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Sentiment Patterns

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"Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than mathematical expectations, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits—a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities"

John Maynard Keynes [1]

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Introduction

Human action is, in a great extent, predictable. Humans are rational and endowed with the ability to weigh benefits and costs in search for the best possible expected outcome. Despite this straightforward evidence, in many circumstances involving decision-making there are evident departures relatively to the strict rational behavior. The complexity of the problems faced by individuals often compels them to adopt simple heuristics, to engage in strategic complementarities and to decide based on instincts or sentiments. As the above quotation by John Maynard Keynes [1] suggests, spontaneous and intuitive reactions govern human behavior in many circumstances. Although difficult to measure and to model, animal spirits are part of the essence of what we are as human beings and they cannot be neglected when assessing how social and economic relations unfold.

From a macroeconomic point of view, animal spirits or sentiments are frequently associated with business cycles, i.e., with the fluctuations aggregate output and other macro variables (e.g., consumption, investment, employment or wages) exhibit. Periods of expansion or 'booms' are attached to phases of generalized optimism and periods of recession or 'busts' correspond to phases of generalized pessimism. Evidently, there are concrete economic reasons that in part govern the formation of waves of optimism and pessimism but most of the times such waves are fuelled by psychological and sociological factors: optimism and pessimism are human characteristics that tend to spread as an epidemic disease, i.e., through the simple contact with others in the population. The interest on the role of animal spirits in the determination of short-run economic outcomes has been renewed in the last few years, as several meaningful academic contributions allow for evidence [2-6].

In this short note, I present a simple modeling structure aimed at illustrating how social interaction within a homogeneous mixing population network might conduct to sentiment fluctuations. The obtained sentiment fluctuations mimic two important pieces of evidence: first, periods of generalized optimism alternate with periods of generalized pessimism and, second, each of the mentioned periods may persist for a relatively long time; these two features are commonly identified when assessing the short-term behavior of the most important macroeconomic variables. The structure of the model is adapted from Gomes [7], which in turn is inspired in the rumor propagation framework, as discussed by several authors [8-10] among many others. The fluctuations will emerge, in the present context, because we associate simple stochastic processes to the probabilities of transition across the considered sentiment states. A standard Wiener process or Brownian motion is all that is required to obtain the already mentioned evolution pattern: a pattern in which relatively persistent periods of generalized optimism and generalized pessimism coexist over time.

The Structure of the Model

Consider five categories of agents: the neutral, the exuberant optimist, the non-exuberant optimist, the exuberant pessimist and the non-exuberant pessimist. At a given date t, there will be a share of agents allocated to each of the mentioned categories; these shares will be represented, respectively, by x(t), y(t), z(t), v(t), w(t); the five categories cover the whole universe of agents, i.e., x(t)+y(t)+z(t)+v(t)+w(t)=1. The following interaction rules across agents are taken (these are adapted from rumor propagation theory and are supposed to be logical and intuitive from the point of view of the spreading of sentiments).

First - when an individual in the neutral state meets an exuberant optimist or an exuberant pessimist, the first will be converted into an exuberant optimist or an exuberant pessimist, respectively, with a given probability;

Second - when an individual in the exuberant optimist or exuberant pessimist category establishes contact with another individual in the optimist (exuberant or not) or pessimist (exuberant or not) category, respectively, the first will shift into the non-exuberance state, with a given probability;

Third - when a non-exuberant individual, optimist or pessimist, meets a neutral agent, the first becomes neutral as well, with a given probability.

To simplify the analysis, one considers that the transition probabilities mentioned in each of the three processes is identical; this will be denoted by $\lambda \in (0,1)$. Taking a homogeneously mixing population and considering the law of mass action, the above rules give place to the following system of five ordinary differential equations,

$$\dot{x}(t) = \lambda\left[\left(z(t)+w(t)\right)x(t) - x(t)\left(y(t)+v(t)\right)\right]$$

$$\dot{y}(t) = \lambda\left[x(t)y(t) - y(t)\left(y(t)+z(t)\right)\right]$$

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$$\dot{v}(t)=\lambda\left[x(t)v(t)-v(t)\left(v(t)+w(t)\right)\right]$$
$$\dot{z}(t)=\lambda\left[y(t)\left(y(t)+z(t)\right)-z(t)x(t)\right]$$
$$\dot{w}(t)=\lambda\left[v(t)\left(v(t)+w(t)\right)-w(t)x(t)\right]$$

x(0), y(0), z(0), v(0), w(0) given

For the above system, under conditions y(0)>0∧v(0)>0, a unique steady-state exists, such that in the long-term the system will rest, in the absence of external perturbations, in the equilibrium point

$$x^*=\frac{1}{3};y^*=z^*=v^*=w^*=\frac{1}{6}$$

Note that, under the assumed configuration of the system, with identical probabilities of transition, optimists, pessimists and neutral agents will be split into equal parts: they all represent one third of the population. Independently of the value of parameter λ, the equilibrium point is stable, i.e., irrespectively of initial conditions and as long as y(0)>0∧v(0)>0, convergence towards the steady-state will take place. The process of convergence is oscillatory, as Figure 1 reveals for λ = 0.25.

Randomness in the Transition Probabilities

Let us consider now that instead of constant transition probabilities, each of the characterized transition processes is subject to a probability governed by a simple stochastic differential equation of the following type,

$$d\lambda_{ij}(t)=\sigma\lambda_{ij}(t)dB(t),\ i,j=x,y,z,v,w;\ i\neq j;\ \lambda_{ij}(0)$$

Parameter σ corresponds to the volatility of the stochastic process and B(t) represents a Brownian motion, i.e., the derivative with respect to time of a white noise random variable. The system of differential equations presented above to describe sentiment dynamics is now adapted to include the stochastic transition probabilities, i.e.,

$$\dot{x}(t)=\lambda_{zx}(t)z(t)x(t)+\lambda_{wx}(t)w(t)x(t)-\lambda_{xy}(t)x(t)y(t)-\lambda_{xv}(t)x(t)v(t)$$
$$\dot{y}(t)=\lambda_{xy}(t)x(t)y(t)-\lambda_{yz}(t)y(t)\left(y(t)+z(t)\right)$$
$$\dot{v}(t)=\lambda_{xv}(t)x(t)v(t)-\lambda_{vw}(t)v(t)\left(v(t)+w(t)\right)$$
$$\dot{z}(t)=\lambda_{yz}(t)y(t)\left(y(t)+z(t)\right)-\lambda_{zx}(t)z(t)x(t)$$
$$\dot{w}(t)=\lambda_{vw}(t)v(t)\left(v(t)+w(t)\right)-\lambda_{wx}(t)w(t)x(t)$$

x(0), y(0), z(0), v(0), w(0) given

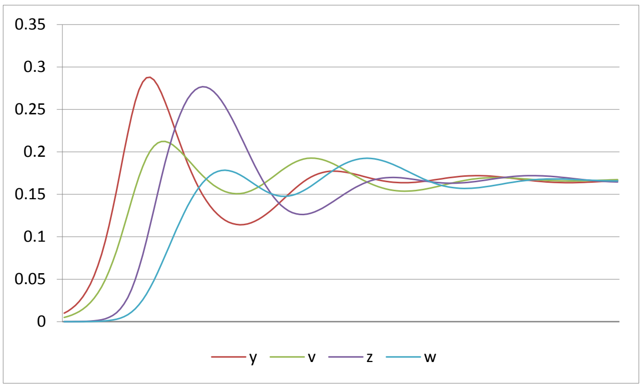

Figure 1: Sentiments' time trajectory for y(0)=0.01,z(0)=0.005,v(0)=w(0)=0 and λ=0.25.

Transition probabilities correspond now to six different variables; their evolution obeys to a same stochastic rule, but they evolve independently from each other. Since the motion is random such probabilities end up by following different time paths. Note, as well, that the values of $\lambda_{ij}(t)$ are bounded from above and from below, i.e., if $\lambda_{ij}(t)=0$ then $d\lambda_{ij}(t)\geq 0$, and if $\lambda_{ij}(t)=1$ then $d\lambda_{ij}(t)\leq 0$.

The introduction of randomness into the model in the described terms will imply persistent fluctuations. The outcome in Figure 1 is replaced by a series of trajectories that behave as illustrated in Figure 2a-2c. The figure clearly shows that everlasting waves of optimism and pessimism set in, with the pattern of the trajectory changing drastically each time one applies the stochastic process to the transition probabilities.

To better assess the aggregate sentiment level, one constructs a straightforward confidence index $conf_{index}$=-(v+w)+(y+z). If the value of this index is positive, then optimism prevails in the population; if it is negative, pessimism will predominate. For the same three scenarios displayed in Figures 2a-2c, Figures 3a-3c presents the time trajectory of the index. As it is visible, periods of dominant optimism and periods of dominant pessimism are observable and the length of such periods might vary significantly.

Conclusion

Sentiment cycles may constitute a meaningful base over which one can approach short-run economic fluctuations; business cycles are surely driven by endogenous economic events, but they are also

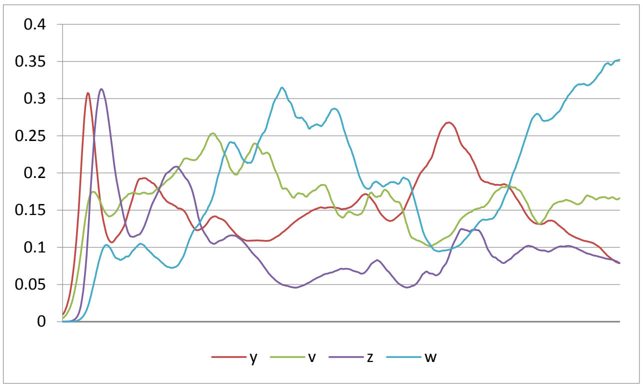

Figure 2a: Sentiments' time trajectories for y(0)=0.01, z(0)=0.005, v(0) =w(0)=0 and λij (0)=0.25, σ=1.

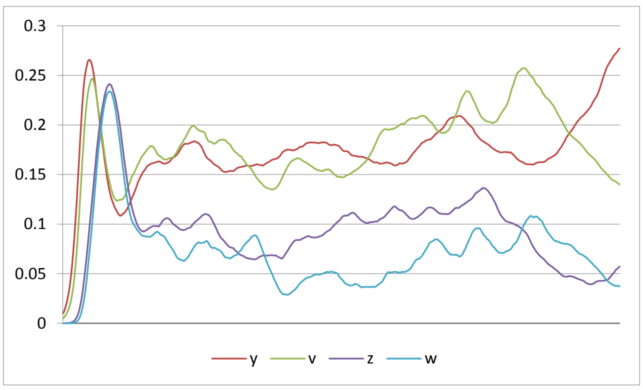

Figure 2b: Sentiments' time trajectories for y(0)=0.01, z(0)=0.005, v(0)=w(0)=0 and λij(0)=0.25, σ=1.

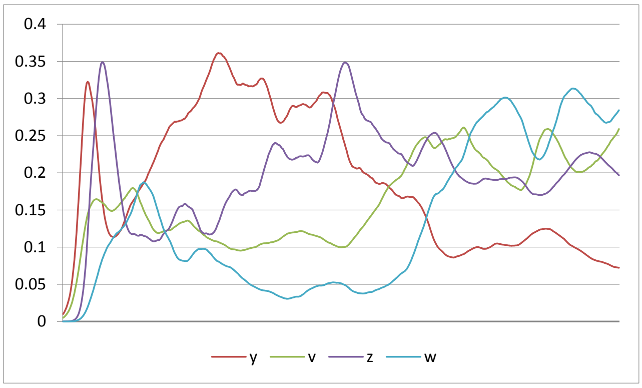

Figure 2c: Sentiments' time trajectories for y(0)=0.01,z(0)=0.005,v(0) =w(0)=0 and $\lambda_{ij}(0)$=0.25,σ=1.

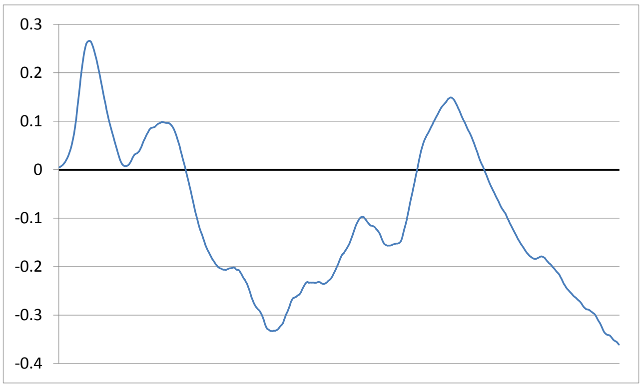

Figure 3a: Confidence index possible long-term time trajectories.

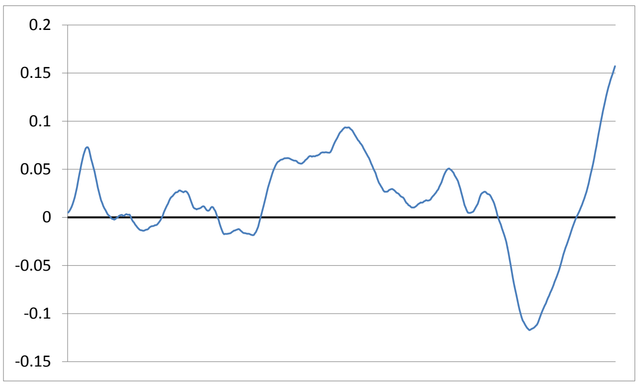

Figure 3b: Confidence index possible long-term time trajectories.

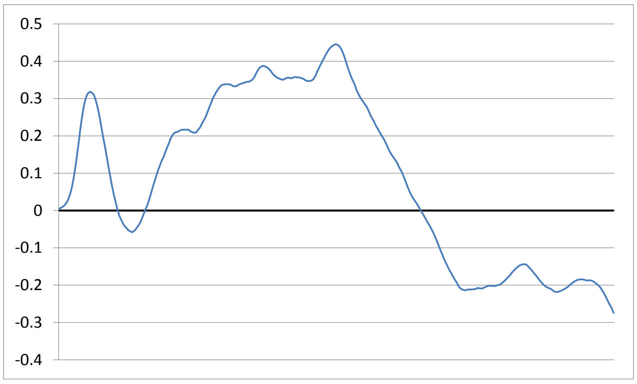

Figure 3c: Confidence index possible long-term time trajectories.

certainly fuelled and amplified by exogenous sentiment changes that, as the above reasoning suggests, eventually emerge from a mechanism of social interaction under random transition probabilities.

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