

# Similitude in Archaeology: Examining Agricultural System Science in PreColumbian Civilizations of Ancient Peru and Bolivia

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## Abstract

Similitude theory adapted to archaeological system analysis provides insight into thought processes underlying agricultural field-system designs used by Andean societies. A basic equation governing the optimum rate of food production dependent upon land, water, labor and technology resources is derived and compared to Chimú, Tiwanaku and Inka field-system designs. Actual designs are close to theoretical optimum designs demonstrating advanced engineering used in decision making underlying field-system designs. Further examples demonstrate how Andean societies managed land, water, labor and technology to provide economic advantage for their populations. The presentation illustrates that agricultural field-system designs were based upon scientific/economic principles and provide a further dimension as to how Andean societies successfully sustained their agricultural development.

**Keywords:** Archaeology; Civilization; Agricultural systems; Andean societies

## Introduction

Determining the cognitive ability of ancient civilizations to conceptualize, design and build water supply systems for agricultural use is examined through mathematical models that predict the optimum use of land, water, labor and technology resources to maximize food production. From the archaeological record of agricultural systems used by several Pre-Columbian societies of ancient Peru and Bolivia, knowledge of agricultural system configurations permits comparison of actual to theoretically optimum agricultural systems. This comparison permits evaluation of the agro-engineering knowledge achieved by societies subject to different ecological conditions and provides insight into their technical achievements produced by evolutionary trial-and-error empirical observation of system improvements and/or engineering foresight to conceptualize an optimum design and put it into use. Use of a basic equation derived from similitude methods provides the basis to replicate the thought process and logical decision making of ancient agricultural engineers albeit in a format different from western science notational conventions. Examples of agricultural system designs from coastal Peru canal-supplied (900-1450 AD) Chimú irrigation systems, groundwater based raised-field agricultural systems of the (300 BC-1100 AD) Tiwanaku society of Bolivia and later (1400-1532 AD) Inka terrace systems are used to illustrate conclusions derived from a first application of similitude methods to archaeological analysis.

Investigation of economic progress and intensification (defined as increased agricultural productivity from land, water, labor and technology resources) in the Pre-Columbian Andean world is examined from the vantage point of technology advances, governmental bottom-up or top-down resource management, environmental and climate factors and societal structure. A summary of progress in the field of agricultural field system designs, strategies and intensification for societies in the Lake Titicaca basin area is available [1-8,15,17,40] as well as for Peruvian north coast and highland societies [14-16,37,39]. For Lake Titicaca basin studies, Scarborough's [9] reference to "wide interpretive distance" between several authors conclusions on aspects of Lake Titicaca basin agricultural strategies and intensification conclusions point to different views of the intensification processes from the same data set. To help quantify different interpretations of

archaeological data, an alternative analytic approach is proposed related to understanding the generation and maintenance of intensification as related to agricultural productivity through use of land, water, labor and technology resources. The similitude approach offers a path to relate resource factors to agricultural production given the many complications related to different societies with different political economies operating in different environments subject to different climate and weather variables. When observed field data coincides with similitude predictions of the optimum use of land, labor, water and technology resources, advances in intensification then have a rational basis based upon an application of agricultural science. Thus, similitude methodology provides insight for discovery of the science base used for intensification by several Andean societies for which extensive archaeological field data exists on their agricultural systems. Additionally, similitude methodology predicts the economic benefits of one agricultural strategy over others; such apparent benefits may lie behind decisions made to implement or alter agricultural strategies again signaling some form of decision making based upon a rational analytic basis.

The proposed similitude methodology examines use of land, water, labor and technology in several Andean societies to determine if observed progress toward intensification noted in the archaeological record follows optimal use of resources predicted by mathematical models based on similitude methods. Provided observed field system design and use strategies from the archaeological record follow predicted optimum usage patterns, then an Andean version of science underlying agricultural strategies exists albeit in different from western science models. While closeness to predicted, optimum conditions may exist through repeated trial-and-error corrections, an alternate

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but complementary interpretation involves cognitive intelligence proceeding from interpretation as to how nature works in an optimal manner. As western science has a similar historical, multi-century experimental path toward discovery of basic scientific principles, this same observational, recording and learning path derived from successful (or failed) agricultural experiments may also be attributed to Andean societies when their agricultural constructions ultimately match science-based optimal use predictions. Provided correlation exists between observed and theoretical predictions, the net result points to an active, societal intelligence at work to provide economic improvements for their populations. This posits a dynamic nature for Andean societies based upon technical achievements similar to the basis of modern societies. The first task then is to determine the optimum use of land, water, labor and technology resources and then determine how closely select Andean societies duplicated this path to maximize agricultural production. Again, if the archaeological record demonstrates closeness to optimum use of resources, then the basis for the intensification process is founded upon some as yet codified form of scientific principles applied to agricultural productivity.

The question of the development societal structure as dependent upon water management techniques has a long history. Clark identified its role with in his publication *Water in Antiquity* which reviewed available archaeological evidence. This treatise on water management and its relationship to societal structure became the foundation for a theoretical hydraulic hypothesis of the composition and structure of an oriental society in Wittfogel's 1957 *Oriental Despotism: A Comparative Study of Total Power* [18] that states societies in Asia depended on the building of large-scale irrigation works as the source of their dominance and bureaucratic structure. Here irrigation required organized, forced labor and a large and complex bureaucracy both of which provided the basis for despotic rule. Steward followed this hypothesis in his irrigation civilization study, claiming that irrigation was the catalyst for state formation. Adams attempted to test the hydraulic hypothesis with regard to the rise of Mesopotamian civilization and found that complex systems of canals and irrigation came *after* the appearance of cities and the indicators of bureaucratic statehood rather than before as Wittfogel had proposed; the same conclusion was found with regard to the emergence of the Mesoamerican archaic state [9,10]. With further research based upon societal development in different parts of the world, it was evident that societies throughout history had developed sophisticated techniques of water management but all did not evolve into states and that many water management systems utilized reservoir management rather than irrigation [11,12]. Thus a far more complex and diverse association between water and society than Wittfogel, Steward and Clark had envisioned now prevailed as exemplified by cross-cultural studies [9,19,20] that illustrated a great variety of methods of water management and their relationship to land, water, labor, technology and societal structure. Here similitude methods provide new insights as to the connection between optimum uses of land, water, labor and technology and the societal structure that attains these ends through utilization of a form of Andean agro-science not previously considered in the literature.

To implement the search for strategies that produce optimal use of resources and maximize agricultural productivity, the use of similitude methods [21-23] is made to derive a basic equation governing optimum ways to maximize food production taking into account the widely varying ecological conditions at different geographic locations experienced by different Andean societies. The basic equation subsequently derived from similitude methods is then used to determine the degree to which major precolumbian societies

of Peru and Bolivia originated technology that conformed to predicted optimum productivity methods; this is achieved through examination of field system designs and strategies from the archaeological record. Where conformance to basic equation predictions exist from the archaeological record, insight into the creativity and engineering science underlying Andean societies' agricultural strategies and field system designs offer perspectives on their innovation, creativity and path towards intensification progress. While many theories exist to explain the connection between societal structures as influenced by their city and agricultural water supply and distribution systems [19,24,25], the present development based on similitude methods provides an alternative quantitative methodology to determine the basis that underlies the decision making of several Andean societies. Thus economic advantage considerations that underlie the decision making of societies influence their social structural development much in the same way that modern technological developments influence paths of present day societal structure. This analog, based upon concrete economic advantages that a society gains through technological advances thus has a counterpart in the ancient world.

## Basic Equation Development

Archaeology examines the cultural remains of a society and through anthropologic models, attempts to interpret the thinking and societal/political/economic structure underlying the creation of ceremonial and secular architectural remains and iconographic material objects abetted by historical reports and surviving literature from ancient sources. These remains inform how a society prioritized resources to support its institutions and was organized to achieve sustainability. Further insight derives from examination of the use of land, labor, water and technology to optimize agricultural production to maintain societal continuity through natural weather/climate/geophysical landscape variations and/or man-generated disasters [13,26]. The development of technologies vital for agricultural yield increase follow from observation, recording and encoding of 'what works best' to increase agricultural production; these heuristic principles refined over time by agricultural experiments, chance discoveries and intuitive insights ultimately improve field system designs toward optimum field system designs and irrigation strategies. While the archaeological record demonstrates the progression of engineering skill applied to agricultural systems as they evolved over time, the science underlying changes in agricultural system evolution remains elusive as the archaeological record shows evolutionary changes but not the development and codification of the science underlying these changes.

To investigate the science underlying evolutionary changes in agricultural systems, one strategy is to determine the theoretical optimum use of land, water, labor and technology given a society's resource base and then determine, from the archaeological record, how close this optimum had been achieved. Given the wide variety of agricultural methods applied by Andean civilizations adapted to different environmental conditions (Figure 1) and archaeological knowledge of societies that occupied pre-Columbian Peru and Bolivia [13], examples illustrating agricultural strategy changes due to climate and geophysical landscape change provide illustrative cases of how these societies modified land, labor, water and technology to maintain agricultural productivity. Given that different productivity optimums exist for different environmental resource conditions in different areas occupied by different Andean societies, different optimum conditions exist. A basic equation derived from similitude methods that incorporates land, water, labor and technology parameters as independent variables and the rate of food production as the dependent

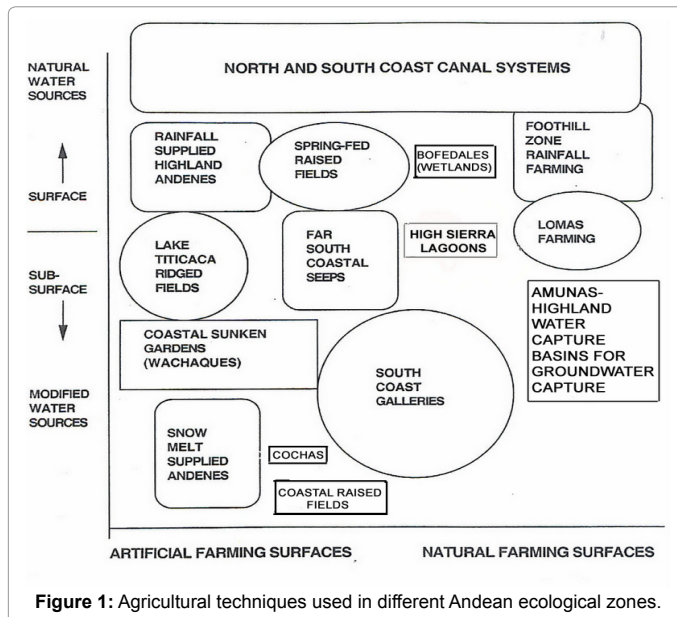


Figure 1: Agricultural techniques used in different Andean ecological zones.

variable is required to evaluate agricultural strategy evolution and the optimum use of resources. When coincidence exists between basic equation predictions and archaeological record observations of agricultural field evolution in terms of use of land, water, labor and technology to achieve optimum food production, then insight into the thinking of ancient Andean societies follows. The derivation of a basic equation that relates optimum use of land, water, labor and technology variables to the rate of food production is achieved by use of Buckingham Pi-Theorem similitude methods [21-23] (Table 1).

The equation for optimum use of land, water, labor and technological resources is expressed in general functional form as:

$$dF/dt = f(L_c, A, V, Q, t, \Phi/\Phi^*)$$

Applying the Pi-Theorem, the equation in nondimensional group form is [15]

$$(A^{3/2}/QF)(dF/dt) = f(L_c/A^{1/2}, V/A^{3/2}, Qt/A^{3/2}, \Phi/\Phi^*)$$

Rearranging and selecting a functional form with population  $P$  related to food supply  $F$  and the food mass consumption per person  $r$  (in a time interval) by  $P = F/r$ , then the basic equation is:

$$[(A^* - A - \Delta A^*)^{3/2} / (Q^* - Q - \Delta Q^*)] F (dF/dt) = [(L_c^* - L_c) / (A^* - A - \Delta A^*)^{1/2} + (V^* - V) / [A^* - A - \Delta A^*]^{3/2}] \cdot [Q - Q^* - \Delta Q^*] t (\Phi/\Phi^*) / (A^* - A - \Delta A^*)^{3/2} \quad (1)$$

where  $\Delta A^*$  = fraction of maximum land area  $A^*$  unusable for cultivation due to non-accessibility by an irrigation canal or landscape contours and  $\Delta Q^*$  is unusable water due to limited canal inlet access. Equation (1) incorporates a constraint condition guaranteeing maximum food production ( $dF/dt = 0$ ) when all land, water, labor and technology resources are maximally (\*) exploited and  $\Delta A^*$  and  $\Delta Q^*$  are small. For this condition, labor is in balance with land, water and technology resources to maximize food production in an economic equilibrium sense. For cases for which  $V > V^*$ , excess labor is being applied to no productive advantage as a subsequent example case details. Similarly, when  $L_c > L_c^*$  and  $Q > Q^*$ , canal length extension and excess water over that required for maximum food production with balanced labor, water and canal lengths in place results. Thus \* conditions represent the optimum employment of  $L_c$ ,  $A$ ,  $V$ ,  $Q$  and  $\Phi$  resources for maximum food production when  $dF/dt = 0$ . While different

functional arrangements are possible, the selection given reproduces known economic laws and provides interpretation of archaeological data related to agricultural field systems and irrigation networks. When  $L_c$ ,  $V$ ,  $A$  and  $Q$  equal their \* counterparts,  $dF/dt = 0$  given that  $0 < \Delta A^* \ll A^*$  and  $\Delta Q$  is small. The zero derivative denotes a maximum value of food production. In an  $F$  vs.  $t$  (time) plot (Figure 2), the zero derivative coincides with zero slope at the maximum of the  $F$  vs.  $t$  curve. A positive slope of the  $F$  vs.  $t$  curve,  $dF/dt > 0$ , indicates potential growth of a society's food supply due to large exploitable resources of land, water, labor and technology ( $L_c < L_c^*$ ,  $V < V^*$  and  $Q < Q^*$ ). For a negative slope  $dF/dt < 0$ , land, water, labor and technology resources are diminishing from flood deposition and erosion events, farming land area contraction due to drought, geophysical land wasting effects, water supply decline and/or labor decline due to plagues or migrations among other degrading causes affecting agricultural production. When  $L_c^*$  decreases rapidly, indicating rapid contraction of the irrigation canal network due to drought or geophysical land wasting inflation/deflation effects, and  $V > V^*$  represents a larger than required labor force working less available land, no options remain to sustain food production as  $A^*$  contracts and  $Q$  can exceed  $Q^*$  indicative of available, but non-usable water, due to contraction of arable land and the canals necessary to support irrigation. In Figure 2, line A-B represents a rapidly developing event (tsunami, massive flood, earthquake, crop pest infestation, sand inundation, invasion, epidemic, etc.) with no anticipated or rapid defensive response to sustain agriculture; such events are in the realm of catastrophe theory [24,27] where  $dF/dt$  becomes highly negative over a short time period with a transition to a collapsed state.

To illustrate application of the basic equation, several scenarios are examined. The first considers too large a labor force ( $V > V^*$ ) exploiting a small farming area ( $A \ll A^*$ ) with low water resources ( $Q \ll Q^*$ ) using a low level of technology in the form of short irrigation canals ( $L_c$  small) emanating from a river source. Here:

$$(A^{3/2}/Q^*F)(dF/dt) = [(L_c^*) / (A^* - \Delta A^*)^{1/2} - V / (A^* - \Delta A^*)^{3/2}] (Q^*) t / A^{3/2} (\Phi/\Phi^*) < 0$$

where the dominant ( $V > V^*$ ) term from Equation (1) is negative and  $A^* - \Delta A^* > 0$  leading to  $(dF/dt) < 0$ . The above expression is a version of the economic *Law of Diminishing Returns* in that excess labor ( $V > V^*$ ) added to balanced, equilibrium resource assets ( $V$ ,  $L_c$ ,  $A$ ,  $Q$ ) does not increase food output without a corresponding increase in the resource assets that utilize the additional labor resources. This law applies to the rightmost branch of the  $F$  vs.  $t$  curve in Figure 2 as excess labor only is not sufficient to exploit limited land, water and technology resources. A second example involves a small labor force exploiting large farming areas with large water resources using a high level of technology. Here  $V \ll V^*$ ,  $A < A^*$ ,  $Q < Q^*$  and  $\Delta A^* \ll A^*$  and the basic equation becomes:

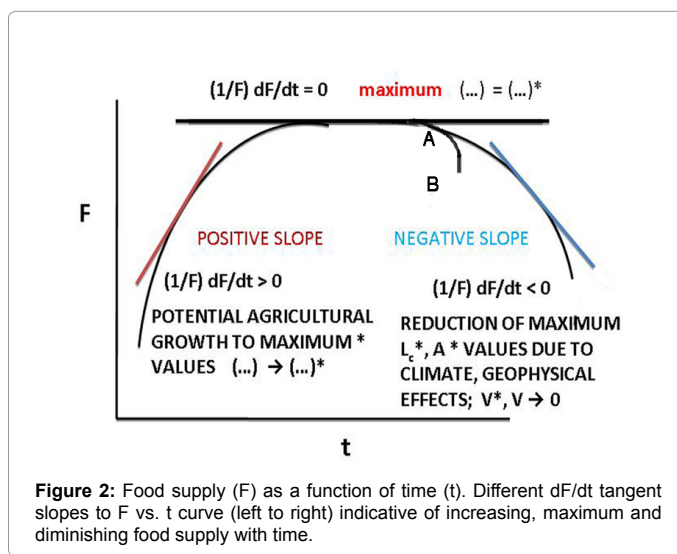
$$(A^*)^{3/2}/Q^*F(dF/dt) = [(L_c^* - L_c) / (\Delta A^*)^{3/2} + V^* / (\Delta A^*)^{3/2}] t Q^* (\Phi/\Phi^*) / (A^*)^{3/2} > 0$$

where  $(1/F)(dF/dt) > 0$  indicates potential to increase food production. This result is typical of the leftmost branch of the  $F$  vs.  $t$  curve in Figure 2. Here the *Law of Diminishing Returns* works to increase food production as excess labor is available to exploit excess land, water and technology resources to \* levels. For both these cases the *Law of Diminishing Returns* is imbedded within the formalism of the basic equation. A further example originates from extreme drought conditions imposed upon a canal irrigation system. Here  $Q \rightarrow 0$  with a large population reacting to incipient drought with  $Q \ll Q^*$  and  $A \ll A^*$  values; the basic equation reduces to a more simplified form with the introduction of variables  $F' = F/F^*$ ,  $A' = A/A^*$ ,  $L_c' = L_c/L_c^*$ ,  $Q' = Q/Q^*$  and



Variable	Description	Units
F	Food supply mass over a time interval	M
dF/dt	Rate of food production over a time interval (time derivative)	M/t
r	Average consumed food mass/person over a time interval (r=constant)	M/r=total population P (number of persons)
L <sub>c</sub>	Total canal or water trough lengths to supply land area A	L
A	Area under cultivation	L <sup>2</sup>
V	Volume of terrain removed/alterd to produce terraces or canals; this parameter relates to the labor force size and soil volume transfer capacity; in terms of units, k1M=labor force=k1rP, k2L3=total soil volume removed and/or altered on a project by the k1M labor force; k1<1.	L <sup>3</sup>
Q	Water volumetric supply rate	L <sup>3</sup> /t
t	Time	t
Φ/Φ*	Technology level (low angle surveying (low) 0<Φ/Φ*<1 (high)	Non-dimensional

**Table 1:** The first step is to express the units of key variables in terms of length (L), mass (M) and time (t) as follows.



**Figure 2:** Food supply (F) as a function of time (t). Different dF/dt tangent slopes to F vs. t curve (left to right) indicative of increasing, maximum and diminishing food supply with time.

$t' = t/t^*$ , where  $t^*$  has dimensions of (one year)<sup>-1</sup>. To render variables nondimensional, the (') notation has been dropped in the development to follow. For severe drought conditions for agricultural technology at an elementary level ( $\Phi/\Phi^* \sim 0.5$ ) with surveying accuracy for canal development limiting exploitable land area using marginal labor and water supplies for agriculture well below that available from a river source, then  $L_c \ll L_c^*$ ,  $A \ll A^*$ ,  $Q \ll Q^*$  and  $V \ll V^*$ . From the general case, after integration of the basic equation with respect to time,

$$F = F^* \exp\{[L_c^*(1-L_c)/A^{1/2}(1-A)^{1/2} + V^*(1-V)/A^{3/2}(1-A)^{3/2}][Q^*t/2\Phi^*(1-A)^3]\}$$

with

$$c = [L_c^*(1-L_c)/A^{1/2}(1-A)^{1/2} + V^*(1-V)/A^{3/2}(1-A)^{3/2}](\Phi/\Phi^*)(Q^*t/2A^3(1-A)^3)$$

Under drought conditions,  $c_d$  simplifies to:

$$c_d \approx [-L_c^*/A^{1/2} - V^*/A^{3/2}](\Phi/\Phi^*)(Q^*t/2A^3)$$

where  $c_d$  can be negative indicating a canal can be of great length to a distant water source, V can be large indicating a large labor force to maintain a long canal and A small indicating less land available for agriculture due to drought. Here t denotes the drought duration in years. For this condition,

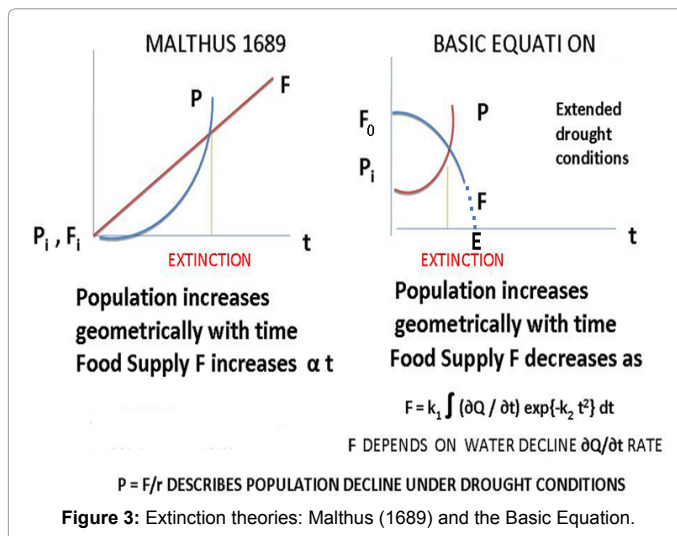
$$F = (1/F^*) \exp\{-|c_d|Q^2t^2\} \quad (2)$$

so that

$$\partial F/\partial Q = -(2|c_d|/F^*)Q^2 \exp\{-|c_d|Q^2t^2\} \quad (3)$$

Equation (2) indicates that the food supply can decrease exponentially as negative (time)<sup>2</sup> under severe and sudden drought conditions in a closed system where food resources are only available from land, water and labor within a closed geographical boundary with no outside import resources available. As  $P = F/r$ , Equation (2) indicates population also exponentially declines as negative time squared as drought deepens. These results are consistent with dual competing (prey-predator) species extinction theories that predict the decline rate of one species after their food supply (the other species) is exhausted [28]. While these closed society, limiting relations apply for rapid onset of drought characteristic of Figure 2, A-B, societal extinction does not necessarily follow if population can adjust to diminishing food supply or if drought slowly develops with intermittent periods of rainfall. Under conditions of variable drought duration and intensity, population may adjust correspondingly but this may lead to selective pruning of key management and labor force personnel in a society that decreases the coherence of a society to function efficiently and collectively. An example from the archaeological record resides in 6<sup>th</sup> century AD Moche settlements in the Jequetepeque Valley [29,30]; here a large Moche population exploiting a shrinking agricultural resource base depleted by climate extremes [31-33] experienced conflict between groups occupying multiple walled settlements competing for resources. The former coherence of Moche society exhibited through common religious practices and rituals at major urban centers appeared fragmented in the post 6<sup>th</sup> century drought environment as scattered settlements replace the major urban centers. This example of conflict represents an extreme, but perhaps common, solution resulting from a population exceeding its resource base resulting from geophysical climate change effects. As a further example of the descriptive content of the basic equation, an increase in low angle surveying technology ( $\Phi/\Phi^*$  increases) implies less labor is required to increase food production (dF/dt) for the same amount of land A, water Q and canal lengths L<sub>c</sub>. For  $V < V^*$  and  $\Phi/\Phi^*$  increasing, then dF/dt increases consistent with the results of previous scholars [34,35]. Of the many theories of state formation as dependent upon population growth, resource availability, warfare, leadership capability, trade and exchange and environmental factors [18,34,36], the present analysis supports technological advancements as the source of defining a hydraulic society. From similitude analysis, technical progress increases the rate of food production (dF/dt) given fixed land, water and labor resources; conversely, an increase in food production leads to an increase in population. Thus, the Boserup hypothesis [34].

(population growth( $V^*$ ) → new farming ( $\Phi/\Phi^*$ ) → increase in agricultural (dF/dt)



technologies production based on introduction of new farming methods and supportive technologies produces increases in agricultural production, apparently works in both directions according to similitude analysis with technological advances key to this process. Here, for  $L_c < L_c^*$ ,  $A < A^*$  and  $V < V^*$ , the basic equation yields

$$(A^{3/2}/QF)(dF/dt) = [L_c^*/(A^* - \Delta A^*)^{1/2} + (V^* - V)] / (A^* - \Delta A^*)^{3/2} Q^* t (\Phi/\Phi^*)$$

confirming the Boserup [34] hypothesis. Alternate hypotheses presented by Netherly's studies [37] of the Chimú (~1000-1400 AD Late Intermediate Period) occupation of the Chicama Valley appears to indicate that there is no direct association between political association and agrarian expansion. In the remote Chicama Valley, Chimú occupation was limited thus permitting more opportunistic farming regions controlled by kin groups outside the direct control and interest of Chimú hierarchy more concerned with Moche Valley agriculture and its supporting water supply networks providing the sustenance for the Chimú capital of Chan Chan. The concentration of canal systems in the Moche Valley area necessary to irrigate vast north and south side field systems (Figure 9) vital for the sustainability of the Chimú capital city of Chan Chan required more sophisticated water, land, labor and logistics management skills combined with hydraulic technology skills sufficient to justify a correlation between irrigation system size and complexity to overriding royal control by irrigation specialists [38].

From the basic equation, a new form of civilization extinction based on declining water supply for agriculture can be derived and compared to Malthusian predictions. Figure 3 shows Malthusian theory where population increases geometrically with time while food resources increase linearly with time. Past the time when food supply and population curves cross, civilization extinction prevails. Although Malthusian theory likely had relevance given limited European 17<sup>th</sup> century agricultural technology and population trends, the growth of the green revolution and more efficient industrial agricultural practices in later centuries matched geometric population growth in advanced countries relegating this theory to historical significance. To implement the basic equation (Figure 3) extinction relation, the food supply change dependent upon water supply change given land area and canal lengths is given by  $\partial F/\partial t = (\partial F/\partial Q)(\partial Q/\partial t)$  with  $A, A^*, L_c, L_c^*$  constant. A relation developed from Equation (2) for extended drought conditions between  $t_0$  and  $t_1$  time limits (in years) describes the decline in the food supply  $F$  as:

$$F = -2|c_d|Q \int_{t_0}^{t_1} t^2 \exp\{-|c_d|Q^2 t^2\} \bullet t^{-2} \exp\{-kt^2\} dt \quad (4)$$

where the water decline rate ( $\partial Q/\partial t$ ) between time limits  $t_0$  and  $t_1$  (in number of years) due to drought may take different forms other than that shown in Equation (4) depending on the average water supply decline time history during extended drought. Specification of Andean water decrease (for agricultural use) over times of extended drought are available [32,33] but require formatting into a mathematical format appropriate for use in Equation (4). As an example case, for rapidly occurring drought, one hypothetical rapid water supply decline relation is given, in terms of nondimensional variables, as  $\partial Q/\partial t = -t^2 \exp\{-kt^2\}$  which describes a severe drought water supply decline decreasing even more rapidly than exponential minus (time)<sup>2</sup>. This rapid water decline case seriously affects the food supply  $F$ . Here  $Q^*$  is the initial water supply at  $t=0$ .

Substituting and integrating Equation (4),

$$F = F_0 - 2|c_d|Q \times F \times (\pi^{1/2}/2a) \text{erf}(at) \Big|_{t_0}^{t_1} \quad (5)$$

where  $a = (|c_d|Q^2 + k)^{1/2}$ .

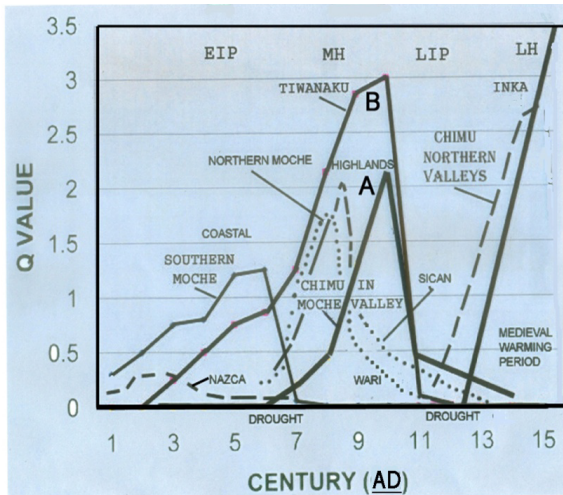
Here  $K$  and  $k$  are constants used in the drought specification term.

In Equation (5), the error function,

$$\text{erf}(x) = (4/\pi)^{1/2} \int_0^x \exp(-s^2) ds \quad (6)$$

is numerically tabulated [39]. Equation (5) then provides the  $F$  vs. time curve for Figure 3. From Figure 3, when the  $P$  (population) and  $F$  (food mass decline) curves cross, insufficient water is available to support food supplies necessary for a geometrically growing population under extended drought conditions causing societal extinction to occur. As the basic equation applies to population decreases commensurate with food supply ( $P=F/r$ ) from either natural or man-generated disasters, societal extinction occurs when population approaches Figure 3. Well before that point, however, questions arise as to the coherence of a society as key leadership, managerial and labor force personnel are randomly subtracted from the population upsetting normal societal task functions leading to secondary collapse mechanisms through political instability. Such a path has been suggested [1,6,40] to describe the collapse of the Tiwanaku society under post 10<sup>th</sup> century drought conditions as well as for other Old World societies subject to climate variations [41].

To illustrate application of the above result to archaeological data, the following illustrative example is given using available field data from an early (5400 BP) coastal Zaña Valley preceramic Peruvian canal system [42] characterized by simplicity. For a group of 100 people using elementary technology ( $\Phi/\Phi^* \sim 0.5$ ), and for ~ 1 kg of food per person per day, an average of ~ 1.0e-1 m<sup>3</sup> per person-day of irrigation water is necessary to sustain the food supply given evaporation and seepage losses that leave only ~ 10-15% of available water available for crop root systems [43]. Given pre-drought conditions were optimal conditions, over a year,  $F_0 = 3.65e4$  kg of food/year is required from  $Q_0 = 3.65e3$  m<sup>3</sup>/year of irrigation water under normal, non-drought water supply conditions at  $t_0=0$  before drought begins. For shallow, short  $L_c \sim 100$  m canals to a field area of ~ 1000 m<sup>2</sup>, some simplifications can be assumed for the example case. Here  $L_c < L_c^*$  as the full potential of arable land is not utilized,  $V < V^*$  as only a fraction of the population is used for agricultural purposes and  $Q < Q^*$  as water for a small agriculture area is only a fraction of that available from a river source.



**Figure 4:** Time spans of major Andean societies based on their sustainability (Q) subject to climate change duress. From Orloff et al. [7].



**Figure 5:** Marsh-filled coastal bay resulting from blocked flood debris by an accumulating beach ridge; area shown located is on the Peruvian north central coast area south of Aspero demonstrating an A' loss of arable land and loss of the marine resource base. Loss event occurred in the Late Preceramic Period 3000-1600 BC.



**Figure 6:** Infilled bay behind a beach ridge in the Peruvian Supe-Huanchaco Valley coastal zone formerly supporting agricultural land in the Late Preceramic Period.

For drought conditions,

$$c_d = \{-|L_c/A^{1/2}| - |V/A^{3/2}|\}(\Phi/\Phi^*)(Q^2/2A^*) = (3.16 - 0.06)(0.5)(1)(0.01) \approx 0.2$$

After  $t_1 = 1.0$  year, using  $\partial Q/\partial t = -t^2 \exp\{-kt^2\}$  to characterize the rapid decline in water supply, only ~ 25% of the initial food supply at  $t_0$  is available for 0.35 of the pre-drought water supply. Further years of decreasing water supply show lower values of yearly food supply as expected. Application of Equation (5) for times  $t > t_0$  then would provide the food supply curve E shown in Figure 3. At a time when the available food supply becomes less than that to sustain the population, population decreases accordingly; if drought continues, then extinction results. Here a closed system is assumed with a stationary population fixed within a closed area with no exterior food sources available for import. Application of Equation (4) to other societies using data taken from their agricultural field area and irrigation canal systems can be used to compute the decline of agricultural output under drought stress. Such calculations aid to understand the canal flow rate contraction drought response patterns mirrored in Figure 8. Here the Peruvian north coast Chimú society experienced continually decreased canal system flow capacity as indicated by time-contracted canal cross-section profiles (Figure 8) to accommodate decreased water supplies in response to continuing 10-11<sup>th</sup> century AD drought [32,33]. Here the food production derived from the irrigation water of the main N1 canal (Figure 9) declined substantially affecting the continuity of the Moche Valley's food supply. Given that other valleys than the Moche Valley with greater water and land resources were available to the Chimú, dispersal of population to satellite valleys was inevitable [34]. Earlier 6<sup>th</sup> century drought [31-33] had similar agricultural contraction effects on the Moche Valley heartland with the collapse of major ceremonial center and agricultural systems [13,29]. Thus from knowledge of drought duration and severity (characterized by a  $\partial Q/\partial t$  term) and knowledge of the field system and supply canal design and drought response data shown in Figure 8, use of Equation (4) can be used to calculate the decline in food availability in the Moche Valley over time as a source of political change and resettlement patterns characteristic of the close of Moche V settlement in the Moche Valley [13,26].

That major mass wasting and drought events influenced Andean history and determined the time dependence and magnitude of the  $\partial Q/\partial t$  term is apparent from the archaeological record. Figure 4 shows the sustainability of major ancient South American civilizations [7] occupying coastal and mountainous highland areas within present-day Peru and Bolivia. Major ~ 30 year 7<sup>th</sup> and ~ 200 year 10-11<sup>th</sup> century AD drought [31-33] caused cultural decline and societal transformation of several major Andean societies in their homelands while other societies were able to cope with changes in irrigation water supply through modification of their canal systems and/or massive construction projects transferring water from distant sources. An example of a coping strategy originates from the (LIP) Late Intermediate Period (~ 1000-1400 AD) Chimú society of north-coast Peru involved construction of the ~ 75 km long Intervalley Canal in the 10-11<sup>th</sup> centuries AD that transferred water from the Chicama River to reactivate Moche Valley desiccated fields [6,7,15,16,44,45] shown in Figure 9 through reactivation of the main Vinchansao canal. In other cases, migration to zones with larger water and land resources occurred exemplified by 7<sup>th</sup> century AD Moche presence and settlement in the northern Jequetepeque and Lambeyeque valleys [29,30] as well as LIP Chimú colonies north and south emanating from the Moche Valley base during 10<sup>th</sup> century drought. Other examples from Andean history demonstrate conquest of weaker states and population transfer (14-15<sup>th</sup> century AD Inka



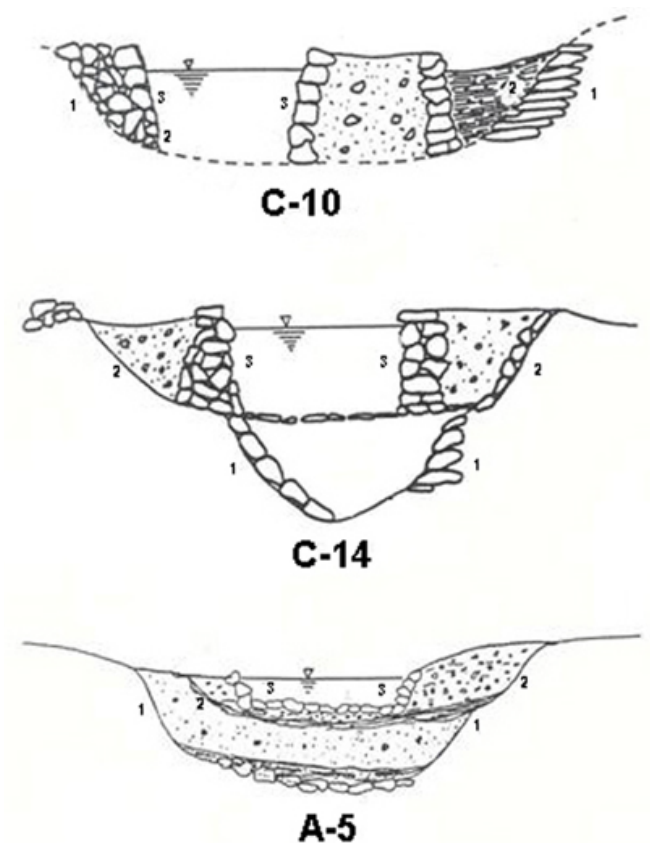
expansion and conquests [46], Chimú military/cultural expansion into north coast Peru valleys in the 10<sup>th</sup> century AD [13]) that were likely influenced to some degree by climate change effects. Examples of societies dispersing and/or disappearing from the archaeological record unable to cope with ecological disasters, geophysical landscape change, drought, and El Niño flood events altering field system's productivity are indicated from the archaeological record. These events include: 6-7<sup>th</sup> century AD drought and sand encroachment into the Moche Valley causing decline of the classic southern Moche V society [26]; the Late Archaic (Preceramic) Period Caral society of the Supe Valley (~3000-1600 BC) experiencing contraction of coastal agricultural lands due to flood erosion, beach ridge formation, bay infilling and aeolian sand transfer compromising their agricultural and marine resource base [15,23,47].

Further examples include 10-11<sup>th</sup> century AD contraction of Chimú Moche Valley agricultural lands and irrigation canals from drought and tectonically induced river downcutting [6,15,44] and the 10-11<sup>th</sup> century AD collapse of Tiwanaku raised-fields and Wari *andenes* farming sites due to drought [6,15] as well as early coastal societies in far south Peru that experienced El Niño mass wasting effects that caused societal collapse [2,15,48]. Figures 5-7 illustrate geomorphic A' landscape changes in the north central region of Peru that started in the Late Archaic Period and marginalized former agricultural and marine resource areas. These landscape changes are still evident and ongoing from continued El Niño and other mass wasting effects observed from satellite imagery taken over time as well as archaeological excavation data [47] on the Peruvian north-central coast. All of these mass wasting geomorphic changes resulting from large El Niño erosion and deposition events over ancient society occupation periods reduced A and A' agricultural lands (and increased  $\Delta A'$ ) that altered the agricultural landscape and induced some form of societal structural change. Figures 8 and 9 illustrate results from archaeological excavations of canal profiles taken from the Huanchaco canal N1, N2 and N3 profiles. Successive contractions of canal profiles mirrored the decline in available water supply from the Moche River through canal systems during the 10<sup>th</sup> century AD drought [6,15,32,33,44,45]. These changes reduced Q and Q' in the 10<sup>th</sup> century AD time period that limited water available for agriculture for field systems adjacent to the Chimú capital of Chan Chan. These examples document A, A' land loss and Q, Q' water loss events originating from natural climate change events that altered and influenced intensification progress and affected food production rates over time, as can be calculated by Equation (4), as changing climate effects on agricultural lands and marine resource areas affected population sustainability (as  $P=F/r$ ) in these regions.

In extreme cases of extended drought (as occurred in 7<sup>th</sup> century AD Peru and 10-11<sup>th</sup> century AD in Bolivia and Peru), Equation (4) provides the mechanism to estimate changes in local population size responding to local changes in irrigation water and/or groundwater supply used for agriculture as dependent upon the severity and duration of a drought. Water availability changes over time likely motivated Andean societies to develop maximum productivity strategies through technology advances to get the highest crop yields per unit amount of water to meet population demands together with developing crop storage housing [29] to sustain population through drought periods. This resource maximization strategy likely drove the pursuit for optimum field system design and use strategies; key to this were advances in technology as the means to increase food productivity given fixed amounts or arable land, water and labor.



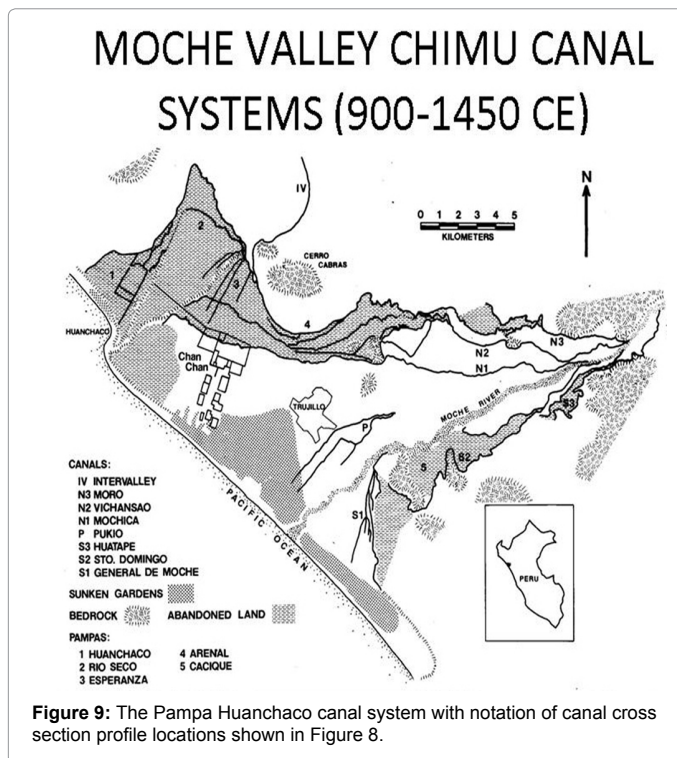
**Figure 7:** Aeolian sand infiltration-Supe Valley agriculture moves from wide, down-valley to narrow, up-valley field areas in the Late Preceramic Period due to sand covering field systems.



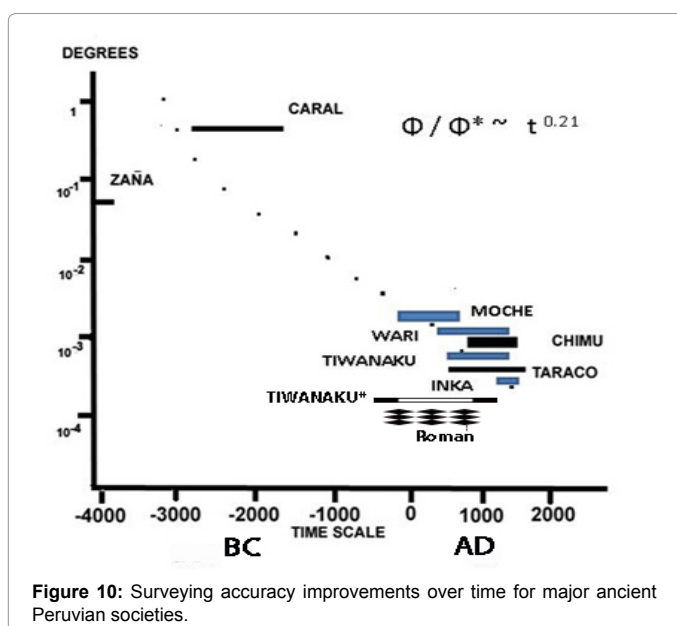
**Figure 8:** Contraction of Chimú Pampa Huanchaco (Figure 9) canal cross-sections due to 9<sup>th</sup>-10<sup>th</sup> century AD drought. Early (1) to late (3) canal profiles.

## Advances in Technology ( $\Phi/\Phi'$ )

The benefit of higher agricultural technology levels is next considered. More low slope coastal field area can be used when lower canal slopes are achievable as low slope canals placed high on hillsides include more downslope farming area. Assuming additional farming land is available for canal placement, then given surveying accuracies sufficient to measure small canal declination slopes, A/A' is assumed



**Figure 9:** The Pampa Huanchaco canal system with notation of canal cross section profile locations shown in Figure 8.



proportional to the technology ratio  $\Phi/\Phi^*$ . Here  $\Phi^*$  represents the minimum canal slope possible given the accuracy achievable by surveying instruments used in conjunction with known *yupana* calculation and *quipu* data storage methods [15,49] to process data. Substituting  $A/A^*$  proportional to  $\Phi/\Phi^*$  into the basic equation and integrating over time, the food supply  $F$  becomes

$$F = \exp\{A^{-1}[(L_c^* - L_c)/\Omega^2 + (V^* - V)/A^* \Omega^3](Q^* - Q)[Q - Q^*]^{1/2} / 2A^* \Omega^3\} \quad (7)$$

where  $\Omega = (\Phi/\Phi^*) - 1$ . As  $\Phi/\Phi^* \rightarrow 1$ ,  $\Omega \rightarrow 0$  and  $F$  increases. For a fixed water resource  $Q < Q^*$ , canal lengths  $L_c < L_c^*$ , labor resources  $V < V^*$  and

$A < A^*$ , as  $\Omega$  decreases toward zero,  $F$  increases exponentially due to small  $\Omega^2$  and  $\Omega^3$  terms. This indicates the importance of surveying technology to improve food output when land, water and labor resources are at their maximum levels. Figure 10 shows surveying accuracies (in degrees) for several major Andean societies in different time periods and illustrates that from early surveying accuracies available at ~2500 BC, refinement in surveying accuracy occurred over time to obtain lower canal slopes. Included in this figure at a comparable time period is a band representing the minimum slope capability of Roman surveying engineers. While less steep slopes are commonplace along the length of many Roman aqueduct constructions [50], one major difference exists between Roman and later Late Intermediate Period (1000-1430 AD) Chimu canal designs: Roman engineers kept the canal cross sectional width constant and varied the sidewall height according to water depth changes brought about by slope decrease sections. Some exceptions to this rule are noted [51] where canal sections are locally widened to reduce flow velocity to precipitate settling of entrained silt and soil particles to facilitate, by decantation, cleaner water to potable status. The Chimu utilized a different canal design strategy that varied canal cross sectional shape, wall roughness and slope in a manner cognizant of sub- and supercritical Froude Number effects on cross sectional shape changes, particularly seen in canal expansion and contraction sections [15,44] in their Chicama-Moche Valley Intervalley canal [44]. The Chimu canal designs thus anticipate formal discovery of open channel hydraulics in western science by ~1000 years. One canal (now destroyed), labeled Tiwanaku\* in Figure 10 emanating from the Tiwanaku River about one kilometer distant from Tiwanaku city (the Waña Jawira canal, [4] had the lowest slope (0.0001degrees) thus far encountered in the ancient Andean world; adjacent to this canal was a drainage canal flowing in the *opposite direction* to the Tiwanaku River. From Figure 10, surveying technology apparently advanced by technical transfer and/or diffusion between societies or from invention within a society with little contact with more advanced societies in the post-Middle Horizon time period. Although disruptions in societal continuity, as indicated in Figure 4, occurred from extended drought in the 7<sup>th</sup> and 10<sup>th</sup> centuries AD, some technical transfer occurred between successive LIP societies sharing the same coastal area sharing common interest in canal based irrigation systems as indicated in Figure 10. Elsewhere, for societies in different ecological zones (the Tiwanaku in particular using raised-field agriculture), surveying accuracy appears to be independently developed to a high degree as evidenced by one canal leading water from the Tiwanaku River to a field system showing a slope of  $10^{-4}$  degrees. Figure 10 indicates a steady progress in surveying accuracy refinement supporting the need for bringing into production more land area to support growing populations.

For any of these paths of technical advancement, guidance as to how Andean technology developed can be obtained from basic equation insights based upon the agricultural system optimization goal. If the rate of technology growth  $\partial(\Phi/\Phi^*)/\partial t$  of an Andean society originates from an existing, highly-developed body of surveying accuracy knowledge, then the rate of technical advancement derives from an extension to the existing technology level. In mathematical terms,  $\partial\Phi/\partial t = k\Phi$  or  $\Phi = Ce^{kt}$  where  $k$  and  $C$  are constants. Substituting into the basic equation, integrating, and examining time dependence, the food supply  $F \sim (e^{kt})^{(kt-1)}$  results from which  $\partial F/\partial t \sim k^2 t e^{kt}$  governs the increased rate in food production with time. This indicates that high levels of technology growth proceeds from a base of advanced technology similar in form to that observed in modern societies where knowledge and invention multiply rapidly over short time periods. Another path for technology improvement over time based on surveying accuracy



of low canal slopes involves technology advancing by a power law  $\Phi = Kt^n$ . From Figure 10, the time exponent  $n$  is small ( $n < 0.2$ ) over time between different Andean societies indicating somewhat limited inter-societal technology exchange. This follows from Andean societies being widely dispersed geographically over different ecological zones with widely-varying water supply conditions (Figure 2) which require agro-technologies only useful for their particular ecological conditions. A further impediment to technical transfer originates from societal collapse/modification periods (Figure 4) originating from climate change episodes which interrupt the continuity and sustainability of societies to limit inter-societal technical transfer. This effect would stimulate intra-societal innovation development. An example [6,52] of innovative technology specific to one society based on its unique ecological conditions (groundwater raised field irrigation) underwrote rapid Tiwanaku development from 600-1100 AD and underscores the  $\Omega^2$ ,  $\Omega^3 \ll 1$  dependencies shown in Equation (7). Extension of groundwater control technology to Tiwanaku's city areas through use of a perimeter canal encircling the ceremonial and monumental structures of the city to promote rapid post-rainy season ground drying, foundation stability of major monumental structures and draining of the Semi-submersible Temple by groundwater height control over seasonal rainfall change is a further example of advanced hydrological technology applied to the Tiwanaku intra-city environment [8,17]. The Chimu development of open channel flow technology to support Peru coastal valley irrigation systems and intervalley water transport by mega-canal construction [6,15,16,44] attest to a specific hydraulic technology development attuned to arid ecological conditions. These examples illustrate that different societies develop surveying technologies specific to their ecological landscape conditions with limited technical transfer from older and distant societies as Figure 10 implies. For different societies occupying the north coast Peru area, technical transfer appears to be a stronger possibility. For stable climate periods between major 7<sup>th</sup> and 10-11<sup>th</sup> century AD drought events, a high rate of internally developed agro-technology apparently existed driven by population increase demands (or conversely, technical development allowed for population increase). In summary, surveying accuracy improvements combined with advanced hydraulic/hydrological technology appears in individual LIP Andean societies sharing common landscape/water resource features with limited technical transfer from earlier sources under different landscape/water resource conditions; this  $\Phi/\Phi'$  increase is a main contributor to intensification progress according to the large effect of the small denominator  $\Omega^2$  and  $\Omega^3$  terms in Equation (7).

## Optimal Agricultural Strategies: Coastal Peru, Highland Bolivian and Inka Field System Designs

Questions arise as to the optimum agricultural strategy given land, water, labor and technology resources to exploit land's agricultural potential. As initial field and irrigation system designs improve as new strategies and technologies were implemented that contributed to increased productivity, agricultural systems evolved toward an optimum design. The question arises as to what is the optimum field system design for different site ecologies and are these designs achieved initially by thoughtful insight guided by a form of scientific and economic principles known to these societies- or in an evolutionary manner by trial and error observations of improvements in agricultural productivity by different societies. Initially, it may be posed that early societies experimenting with agriculture developed individual field plots (Figure 11) that had individual water sourcing and represented lands farmed by individual kin-groups. New population transfers from outside locations migrating into the same area and/or additional people

from the expanding population base desiring to duplicate similar plots to those in existence may have had access only to marginal lands sourced by long canals to these regions. Here the assumption is that the more easily productive lands requiring the least amount of labor were initially occupied by groups exploiting readily available field areas conveniently close to the water sources (Figure 11).

Subsequent groups arriving at later times thus only had land areas requiring longer canal paths to distant fields (Figure 10) requiring increased labor demand to farm. If realization that cooperation rather than competition between resource competing groups yielded higher productive use of land/water resources through canal path redesign under a management consensus structure, then societal integration proceeds to the first step in a bottom-up manner. While such Strategy 1→2 developments appear a logical step in the path toward state formation, the basic equation provides a qualitative basis to illustrate that cooperation rather than competition leads to advantages for all and provides a logical path with economic advantages to more intricate social structures preceding state development. The combined basic equation is given as:

$$(1/F)(dF/dt) = \{[(L_c' - L_c)/(A' - A - \Delta A')^5 + (V' - V)/(A' - A - \Delta A')^6] \cdot (Q' - Q)/[Q - Q']t(\Phi/\Phi')\} \quad (8)$$

Non-cooperative Strategy 1 requires  $L_c > L_c'$  and  $V > V'$  so  $dF/dt < 0$  meaning that Strategy 1 is inefficient compared to Strategy 2 for which  $L_c \leq L_c'$  and  $V \leq V'$ . For Strategy 1 to be sustained for new, incoming migrant groups and/or increased population derived from original kin groups, long canals to outlying land areas require increased labor to manage leading to  $(1/F)dF/dt < 0$ . Here an inefficient bottom-up managed system by a core of kin-based farmers can evolve into a cooperatively managed top-down Strategy 2 provided concession to an oversight management body provided tangible productivity benefits to all through increased field system yields. This process involves original and newcomer groups exploiting agricultural resources collectively and sharing common societal objectives. This development process, applied to the Tiwanaku society [4,40], can be characterized [9,10] as *labor-tasking* evolving into *techno-tasking* as early, labor intensive development of separate raised-field plots yielded to field system development with integrated global features such as groundwater height management by large scale canal bypass water supply and drainage features and mega-construction projects such as the Koani trans-pampa road system [2-4] providing food production benefits obtained through technical advances commensurate with progress from bottom-up to top-down elite management structures. Management changes to top-down authority usually involve construction of elaborate elite palaces, religious edifices and ceremonial centers only

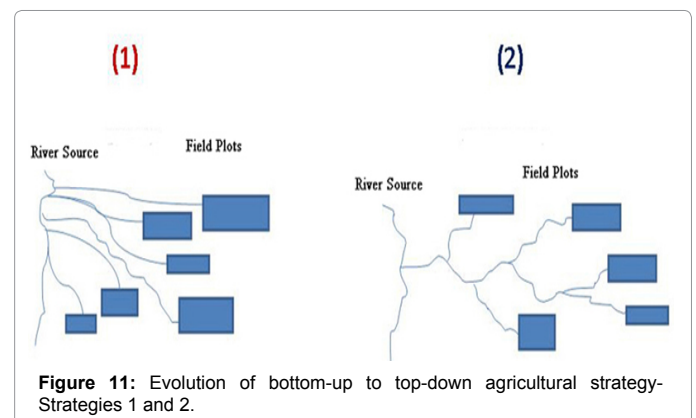


Figure 11: Evolution of bottom-up to top-down agricultural strategy- Strategies 1 and 2.

possible from top-down control of populations. An example of this process characterizes management of the (MH) Middle Horizon (AD 700-1000) Tiwanaku raised-field systems [4,40] where the technology for surface and groundwater control to support agriculture during altiplano long term climate and short term weather fluctuations demanded broad managerial oversight for the collective good of all members of the farming community rather than for individual member groups. For this example, multiple individual operational center structures occupied by small *ayllu* groups successively were incorporated into higher level, larger scale oversight management structures on the Koani plain with “*ayllu* pre-state social formations that functioned by community consensus” ultimately acceding authority to top-down management oversight for major projects that required enlisted labor from all groups to accomplish [4]. While some kin-groups would undoubtedly concede independence to a controlling, oversight group to govern projects based upon best use of labor, other non-cooperative groups wishing to maintain their independence and self-rule identity limit evolution from a bottom-up to a top-down societal structure as evidenced by LIP societies in the Huarochiri area of north central Peru [11,12,53]. An example of kin-group control of productive areas using this labor efficiency advantage to rise to political power characterized the Tiwanaku elite political structure [1,3,4,40,53]. As locally-controlled management is sometimes better than elite corporate-level management, bottom-up structure may prevail as elite management chooses not to disturb a process that functions efficiently without their intervention [11]. A further alternative relies on elite groups exercising control and mandating organization of individual farming groups into a collective. In his case, top-down management ceases to be a cooperative evolutionary end but rather an element of elite group control. An example of this is manifest in Inka societal structure where top-down control had been institutionalized into state controlled lands with elaborate labor taxation imposed upon society members [46,54-57] by the elite governing class. This dominant, top-down, elite controlled social structure prevailed in territories conquered by the Inka where *m’ita* labor service was imposed upon conquered populations to maximize their agricultural productivity to state ends. While specific to the Inka society, rule through elite manipulation of culture, traditions, resource management, traditional symbols and rituals applied to underwrite elite control of many New and Old World societies throughout history [19,20,25,58].

As climate change in the 13-14<sup>th</sup> century AD resulted in increased rainfall [32,33,41], Inka administrators developed lowland agriculture to redistribute restive populations of conquered territories to farm lands distant from their homeland [25,46,54,56,57]. This reorganization process was an integral part of Inka top-down management of resources to promote intensification. Some change from *andenes* terraces to more productive bottomland agriculture occurred due to well-understood canal irrigation practices, predictable and longer duration river water supplies than rainfall-fed terraces, more fertile bottomland soils, high productivity per unit labor input, more available land area and wider crop varieties less subject to temperature extremes. A change in food production rate going from terrace to lowland agriculture involves, for the same labor input, farming area and water supply, the following result from the basic equation:

$$F_{\text{bottomland agriculture}} - F_{\text{terrace agriculture}} = \exp\{[(L_c^* - L_c)(Q - Q^*)^2(\Phi/\Phi^*)/2(A^* - A - \Delta A^*)^5]\} > 0$$

The difference is largest when  $L_c$  is smaller (the case when supply canals originate from a nearby river),  $\Phi/\Phi^*$  is large (canal water distribution technologies are more advanced, controllable, and have

longer water supply duration than terrace intermittent-rainfall systems) and  $A \rightarrow A^*$  (easier agrosystem construction and maintenance on flat terrain with better bottomland soil fertility than on steep mountainside terraces to increase productivity of different crop types in a more temperate climate). From the above equation, the RHS is positive implying an increase in food production by transferring *mit’a* labor assets from terrace to bottomland agriculture thus supporting Inka management decisions for best use of labor to increase the productivity of lands under state control. While not all labor was channeled in this manner owing to lack of large bottomland areas close to highland Inka administrative centers, much terrace agriculture continued as it was well-adapted to the mountainous terrain that prevailed over much of the highland Inka domain. Cases exist for which Andean societies are characterized by dispersed villages emphasizing their independence from central authority or alternatively, opportunistically adapting to a changing resource base to maintain individual survivability. An example of the former lies in Early Intermediate Period Moche occupation of the Jequetepeque Valley characterized by little central authority [30]; a latter example resides in late post-Tiwanaku V society abandoning the central city for dispersed local sites near water resources to support farming [2]. While structural anthropology theory [59,60] provides general conclusions on social phenomena and societal behavior patterns from observable phenomena, research specific to Tiwanaku societal structure [1-4,8,15,40] indicates the presence of late governing hierarchical structures in the form of elite palaces and monuments segregated by a perimeter canal from secular urban housing districts [8]. The proposed transition from bottom-up to top-down management of agricultural resources reflects limited group development of early agricultural management to a later formalization by an elite management and ruling class- this based on higher agricultural production efficiency beneficial to all societal classes [1,40].

## Optimum Field System Designs

A further question asks how ideas are generated to improve agricultural system design. This posits a science that governs decision making perhaps codified from observations deriving from agricultural experiments (or intuition) that produced either positive or negative results related to field system productivity. Given that an optimum configuration exists for an agricultural system given area ecological conditions, the basic equation is instructive to determine these designs. The closer to optimum field system/irrigation network designs that Andean farmers initially chose (or evolved to) provides insight into their science. Key questions are: What is the optimum way to irrigate a field system by subdivided water-path networks? How close to an optimum methodology was practiced by Peruvian coastal and Bolivian highland societies from early to late times? To address these questions by use of the basic equation, first divide resources as follows:  $A/n$ , where  $n$  is the number of field plot subdivisions with their required amount of water supply  $Q/n$  and supply canals  $L_c/n$ . Here sufficient labor resources are assumed distributed as required to service  $n$  land plots, or equivalently,  $V/n$ . Substituting into the basic equation, insight into how land is to be subdivided for maximum access to water supply is given by:

$$[(1/F)dF/dt]_{\text{divided}} \sim n^{5/2} [(1/F)dF/dt]_{\text{undivided}} \quad (9)$$

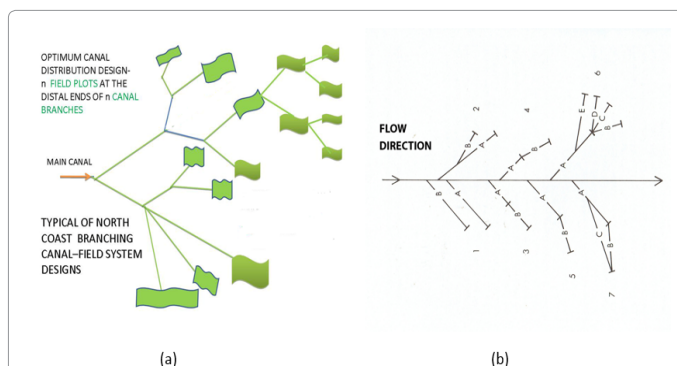
The interpretation of this result is that if  $A$  is divided into  $n$  partitions supplied by  $n L_c/n$ ,  $V/n$  labor and  $Q/n$  water segments then food production in the divided segments is increased by  $n^{5/2}$ . In other words, the best way to efficiently section an agricultural area into divided subplots with individual water access involves a high number ( $n$ ) of smaller plots each with a proportionate amount of water and

labor to optimize production. For increasing  $n$  subdivisions,  $\{(1/F) dF/dt\}_{divided}$  experiences an increased food production rate. Practically a limit exists for the number of  $n$  subdivisions given constraints on the number of exposed water channel surface areas promoting evaporation loss. For typical irrigation systems (Figure 12b) of north-coast Peru [6,13,15,26,37,38], the optimum field system configuration consists of many sequential daughter branches from a main canal each serving an accessible field plot. This type of canal branching system was previously noted [38] in their studies of the Pampa Esperanza canal system adjacent to the Chimú capital city of Chan Chan. Figure 12a illustrates their version of the canal daughter branching network leading to individual field plots similar to those shown in Figure 12. Here the canal subdivisions are in the form of branches from a main water supply canal as a main canal subdivides into multiple daughter branches that further subdivide and feed water into terminal field plots which further branch into multiple channels each serving elevated berm planting areas. Each field plot is further divided into smaller sections served by individual water channels (Figure 12b). Here the optimum water distribution system is provided by an example from nature given by water distribution in a leaf. A leaf contains multiple fractal distribution branches (Figure 16) that optimally (least water transfer energy involved) to deliver water to leaf subareas. This conceptual ideal underlies the canal network designs of coastal Peru systems that provided water to land areas in a manner similar to how nature distributes water through elaborate canal systems within by the Chimú society. (It may be suggested that such an observation from nature inspired Chimú thinking on how to design irrigation canal networks efficiently.) For field system and irrigation network designs typical of the Chimú society of north coast Peru in the 9<sup>th</sup> to 14<sup>th</sup> centuries AD (Figures 9 and 12), field system designs existed based on the presence of gradually sloping farming land within alluvial river delta valleys using a river source to provide irrigation water through canal networks [6,15,16,45]. Figure 12 indicates that multiple-branch, daughter canals emanating from a main canal individually supplying multiple sub-sections with water channels provided the optimal way to increase food productivity as the basic equation predicts. The more field plots (high  $n$ ) with their individual daughter canal water supply, the closer to optimal food production exists. This system reinforces the fractal efficiency of water delivery to individual field plot areas given the constraint that a land area fraction ( $\Delta A'$ ) is inaccessible due to topographic features that prevent access by canal branches. This example reinforces a vision

of Andean agricultural science that supported intensification involving complex field system design land-water use strategies evolved from observation and field trial results that led to optimum use strategies of land, water and labor resources.

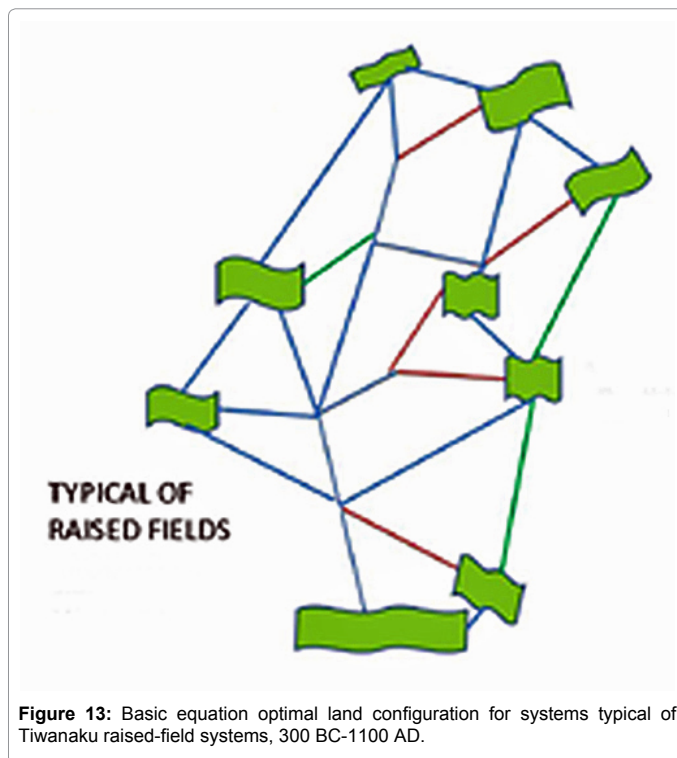
For Tiwanaku raised-field systems based on multiple agricultural mounds (berms) surrounded by water filled swales that penetrated the groundwater level, the question is how to optimally distribute sectioned land areas so that each berm mound has an adequate capillary infused water supply to root systems while maximizing the farming surface area of the mounds per unit surface area. If, for example, field system designs were to be configured as a series of individual circular mounds then the ratio of the exposed water perimeter surrounding a farming area berm is  $4/D$  where  $D$  is the circular area diameter. Clearly if  $D$  is large then the near center part of the mound has limited access to capillary water from surrounding water channels to reach crop roots. This presents a mathematical problem to field system designers to figure out how best to maximize farming field plot area configurations with access to a minimum wetted perimeter surrounding channel water supply so that all of the farming area berm has equal access to water supply to maximize crop productivity and limit evaporative water losses. For cases for which water is supplied by surface irrigation canals, field system designs evolve about the number of  $n$  sequential subdivisions to effectively use limited water resources to field systems; for agricultural systems that depend upon groundwater, somewhat different field system designs are required as the next example case demonstrates.

Contrary to the optimum water supply design to surface canals by a canalized river source, here groundwater lies at an approximate constant depth below the surface of Tiwanaku raised-fields estimated to be 75 km<sup>2</sup> in the Pampa Koani raised-field area [2-4]. Thus while more sub-divided raised-field mounds surrounded by swale water increase food production, there is a minimum raised-field width sufficient to capillary-transfer swale water to root systems and to preserve mound heat-storage capability [5]. Figure 13 shows a field system design where  $n$  interconnecting water channels distribute water to  $n$  raised-field ridges, here the higher  $n$  is and the more subdivision ridges exist served by swales, the higher productivity results. The key to field system design is to obtain the maximum farming land area under these constraints to increase yields. As an example of typical raised-field designs, the early raised-fields at Taraco on the northwest boundary of Lake Titicaca (Figure 14) [61] conform to a near optimum design reinforcing that Tiwanaku agricultural science involved insight into earlier raised-field designs for productivity optimization. Within the Taraco raised-fields are occurrences of imbedded shorter wavelength field systems within larger wavelength field systems indicative of water and heat storage requirements for specialty crops. Thus the partitioning of raised-fields had different optimum configurations for different crop types with different water requirements. This type of field area division was also evident at the Tiwanaku satellite farming area at Pajchiri [62] where different crop types required different water designs supply systems. Similar to the earlier Taraco field system are the later Tiwanaku field system designs [4] that mirror the same closely spaced, elongated farming berm land design indicative of some form of technical transfer over time of an optimal agricultural strategy (Figure 15). While the surface berm configurations were vital to optimize agricultural productivity in some way, sub-surface excavations performed at the Lukurmata area of Tiwanaku indicated that experimentation related to the geometry of raised- fields had occurred. Profiles at depth below the ground surface showed earlier raised-fields had much shorter widths and were more closely spaced together compared to later raised-field



**Figure 12:** Basic equation optimum land partitioning configuration for canal supplied irrigation systems typical of Late Intermediate Period north coast Peru. (a) Canal branching in the Pampa Esperanza adjacent to Chan Chan (Moseley, Deeds 1982); distal ends of daughter branches associated with field systems shown in (b). (b) Typical Chimú field system sinusoidal water branching to agricultural berms.





designs.

Research conducted on Tiwanaku's raised-field agricultural systems in the Pampa Koani region and systems under Tiwanaku influence on the northwest regions of Lake Titicaca indicated use of advanced hydrological methodology underlying crop sustainability and yield improvement. Among the advances in agricultural science are usage of heat transfer technology to limit crop destruction by freezing during cold altiplano nights [5] as well as hydrological/hydraulic control mechanisms providing groundwater height control to stabilize raised-field swale water height through seasonal changes in water availability [8,17]. Additionally, raised field technology has been shown the most efficient design choice to limit short term drought effects on crop yield due to continual groundwater supply from intercepted rainfall over vast collection areas continually flowing to the Lake Titicaca basin [5] that stabilized the groundwater height in swales between farming berms. Analysis of groundwater control mechanisms in the Pajchiri agricultural area [3,62] reveals different berm heights and swale depths appropriate to the needs of different crop types. All these observations point to an advanced agricultural science used to maximize and sustain crops in the Tiwanaku heartland. In the Pampa Koani and northwest Taraco regions of Lake Titicaca, different patterns of raised-field lengths, widths and orientation are frequently inserted within more regular patterns- each pattern appropriate for the water needs of different crop types [3,61,62]. Excavation raised-field berms in the Pampa Koani area has indicated stone-base lining and clay layers [3] to limit cold capillary water transfer from deep groundwater regions into berm interior solar heated swale regions [5,15]; capillary water to the berm interior region is provided from adjoining swale water. Due to higher swale water temperatures from solar radiation input [5], the additional storage heat to berm interior regions limits convection and radiation heat withdrawal during cold altiplano nights to prevent freezing damage to crop root systems. In other words, the latent heat removal for water to ice transition within berm interiors during cold altiplano nights is

limited by additional heat transfer from elevated temperature swale water capillary heat transfer into berm interiors [15]. Examination of Taraco raised-field berm patterns [61], Figure 15 as well as Tiwanaku raised fields [4], Figure 16 reveals a berm shape consistency. These figures show that swales are interconnected leading to a continuous water path surrounding berms. When a typical berm is described as an elongated ellipse with major axis  $a$  and minor axis  $b$ , ( $a \gg b$ ; Figure 14), then an  $a/b$  ratio from 10 to 12 appears to characterize average of berm geometries. This ratio for an elongated ellipse ( $a \gg b$ ) is significant in that the ellipse perimeter is a maximum for the given berm surface area ( $\pi ab$ ) for this class of ellipse. This indicates that the average berm pattern configuration yields the maximum wetted berm perimeter and thus requires a minimum of interconnected swale lengths and widths and that narrower swale channels serve to provide capillary water transfer to narrow berms. Here the narrow berms ( $b \ll a$ ) provide an easy path for elevated temperature capillary water to reach berm interiors. This, in turn, reduces the exposed water surface area of the interconnected swales reducing evaporation loss that helps to (locally) maintain a constant groundwater profile to maintain swale water height. The net effect is that a greater number of closely spaced berms can be watered properly to maintain the crop freezing defense while the greater farming berm area per unit raised-field area produces more food. In other words, more closely spaced berms similar in design to Figures 14 and 15 provide more farming area while maintaining crop protection advantages. These advantages are a key indicator of an advanced agricultural science being employed to protect and increase the yields of raised-field agricultural systems. Thus the  $a/b$  ratio of individual berms contains important information related to Tiwanaku engineering practice as their design incorporates a further level of optimization to limit swale water area to reduce evaporation losses and provide additional crop yield benefits.

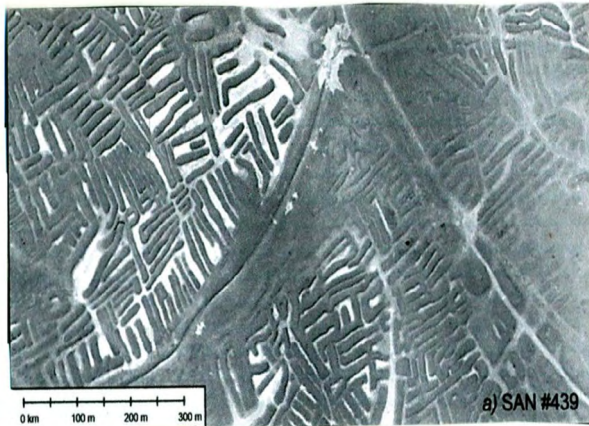
As an example of use of Equation (9), say an original farming area consists of a circular region with an area of 3142 m<sup>2</sup>. This region is surrounded by a water path to provide seepage through the berm aquifer water to root systems. The interior reaches of the circular farming area are water deficient due to the long percolation distance through the berm aquifer from the watered circular region perimeter. If the circular region  $[(1/F)dF/dt]_{\text{undivided}}$  farming area produces an agricultural product of (say) ten units, assume next that if the same circular region area is replaced by ten individual berm strips with the same total farming area with each berm having an  $a/b$  ratio of ten with each berm having optimized agricultural performance due to better and more uniform access to water transferred from the surrounding water path through the aquifer to root systems. For this more agriculturally efficient configuration,  $[(1/F)dF/dt]_{\text{divided}}$  is assumed equal to 100 product units. Forming the ratio given by Equation (9),  $n \sim 2.51$ .

Thus, the same farming area subject to a better berm design is capable of higher agricultural output given the advantages of the elongated, stretched elliptical berm design discussed above. While typical unit numbers are advanced for purposes demonstrating the use of Equation (9), supplemental calculations involving aquifer water distribution within the circular region aquifer subject to surface evaporation can provide more accurate unit numbers related to water deficiency to root systems located near the center of the circular farming area to refine calculation of  $n$  values.

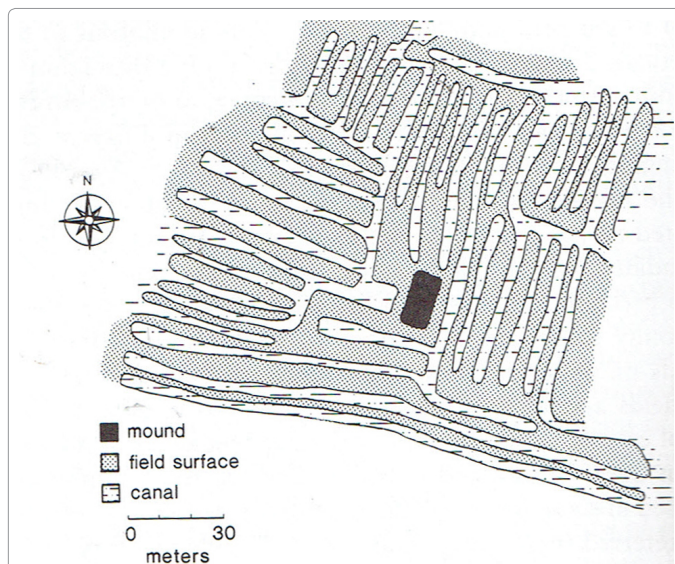
From the basic equation,

$$\frac{[(A^* - A - \Delta A^*)^{3/2}]/(Q^* - Q - \Delta Q^*)F](dF/dt)}{[(L_c^* - L_c)/(A^* - A - \Delta A^*)^{1/2} + (V^* - V)/[A^* - A - \Delta A^*]^{3/2}] \cdot [Q - Q^* - \Delta Q^*]t(\Phi/\Phi^*)/(A^* - A - \Delta A^*)^{3/2}}$$

## TARACO RIDGED FIELDS



**Figure 14:** Aerial view of Taraco raised-field designs in the northeast Lake Titicaca area of Bolivia [43].



**Figure 15:** Raised field system geometry from the Lakaya sector of Koani [3].

for  $V=V^*$ ,  $Q<Q^*$ , when  $dF/dt$  is a positive constant, then  $(L_c^*-L_c)/(A^*-A-\Delta A)^{1/2}$  must be positive. As  $L_c \rightarrow L_c^*$  then for small  $\Delta A$ ,  $A \rightarrow A^*$ . Thus maximum berm farming area is achieved when maximum wetted berm perimeter is achieved. This occurs at  $a/b \sim 10-14$  as the above discussion indicates.

### Additional Similitude Application to Nabataean Petra's (Jordan) Water Systems

Similitude methods have been applied to deepen understanding the underlying thought process of Andean societies and their relation to agricultural science. The methodology employed is universal and can be applied to other hydraulic societies. To illustrate further applications, the agricultural systems of Nabataean Petra in Jordan are next considered.

These systems have been discussed in the literature [15,63-69] and basically rely on dams that trap rainfall runoff into *wadis* (streambeds)

to recharge groundwater used for agriculture; such systems occupy many of hill areas surrounding Petra. These systems rely on rainy season runoff collection guided into terraces and may have had early connections to large, spring-fed reservoirs in early phases of agricultural development but show later modifications with the advent of Roman control starting at 106 AD. Starting from the basic equation,

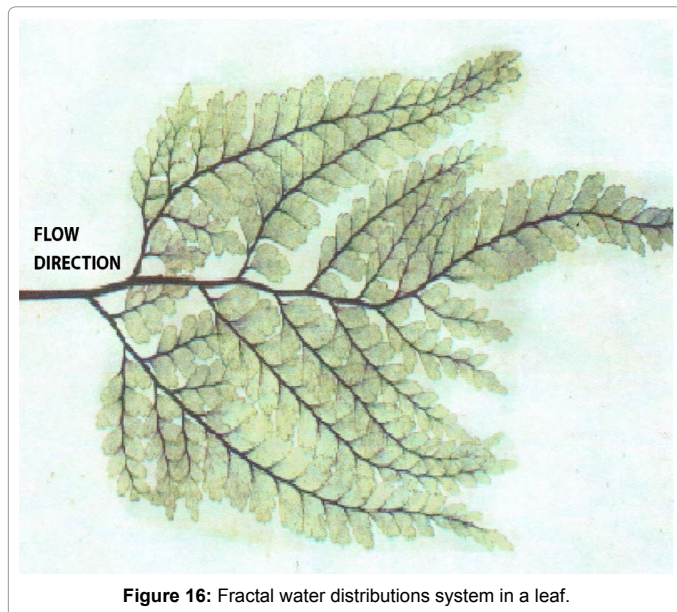
$$\frac{[(A^*-A-\Delta A)^{3/2}]/(Q^*-Q-\Delta Q)F}{(dF/dt)} = \frac{[(L_c^*-L_c)/(A^*-A-\Delta A)^{1/2} + (V^*-V)/[A^*-A-\Delta A]^{3/2}] \cdot [Q^*-Q-\Delta Q]t(\Phi/\Phi^*)}{(A^*-A-\Delta A)^{3/2}}$$

maximum food production rate ( $dF/dt$ ) is achieved for a population estimated to be 20,000 to 30,000 by a large farming area served by use of extensive  $L_c^*$  channels collecting rainfall runoff water flow into *wadis* (streambeds) subject to intermittent rainfall collection ( $Q<Q^*$ ) over short time periods. Farming areas were extensive to support large populations but only a fraction of  $A^*$  was useful due to limited *wadi* channels that could direct limited rainfall runoff water (large  $\Delta Q^*$ ) supplies to limited distribution areas within their reach. Thus while the potential farming area  $A^*$  was large, available land area was limited. With  $A^*$  and  $\Delta A^*$  large, thus available farming area is limited. Thus agriculture appears to be concentrated close to the rainy season implying that food production zones were scattered over large areas with limited productivity for comestibles. This further implies use of agricultural products that do not need continual watering (olives, for example) or products that have a short maturation time after watering (grapes for wine production, for example). As the Nabataean empire covered vast areas from the Negev well into Iraq and Jordan, many ecological zones with varying water supplies existed- this may imply that trade in comestible products must have been an important feature of Nabataean society.

### Conclusions

Andean agriculturalists improved field system designs with land, water, labor and technology strategies that optimized food production. Underlying their initial choice of a field system design were optimum land/water partitioning configurations close to optimum configurations predicted by the basic equation. The science governing these designs likely derived from field trials conducted over time to codify advances in technology that improved agricultural productivity as well as from intuitive insights. As agricultural improvements follow basic equation optimal prediction trends, an argument may be made for Andean societies developing an intuitive science underlying their agricultural system designs. Use of the basic equation provides the foundation to replicate the thought process and logical decision making of ancient agricultural engineers albeit in a format different from western science notational conventions. Here economic advantages in the form of increased agricultural production and efficient use of labor and technology guide decision making to produce economic advantage for a society much in the same way as done in modern society. Similarly, refined surveying technology used to increase farmland area provided addition food resources that supported population growth. As a fraction of this population growth is the labor source to maintain the additional agricultural land, an equilibrium condition is achieved in accordance with the *Law of Diminishing Returns*. Use of the basic equation provides a quantitative way to compute gains in food supply through adjustments in land, water, labor and technology by Andean societies; the similitude methodology demonstrates that basic economic principles apply to decision making. In contrast to many theories on intensification based on religious beliefs, political/economic structure and elite governance characteristics of societies, the present methodology demonstrates that economic advantage is a key factor in decision making of several Andean societies and that these





**Figure 16:** Fractal water distributions system in a leaf.

advantages can be calculated using similitude methodology. While the validity of judgments is likely governed by this rational basis, allowance must be made for judgments compromised by cognitive biases and the societal structure that permits ideas to generate into reality from all members of the population.

Similar to modern concerns, climate change has profound effects on water supply for agricultural lands. Here a version of similar concerns and defenses against drought and excessive rainfall flooding periods occurs in the Andean world mirroring similar defensive construction activity as in the modern world. For Chimú north coast canal irrigation systems constructed during the Late Intermediate Period, extended 10-11<sup>th</sup> century AD drought together with ongoing tectonic/seismic geophysical landscape change and river downcutting required continual modification of canal systems [6,15,44]. The history of canal development within the Moche Valley Chimú heartland reveals abandonment of early canals having inlets placed far up-river at the Moche River entry location into the Moche Valley later replaced by canals with down-river inlets [44] shown in Figure 9. The sequential down-river canal inlet placement from the Moche River was a consequence of tectonically-induced river downcutting that successively stranded higher canal inlets and left little Moche Valley land available for agriculture. Such a reducing agricultural landscape change originating from geophysical tectonic/seismic forces inducing inflation/deflation landscape effects combined with climate changes affecting water supplies support the conclusion that the Peruvian agricultural environment experienced instabilities that required continual agrosystem modifications to balance food supplies to population growth. Examination of the initial configuration of Chimú and Tiwanaku agrosystems demonstrates most later designs evolved close to high productivity optimum configurations and possessed sustainability features (Tiwanaku raised-fields invulnerable to short-term drought; Chimú intervalley canal construction projects to bring in adjacent-valley water resources) indicative of application of their advanced hydraulic science to match the Intervalley Canal's flow rate to activate the main Moche Valley Vinchansao canal flow rate to activate north side Moche Valley agrosystems. Chimú science demonstrates origination of open channel flow water control technology only formally discovered by western science many years later [15,44]. Where

geophysical/climate change altered agrosystems, different strategies to manage these systems evolved- but as Figure 4 indicates, no defense under long-term drought was possible for some coastal societies dependent upon highland runoff into rivers supplying their irrigation systems or for societies relying on constant groundwater levels [6,15].

As complex issues related to agricultural productivity and intensification existed in ancient societies, much intellectual effort was devoted to promote sustainability- this result follows from predictions from the basic equation whose principals were known and used (in some pre-scientific form) by major ancient Andean societies.

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