes for various values of n in case of the bivariate normal distribution.

Tests	n=10	n=30	n=50	n=80	n=100	
t	0.0896	0.0608	0.0496	0.0436	0.0312	
Fisher	0.0904	0.0608	0.0504	0.0436	0.0312	
GL1	0.1028	0.0620	0.0508	0.0436	0.0312	
GL2	0.0960	0.0608	0.0508	0.0436	0.0316	
GL3	0.0876	0.0604	0.0488	0.0436	0.0312	
GL4	0.0792	0.0600	0.0488	0.0436	0.0312	
SK	0.0896	0.0608	0.0496	0.0436	0.0312	
tBoot	0.0464	0.0312	0,0252	0.0260	0.0204	
FBoot	FBoot 0.0228		0.0240	0.0320	0.0200	
SKBoot	0.0812	0.0436	0.0296	0.0340	0.0208	
BCABoot	BCABoot 0.0576		0.0360	0.0368	0.0340	

Table 2: Estimated sizes at the 5% level of significance for bivariate lognormal data

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Tests	n=10	n=30	n=50	n=80	n=100	
t	0.0520	0.0500	0.0536	0.0468	0.0420	
Fisher	0.0524	0.0504	0.0540	0.0472	0.0420	
GL1	0.0924	0.0588	0.0564	0.0472	0.0412	
GL2	0.0652	0.0532	0.0560	0.0484	0.0440	
GL3	0.0500	0.0484	0.0492	0.0440	0.0396	
GL4	0.0288	0.0432	0.0480	0.0452	0.0408	
SK	0.0520	0.0500	0.0536	0.0468	0.0420	
tBoot	tBoot 0.0152		0.0184	0.0252	0.0228	
FBoot	FBoot 0.0348		0.0508	0.0436	0.0348	
SKBoot	0.0852 0.1804		0.2292	0.1048	0.1552	
BCABoot	BCABoot 0.0412		0.0436	0.0384	0.0380	

Table 3: Estimated sizes at the 5% level of significance for the bivariate gamma data.

Tests	n=10	n=30	n=50	n=80	n=100	
t	0.0456	0.0452	0.0560	0.0544	0.0468	
Fisher	0.0472	0.0456	0.0568	0.0548	0.0468	
GL1	0.0912	0.0504	0.0596	0.0548	0.0468	
GL2	0.0608	0.0480	0.0592	0.0568	0.0476	
GL3	0.0420	0.0428	0.0520	0.0500	0.0448	
GL4	0.0220	0.0388	0.0516	0.0516	0.0460	
SK	0.0456	0.0452	0.0560	0.0544	0.0468	
tBoot	0.0064	0.0008	0.0228	0.0312	0.0224	
FBoot	0.0348	0.0432	0.0400	0.0408	0.0424	
SKBoot	0.2108	0.1828	0.2000	0.1744	0.1700	
BCABoot	BCABoot 0.0364		0.0500	0.0500 0.0488		

Table 4: Estimated sizes at the 5% level of significance for the bivariate chi-square data.

powers as compare to other test statistics. For sample sizes 50 or above, (Figure 6 for n=50) we found that all test statistics have good powers except for ρ =0.3. We noted that for weak positive correlation, SKboot has highest power as compare to rest of the test statistics.

In Figure 7, we have presented estimated powers when data are generated from the bivariate lognormal distribution for n=10. We observed that for strong negative correlation, t, SK, tBoot and BCABoot tests statistics have very poor power compare to other tests statistics. For positive correlation, we found that all tests have good powers but tBoot and BCABoot. We observed that these two tests have very low powers as compare to other test statistics.

Figure 8 presents estimated powers when data are generated from

the bivariate lognormal distribution for n=80. We observed that all tests have good powers except bootstrap versions of t, Fisher, SK and BCA for ρ =0.3.

we have tabulated estimated power of various test statistics when data are generated from bivariate gamma and bivariate chi-square distributions (see Figure 9 (n =10) and Figure 10 (n=30) for better understanding). It is observed that power properties of the selected tests are similar when data generated from bivariate normal distribution or lognormal.

In Figures 11 and 12, we have plotted estimated powers for various values of n for ρ =-0.5 and ρ =0.8 to check effects of n on the selected tests. It is observed from these graphs that as n increases powers are





	t	Fisher	GL1	GL2	GL3	GL4	SK	tBoot	FBoot	SKBoot	BCABoot
						n=10					
pho=-0.5	0.2360	0.3300	0.3680	0.2860	0.3140	0.2200	0.2360	0.1440	0.2900	0.5800	0.2760
pho=-0.9	0.6000	0.9740	0.7780	0.6820	0.9740	0.9640	0.6000	0.4740	0.9840	0.9700	0.7400
pho=0.3	0.0940	0.1040	0.1680	0.1080	0.1020	0.0660	0.0940	0.0420	0.0940	0.3280	0.1080
pho=0.8	0.4680	0.8120	0.6260	0.5300	0.8000	0.7220	0.4680	0.3500	0.8460	0.8760	0.5600
pho=0.99	0.6620	1.0000	0.8520	0.7620	1.0000	1.0000	0.6620	0.4860	1.0000	0.9420	0.6260
						n=30					
pho=-0.5	0.7720	0.8420	0.7940	0.7760	0.8320	0.8260	0.7720	0.6660	0.8260	0.8800	0.7320
pho=-0.9	0.9080	0.9080	0.9620	0.9200	0.9080	0.9580	0.9080	0.7560	0.9560	0.9580	0.9080
pho=0.3	0.2480	0.2800	0.2720	0.2520	0.2640	0.2480	0.2480	0.1740	0.3180	0.5380	0.2140
pho=0.8	0.8440	0.9300	0.8600	0.8500	0.9180	0.9120	0.8440	0.6820	0.8940	0.9300	0.8400
pho=0.99	0.9012	0.9745	0.9340	0.9310	0.9235	0.9205	0.9012	0.7930	0.9012	0.9745	0.8923
						n=50					
pho=-0.5	0.9500	0.9740	0.9520	0.9520	0.9660	0.9660	0.9500	0.9240	0.9440	0.9740	0.9340
pho=-0.9	0.9920	0.9980	0.9920	0.9920	0.9980	0.9980	0.9920	0.9800	0.9960	0.9980	0.9920
pho=0.3	0.4420	0.4780	0.4500	0.4460	0.4600	. 0.4560	0.4420	0.3720	0.5220	0.6960	0.4260
pho=0.8	0.9660	0.9900	0.9740	0.9720	0.9900	0.9900	0.9660	0.9420	0.9920	0.9980	0.9540
pho=0.99	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
						n=80					
pho=-0.5	0.9940	0.9980	0.9960	0.9960	0.9980	0.9980	0.9940	0.9860	0.9980	0.9980	0.9940
pho=-0.9	0.9960	1.0000	0.9960	0.9960	1.0000	1.0000	0.9960	0.9940	1.0000	1.0000	0.9940
pho=0.3	0.5480	0.5860	0.5540	0.5560	0.5740	0.5740	0.5480	0.4420	0.5920	0.7780	0.4780
pho=0.8	0.9900	0.9980	0.9900	0.9920	0.9960	0.9960	0.9960	0.9900	0.9780	1.0000	0.9840
pho=0.99	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
						n=100					
pho=-0.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
pho=-0.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
pho=0.3	0.7200	0.7620	0.7200	0.7260	0.7440	0.7540	0.7200	0.5820	0.7020	0.8480	0.6860
pho=0.8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
pho=0.99	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

 Table 5: Powers at the 5% level of significance for bivariate normal data.

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also increases for all selected tests. We noted that for small sample sizes, Fisher, GL2, Fboot and SKboot tests are more powerful than the other considered tests. It is also noted that our proposed bootstrap version SKboot is more powerful than the other considered tests.

In Figures 13 and 14, we have plotted estimated powers for selected values of ρ . Here also we observed same

patterns like Figures 11 and 12 as sample size increases, estimated

powers also increases. As compare to the Figure 11, we noted very

low powers n=10 when data generated from the bivariate lognormal

distribution. We noted that tboot and BCAboot tests have very low

power compared to the other tests.

Figures 15-18 present estimated powers for various values of n and two selected values of ρ when data are generated from the bivariate gamma distribution and bivariate chi-square distribution respectively. Similar interpretation can be drawn from these figures, as we observed when data are generated from the bivariate normal distribution.

Conclusion

In this paper, we study the performance of several methods for testing the population correlation coefficient by means of a simulation study. Data were generated randomly from several bivariate distributions, namely, bivariate normal, bivariate lognormal, bivariate









gamma and bivariate chi-square with a range of sample sizes. Overall, we found that test statistics, t, Fisher, GL2, GL3, GL4, SK and FBoot have sizes close to the 5% nominal level. Fisher, GL3, GL4, FBoot and SKBoot have good powers as compare to other test statistics. It appears from the simulation study is that the test statistics, Fisher, GL3, GL4, FBoot and SKBoot can be recommended for practitioners because these test statistics have good sizes and powers compare to the rest of selected test statistics.

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