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Shape Optimization and Applications

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Since the pioneering work of Hadamard [1], shape optimization techniques have become an important tool of design of new devices and improving their performance. The area of shape optimization, which has gained a huge momentum recently due to the rise of the number of applications, wraps several directions.

Analysis

This part mainly deals with the existence (and uniqueness, if possible) of optimal shapes and their regularity. The topics have been marked by the pioneering work of Caffarelli and Alt [2], and presently are intensively developed, in particular by French and Italian schools. In general, the problem of the existence of an optimal shape is complex and difficult due to the lack of appropriate compactness in the set of domains and to poor behavior of the shape functionals with respect to the topology in the set of domains [3].

While the uniqueness of optimal shape does not hold in general, there are fine regularity results of optimal shapes, see the works of L.A. Caffarelli, mostly based on a geometrical approach, and for more recent results based on an analytical approach the reader may consider [4] and the references within.

Applications

This part includes a list of topics, some of them of quite analysis nature, such as: (i) the shape (and topological shape) differentiation of state solutions and shape functionals, (ii) structure of shape derivatives and (iii) numerical methods and applications.

For the shape (and topological shape) derivatives of state solutions and shape functionals, the reader may consider [5,6]. The shape differentiability is based on the implicit function theorem and asymptotic expansion methods, and on the existence and uniqueness results of the relates state problem. For the structure of shape derivatives, one can see the pioneering work in [1], and more recent results in [7]. The numerical methods in shape optimization are mainly of gradient type, and include the shape and topological derivatives, which may be combined with the levelset method, see for example [6,8,9]. For faster convergent algorithms, one can consider the Newton method, see [10]. The applications of the topological shape optimization techniques are endless, to mention a few: optimal shape of an airplane wing, boundary control of Navier-Stokes equations, optimal shape design of hydrogen fuel cells, etc, and the reader may consider for example [11,8], and the references within.

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