Journal of
Applied & Computational Mathematics

Salahuddin, J Appl Computat Math 2015, 4:2
DOI: 10.4172/2168-9679.1000206

Research Article | Open Access

Sensitivity Analysis for General Nonlinear Nonconvex Variational Inequalities

Salahuddin*

Department of Mathematics, Jazan University, Jazan, Kingdom of Saudi Arabia

Abstract

In this communication, we proved that the parametric general nonlinear nonconvex variational inequalities are equivalent to the parametric general Wiener-Hopf equations. We use this alternative equivalence formulation to studied the sensitivity analysis for general nonlinear nonconvex variational inequalities without assuming the differentiability of the given data.

Keywords: Sensitivity analysis; Parametric general nonlinear nonconvex variational inequalities; Fixed point; Parametric general Wiener-Hopf equations; (φ,ψ)-relaxed cocoercive mapping; Lipschitz continuous mappings; uniformly r-prox regular sets; Hilbert spaces

AMS Mathematics Subject Classification: 49J40, 47H06

Historical background

The variational inequality theory was introduced by Stampacchia [1] has become a rich source of inspiration and motivation for the study of a large number of problems arising in economics, finance, transportation, networks, structural analysis and optimizations [2-5]. It should be pointed that almost all the results regarding the existence and iterative scheme for solving variational inequalities and related optimization problems are being considered in the convex setting. Consequently all the techniques are based on the properties of the projection operators are convex sets which may not hold in general when the sets are nonconvex. It is known that the uniformly r-prox regular sets are nonconvex and included the convex sets as a special cases [6-9].

Over the last decade there has been increasing interest in studying the sensitivity analysis of variational inequalities and variational inclusions. Sensitivity analysis for variational inclusions and inequalities have been studied extensively [2,3,10-13].

The techniques suggested so far vary with the problems being studied. Dafermos used the fixed point formulation to considered the sensitivity analysis of the classical variational inequalities. These techniques have been modified and extended by many authors for studying the sensitivity analysis of the other classes of variational inequalities and variational inclusions. It is known that the variational inequalities are equivalent to Wiener-Hopf equations [14]. This alternative equivalence formulation has been used by Noor [15-17] to developed the sensitivity analysis frame work for various classes of (quasi) variational inequalities.

In this paper we develop the general frame work of sensitivity analysis for general non-linear nonconvex variational inequalities. First we establish the equivalence between the parametric general nonlinear nonconvex variational inequalities and the parametric general Wiener-Hopf equations by using the projection techniques. By using the fixed point formulation, we obtain an approximate rearrangement of the Wiener-Hopf equations. We use this equivalence to developed the sensitivity analysis for general nonlinear nonconvex variational inequalities without assuming the differentiability of the given data.

Preliminaries

Let H be a real Hilbert space whose inner product and norm are denoted by $\langle .,. \rangle$ and $\|.\|$ respectively. Let K be a nonempty closed subset of H.

Definition 2.1 The proximal normal cone of K at a point u $\in$ H with u $\notin$ K is given by $N_k^p(\mathrm{u})=\{\xi\in \mathrm{H}:\mathrm{u}\in \mathrm{P_k}(\mathrm{u}+\alpha\xi)$ for some $\alpha>0\}$

Where $\alpha>0$ is a constant and

$$P_k(\mathrm{u})=\{\mathrm{v}\in \mathrm{k}:\mathrm{d_k}(\mathrm{u})=\|\mathrm{u}-\mathrm{v}\|\}$$

Where dK(.) or d(.;K) is the usual distance function to the subset of K, that is

$$d_k(\mathrm{u})=\inf\|\mathrm{u}-\mathrm{v}\|.$$

The proximal normal cone $N_k^p(\mathrm{u})$ has the following characterizations:

Lemma 2.2 Let K be a nonempty closed subset in H. Then $\varsigma\in N_k^p(\mathrm{u})$ if and only if there exists a constant $\alpha=\alpha(\varsigma,\mathrm{u})>0$ such that

$$\langle \varsigma, v-u\rangle \le \alpha\|\mathrm{v}-\mathrm{u}\|^2, \forall v\in k$$

Lemma 2.3 Let K be a nonempty closed and convex subset in H. Then $\varsigma\in N_k^p(\mathrm{u})$

$$\langle \varsigma, v-u\rangle \le 0, \forall v\in k$$

The Clarke normal cone denoted by $N_k^c(\mathrm{u})$ is defined by

$$N_k^c(\mathrm{u})=\mathrm{co}[\mathrm{N}_k^p(\mathrm{u})]$$

where co mean the closure of the convex hull.

Clearly $\mathrm{N}_k^p(\mathrm{u})\subseteq \mathrm{N}_k^c$ but the converse is not true in general. Note that $N_k^c(\mathrm{u})$ is always closed and convex cone where as $N_k^c(\mathrm{u})$ is convex

***Corresponding author:** Salahuddin, Department of Mathematics, Jazan University, Jazan, Kingdom of Saudi Arabia, Tel: 0922-5291501-502; E-mail: salahuddin12@mailcity.com

Received December 09, 2014; **Accepted** February 19, 2015; **Published** February 28, 2015

Citation: Salahuddin (2015) Sensitivity Analysis for General Nonlinear Nonconvex Variational Inequalities. J Appl Computat Math 4: 206. doi:10.4172/2168-9679.1000206

Citation: Salahuddin (2015) Sensitivity Analysis for General Nonlinear Nonconvex Variational Inequalities. J Appl Computat Math 4: 206. doi:10.4172/2168-9679.1000206

but may not be closed, see [5, 12].

Definition 2.4 For any given $r \in (0, +\infty]$ a subset Kr of H is said to be normalized uniformly r-prox regular (or uniformly r-prox regular) if and only if every nonzero proximal normal to Kr can be realized by an r-ball that is for all $u \in k_r$ and $0 \neq \varsigma \in N_{kr}^{p}(\mathrm{u})$ with $\|\varsigma\| = 1$

$$\langle \varsigma, v-u \rangle \leq \frac{1}{2r} \| \mathrm{v-u} \|^2, v \in k_r$$

Lemma 2.5 A closed set K ⊆ H is convex if and only if it is proximally smooth of radius r for every r>0:

If r=1 then uniformly r-prox regularity of Kr is equivalent to the convexity of K: If Kr is uniformly r-prox regular set, then the proximal normal cone N_{kr}^{p} (u) is closed as a set valued mapping. If we take $n = \frac{1}{2r}$ it is clear that r $\to \infty$ then n=0:

Proposition 2.6 [12] For r>0, let Kr be a nonempty closed and uniformly r-prox regular subset of H. Set

$$u = \{\mathrm{u} \in \mathrm{h} : 0 \leq \mathrm{d}_{kr}(\mathrm{u}) < \mathrm{r}\}$$

Then the following statements are holds:

for all $u \in u, p_{kr}(\mathrm{u}) \neq \varnothing$

for all $r' \in (0, \mathrm{r}), \mathrm{p}_{kr}$ is a Lipschitz continuous mapping with constant $\delta = \frac{r}{r-r'}$ on

$$u = \{\mathrm{u} \in \mathrm{h} : 0 \leq \mathrm{d}_{kr}(\mathrm{u}) < \mathrm{r}'\}$$

(i) the proximal normal cone is closed as a set valued mapping.

Assume that F; T : H ! 2H are set valued mappings, g; h : H ! H the nonlinear single valued mappings such that $k_r \subseteq g(\mathrm{h})$ and N : H X H ⟶ H the mapping. For any constants n>0 and p>0, we consider the problem of finding $u \in h, x \in T(\mathrm{u}), \mathrm{y} \in \mathrm{F(u)}$ such that $h(\mathrm{u}) \in \mathrm{k}_r$ and

$$\langle pN(\mathrm{x,y}) + \mathrm{h(u)} - \mathrm{g(u)}, \mathrm{v} - \mathrm{h(u)} \rangle + n \| \mathrm{v} - \mathrm{h(u)} \|^2 \geq 0, \forall v \in k_r$$

The equation (2.1) is called general nonlinear nonconvex variational inequalities. Now we consider the problem of solving general Wiener-Hopf equations. To be more precise, let QKr = I – h^{-1}PKr where PKr is the projection operator, h^{-1} is the inverse of nonlinear mapping h and I is an identity mapping. For given nonlinear mappings T; F; h; g; consider the problem of finding $z, u \in h, x \in T(\mathrm{u}), \mathrm{y} \in \mathrm{F(u)}$ such that $N(\mathrm{x,y}) + p^{-1} Qk_r z = 0$ is called general Wiener-Hopf equations.

Lemma 2.7 $u \in H, x \in T(\mathrm{u}), \mathrm{y} \in \mathrm{F(u)}, \mathrm{h(u)} \in \mathrm{k_r}$ is a solution of (2.1) if and only if $u \in H, x \in T(\mathrm{u}), \mathrm{y} \in \mathrm{F(u)}, \mathrm{h(u)} \in \mathrm{k_r}$ satisfies the relation h(u) = PKr [g(u) _N(x; y)] where PKr is a projection of H onto the uniformly r-prox regular set Kr:

Lemma 2.7 implies that the general nonlinear nonconvex variational inequality (2.1) is equivalent to the fixed point problem (2.3).

Now we consider the parametric version of equations (2.1), (2.2) and (2.3). To for- mulate the problem, let Ω be an open subset of H in which parameter λ takes values. Let $T, F : \Omega XH \to 2^h$ be the set valued mappings, N : H X H ⟶ and g; h : Ω _ H ⟶ H the nonlinear single valued mappings such that $K_r \subseteq g(\mathrm{h})$ and N : H X H ⟶ H the mapping. For any constants n>0 and p>0, we consider the problem of finding $u \in H, x \in T(\mathrm{u}), \mathrm{y} \in \mathrm{F(u)}$ such that $h(\mathrm{u}) \in \mathrm{K_r}$ and

$$\langle pN(\mathrm{x,y}) + \mathrm{h(u)} - \mathrm{g(u)}, \mathrm{v} - \mathrm{h(u)} \rangle + n \| v - h(\mathrm{u}) \|^2 \geq 0, \ \forall v \in K_r$$

The equation (2.1) is called general nonlinear nonconvex variational inequalities. Now we consider the problem of solving general Wiener-Hopf equations. To be more precise, let $Q_{Kr} = I - h^{-1} P_{Kr}$ is the projection operator, h^{-1} is the inverse of nonlinear mapping h and I is an identity mapping. For given nonlinear mappings T; F; h; g; consider the problem of finding $z, u \in H, x \in T(\mathrm{u}), \mathrm{y} \in \mathrm{F(u)}$ such that $N(\mathrm{x,y}) + \mathrm{P}^{-1} Q_{kr} Z = 0$ is called general Wiener-Hopf equations.

Lemma 2.7 $u \in H, x \in T(\mathrm{u}), \mathrm{y} \in \mathrm{F(u)}, \mathrm{h(u)} \in \mathrm{K_r}$ is a solution of (2.1) if and only if $u \in H, x \in T(\mathrm{u}), \mathrm{y} \in \mathrm{F(u)}, \mathrm{h(u)} \in \mathrm{K_r}$ satisfies the relation

$$h(\mathrm{u}) = \mathrm{P_{kr}}[\mathrm{g(u)} - \mathrm{pN(x,y)}] \quad (2.3)$$

where PKr is a projection of H onto the uniformly r-prox regular set Kr: the single valued mappings. We define $g\lambda(\mathrm{u}) = \mathrm{g(u}, \lambda), \mathrm{h}\,\lambda(\mathrm{u}) = \mathrm{h(u}, \lambda), \mathrm{x}\,\lambda(\mathrm{u}) = \mathrm{x(u}, \lambda) \in \mathrm{T}_\lambda(\mathrm{u}), \mathrm{y}_\lambda(\mathrm{u}) = y(\mathrm{u}, \lambda) \in \mathrm{F}_\lambda(\mathrm{u})$ unless otherwise specified. The parametric general non-linear nonconvex variational inequality is to find $(\mathrm{u}, \lambda) \in \mathrm{HX}\,\Omega, \mathrm{x}\,\lambda(\mathrm{u}) \in \mathrm{T}_\lambda(\mathrm{u}), \mathrm{y}_\lambda(\mathrm{u}) \in \mathrm{F}_\lambda(\mathrm{u}), \mathrm{h}_\lambda(\mathrm{u}) \in \mathrm{K_r}$ such that

$$\langle PN(\mathrm{x}\,\lambda(\mathrm{u}), \mathrm{y}\,\lambda(\mathrm{u})) + \mathrm{h}\,\lambda(\mathrm{u}) - \mathrm{g}_\lambda(\mathrm{u}), \mathrm{v} - \mathrm{h}_\lambda(\mathrm{u}) \rangle \geq 0, \forall v \in K_r \quad (2.4)$$

We also assume that for some $\overline{\lambda} \in \Omega$ problem has a unique solution $\overline{u}$ Related to the parametric general nonlinear nonconvex variational inequality (2.4), we consider the parametric general Wiener-Hopf equation. We consider the problem of finding $(\mathrm{z,u}, \lambda) \in \mathrm{HXHC}\,\Omega, \mathrm{x}\,\lambda(\mathrm{u}) \in \mathrm{T}_\lambda(\mathrm{u}) \in \mathrm{F}_\lambda(\mathrm{u})$ such that

$$N(\mathrm{x}_\lambda(\mathrm{u}), \mathrm{y}_\lambda(\mathrm{u})) + \mathrm{p}^{-1} Q_{kr} Z = 0 \quad (2.5)$$

where p > 0 is a constant and QKr (z) is define on the set (z, λ) with $\lambda \in \Omega$ and takes values in H. The equation (2.5) is called parametric general Wiener-Hopf equation.

Lemma 2.8 If H is a real Hilbert space. Than the following two statements are equivalent: an element $u \in H, x_\lambda(\mathrm{u}) \in \mathrm{T}_\lambda(\mathrm{u}), \mathrm{y}_\lambda(\mathrm{u}) \in \mathrm{F}_\lambda(\mathrm{u}), \mathrm{h}_\lambda(\mathrm{u}) \in \mathrm{K_r}$ is a solution of (2.4), the mapping $E\lambda(\mathrm{u}) = \mathrm{u} - \mathrm{h}\,\lambda(\mathrm{u}) + \mathrm{P_{kr}}[\mathrm{g}\,\lambda(\mathrm{u}) - \mathrm{pN}(\mathrm{x}\,\lambda(\mathrm{u}), \mathrm{y}\,\lambda(\mathrm{u}))]$ has a fixed point.

One can established the equivalence between (2.4) and (2.5), by using the projection techniques, see Noor [10,11].

Lemma 2.9 Parametric general nonlinear nonconvex variational inequality (2.4) has a Solution $(\mathrm{u}, \lambda) \in \mathrm{Hx}\,\Omega, \mathrm{x}\,\lambda(\mathrm{u}), \mathrm{y}\,\lambda(\mathrm{u}) \in \mathrm{F}\,\lambda(\mathrm{u})$ if and only if parametric general Wiener-Hopf equation (2.5) has a solution $(\mathrm{z,u}, \lambda) \in \mathrm{HxHx}\,\Omega, \mathrm{x}_\lambda(\mathrm{u}) \in \mathrm{T}\,\lambda(\mathrm{u}), \mathrm{y}\,\lambda(\mathrm{u}) \in \mathrm{F}\,\lambda(\mathrm{u})$

$$h_\lambda(\mathrm{u}) = \mathrm{P_{krz}} \quad (2.6)$$

$$z = g\lambda(\mathrm{u}) - \mathrm{pN}(\mathrm{x}\,\lambda(\mathrm{u}), \mathrm{y}\,\lambda(\mathrm{u})) \quad (2.7)$$

From Lemma 2.9, we see that Parametric general nonlinear nonconvex variational inequalities (2.4) and parametric general Wiener-Hopf equations (2.5) are equivalent. We use these equivalence to study the sensitivity analysis of general nonlinear non-convex variational inequalities. We assume that for some $\overline{\lambda} \in \Omega$ problem (2.5) has a solution $\overline{Z}$ and X is a closure of a ball in H centered at $\overline{Z}$. We want to investigate those conditions under which for each λ is a neighbourhood of $\overline{\lambda}$ then (2.5) has a unique solution $z(\lambda)$ near $\overline{z}$ and the function $z(\lambda)$ is (Lipschitz) continuous and differentiable.

Definition 2.10 For u; v $\in H, \lambda \in \Omega$ the mapping $N : HXH \to H$

Citation: Salahuddin (2015) Sensitivity Analysis for General Nonlinear Nonconvex Variational Inequalities. J Appl Computat Math 4: 206. doi:10.4172/2168-9679.1000206

is said to be (φ,ψ) relaxed cocoercive with respect to first argument and $g:\Omega XH\to H$ with constants $\varphi>0,\psi>0$, and Lipschitz continuous with respect to first and second argument if there exists a constants $\alpha>0,\beta>0$ such that

$$\langle N(x_\lambda(u),y\lambda(u))-N(x_\lambda(v)),g_\lambda(u)-g\lambda(v)\rangle\geq-\varphi\|N(x_\lambda(u),y_\lambda(u))-N(x_\lambda(v),y\lambda(v))\|^2$$

$$+\psi\|g\lambda(u)-g\lambda(v)\|^2$$

And

$$\|N(x\lambda(u),y\lambda(u))-N(x\lambda(v),y\lambda(v))\|\leq\alpha\|x\lambda(u)-x\lambda(v)\|+\beta\|y\lambda(u)-y\lambda(v)\|$$

$$\forall x\lambda(u)\in T\lambda(u),x\lambda(v)\in T\lambda(v),y\lambda(u)\in F\lambda(u),y\lambda(v)\in F\lambda(v)$$

Definition 2.11 A single valued mapping $g:HX\Omega\to H$ is said to be Lipschitz continuous if there exists a constant $\gamma>0$ such that

$$\|g\lambda(u)-g\lambda(v)\|\leq\gamma\|u-v\|,\forall u,v\in H$$

Definition 2.12 The set valued mapping $T:HX\Omega\to 2^H$ is said to be D-Lipschitz continuous if there exists a constant v > 0 such that

$$D(T_\lambda(u),T_\lambda(v))\leq v\|u-v\|,\forall u,v\in H,\lambda\in\Omega$$

where D is the Hausdorff metric.

Definition 2.13 Let $h:\Omega XH\to H$ be a single valued mapping. Then hλ is said to be ξ-relaxed cocoercive if there exists a constant $\xi<0$ such that

$$\langle h\lambda(u)-h\lambda(v),u-v\rangle\geq\xi\|h\lambda(u)-h\lambda(v)\|^2$$

and Lipschitz continuous if there exists a constant $\mu>0$ such that

$$\|h_\lambda(u)-h\lambda(v)\|\leq\mu\|u-v\|,\forall u,v\in H,\lambda\in\Omega$$

Main Results

In this section, we consider the case when the solution of the parametric general Wiener-Hopf equations (2.5) lies in the interior of X. We consider the map

$$E\lambda(z)=Pkrz-pN(x\lambda(u),y\lambda(u))=g\lambda(u)-pN(x\lambda(u),y\lambda(u)),\forall(z,\lambda)\in X\Omega \quad (3.1)$$

Where

$$h_\lambda(u)=Pk_rz \quad (3.2)$$

We have to show that the map $E\lambda(z)$ has a fixed point which is a solution of parametric general Wiener-Hopf equations (2.5). First of all we prove the map $E\lambda(z)$ defined by (3.1) is a contraction map with respect to z uniformly in $\lambda\in\Omega$ by using the techniques of Noor [10].

Lemma 3.1 Let P_{kr} be a Lipschitz continuous operator with constant $\delta=\dfrac{r}{r-r^1}$ Let $N:H\times H\to H$ be the Lipschitz continuous with first argument and second argument with Constants $\alpha>0,\beta>0$ respectively. Let h; g : $H\times\Omega\to H$ be the Lipschitz continuous with constants $\mu>0,\gamma>0$ respectively and $h\lambda$ be the ξ relaxed cocoercive with respect to the constant ξ < 0: Let T; F : $\Omega\times$ H ⟶ 2H be the D-Lipschitz continuous with constants v; x > 0; respectively. Let N be the (φ,ψ) relaxed cocoercive with respect to first argument and $g\lambda$ with constants $\varphi,\psi>0$ respectively. We have

$$\|E_\lambda(z_1)-E_\lambda(z_2)\|\leq\theta\|z_1-z_2\|$$

Where

$$\theta=\delta\frac{\sqrt{\gamma^2-2p(-\varphi(\alpha v+\beta x)^2+p^2(\alpha v+\beta x)^2}}{1-k} \quad (3.3)$$

$$k=\sqrt{1-2\xi\mu^2+\mu^2},\delta>0 \quad (3.4)$$

For

$$|p-\frac{\psi\gamma^2-\varphi(\alpha v+\beta x)^2}{(\alpha v+\beta x)^2}|<\frac{\sqrt{\delta^2(\psi\gamma^2-\varphi(\alpha v+\beta x)^2)^2-(\alpha v+\beta x)^2(\delta^2\gamma^2-(1-k)^2)}}{\delta(\alpha v+\beta x)^2}$$

$$\delta(\psi\gamma^2-\varphi(\alpha v+\beta x)^2)>(\alpha v+\beta x)\sqrt{(\delta\gamma-1+k)(\delta\gamma+1-k)}$$

$$\delta(\psi\gamma^2-\varphi(\alpha v+\beta x)^2)>(\alpha v+\beta x)$$

$$\psi\gamma^2>\varphi(\alpha v+\beta x)^2 \quad (3.5)$$

Proof. For all z1; z2 $\in x,\lambda\in\Omega$ from (3.1) we have

$$\|E_\lambda(z_1)-E_\lambda(z_2)\|=\|g_\lambda(u)-g_\lambda(u)-g_\lambda(v)-p(x_\lambda(u),y_\lambda(u))-N(x_\lambda(v),y_\lambda(v)))\| \quad (3.6)$$

Now

$$\begin{aligned}&\|g\lambda(u)-g_\lambda(v)-p(N(x_\lambda(u),y_\lambda(u))-N(x_\lambda(v),y_\lambda(v)))\|^2\\&\leq\|g_\lambda(u)-g_\lambda(v)\|^2-2p\langle N(x_\lambda(u),y_\lambda(u))-N(x_\lambda(v),y_\lambda(v)),g_\lambda(u)-g_\lambda(v)\rangle \quad (3.7)\\&+p^2\|N(x_\lambda(u),y_\lambda(u))-N(x_\lambda(v),y_\lambda(v),y_\lambda(v))\|^2\end{aligned}$$

Since N is Lipschitz continuous with respect to _rst and second argument and T; F are D-Lipschitz continuous with constants v; x > 0 respectively, we have

$$\begin{aligned}\|N(x_\lambda(u),y_\lambda(u))-N(x_\lambda(v),y_\lambda(v))\|&\leq\alpha\|x_\lambda(u)-x_\lambda(v)\|+\beta\|y_\lambda(u)-y_\lambda(v)\|\\&\leq\alpha D(T_\lambda(v))+\beta D(F_\lambda(u)-y_\lambda(u),F_\lambda(v))\\&\leq\alpha v\|u-v\|+\beta x\|u-v\|\\&\leq(\alpha v+\beta x)\|u-v\|\end{aligned} \quad (3.8)$$

And

$$\|g_\lambda(u)-g_\lambda(v)\|\leq\gamma\|u-v\| \quad (3.9)$$

From the (φ,ψ))-relaxed cocoercive mapping of N with respect to first argument and g_λ we have

$$\begin{aligned}\langle N(x_\lambda(u),y_\lambda(u))-N(x_\lambda(v),y_\lambda(v)),g_\lambda(v)\rangle&\geq-\varphi\|N(x_\lambda(u),y_\lambda(u))-N(x_\lambda(v),y_\lambda(v))\|^2\\&+\psi\|g_\lambda(u)-g_\lambda(v)\|^2\\&>-\varphi(\alpha v+\beta x)^2\|u-v\|^2+\psi\gamma^2\|u-v\|^2\\&\geq(-\varphi(\alpha v+\beta x)^2+\psi\gamma^2)\|u-v\|^2\end{aligned} \quad (3.10)$$

Hence from (3.7)-(3.10), we have

$$\begin{aligned}&\|g_\lambda(u)-g_\lambda(v)-p(N(x_\lambda(u),y_\lambda(u))-N(x_\lambda(v)))\|^2\leq\gamma^2\|\gamma^2\|u-v\|^2\\&-2p(-\varphi(\alpha v+\psi\gamma^2))\|u-v\|^2+p^2+p^2(\alpha v+\beta x)^2\|u-v\|^2\\&\leq(\gamma^2-2p(-\varphi(\alpha v+\beta x)^2+p^2(\alpha v+\beta x)^2\|u-v\|^2\end{aligned} \quad (3.11)$$

Therefore from (3.6) and (3.11), we have

$$\|E_\lambda(z_1)-E_\lambda(z_2)\|\leq\sqrt{\gamma^2-2p(-\varphi(\alpha v+\beta x)^2+\psi\gamma^2)+p^2(\alpha v+\beta x)^2}\|u-v \quad (3.12)$$

Also from (3.2) and Lipschitz continuity of projection mapping PKr with constant δ; we

Have

$$\begin{aligned}\|u-v\|&\leq\|u-v-(h_\lambda(u)-h_\lambda(v))\|+\|p_{kr}(z_1)-p_{kr}(z_2)\|\\&\leq\|u-v-(h_\lambda(u)-h_\lambda(v))\|+\delta\|z_1-z_2\|\end{aligned} \quad (3.13)$$

Since h is Lipschitz continuous with constant $\mu>0$ and ξ relaxed cocoercive with constant ξ <0 we have

$$\begin{aligned} &\| u-v-(h_\lambda(u)-h_\lambda(v))\|^2 \le \| u-v\|^2 - 2\langle h_\lambda(v), u-v\rangle + \| h_\lambda(u)-h_\lambda(v)\|^2 \\ &\le \| u-v\|^2 - 2\xi \| h_\lambda(u)-h_\lambda(v)\|^2 + \| h_\lambda(u)-h_\lambda(v)\|^2 \\ &\le \| u-v\|^2 - 2\xi\mu^2 \| u-v\|^2 + \mu^2 \| u-v\|^2 \\ &\le \| u-v-(h_\lambda(u)-h_\lambda(v))\| \le k \| u-v\| \end{aligned} \tag{3.14}$$

Where $k=\sqrt{1-2\xi\mu^2+\mu^2}$ From (3.13) and (3.14) we have

$$\| u-v\| \le \frac{\delta}{1-k}\| 2_1 - z_2\| \tag{3.15}$$

Combining (3.12),(3.15) and using (3.3) we have

$$\begin{aligned} &\| E_\lambda(z_1)-E_\lambda(z_2)\| \le (1-\sigma_n)\| z_1-z_2\| \\ &+\sigma_n\delta\frac{\sqrt{\gamma^2-2p(-\varphi(\alpha v+\beta x)^2+p^2(\alpha v+\beta x)^2}}{1-k}\| z_1-z_2 \\ &=(1-\sigma_n)\| z_1-z_2\| + \sigma_n\theta\| z_1-z_2\| \end{aligned} \tag{3.16}$$

Where

$$\theta=\delta\frac{\sqrt{\gamma^2-2p(-\varphi(\alpha v+\beta x)^2+\psi\gamma^2)+p^2(\alpha v+\beta x)^2}}{1-k}$$

From (3.5) it follows that $\theta<1$ and consequently the map $E_\lambda(z)$ de_ne by (3.12) is a contraction map and has a fixed point $z(\lambda)$ which is the solution of parametric general Wiener-Hopf equations (2.5).

Remark 3.2 From Lemma 3.1, we see that the map $E_\lambda(z)$ define by (2.1) has a unique fixed point $z(\lambda)$ that is $z(\lambda) = E_\lambda(z)$ Also by assumption the function $\overline{z}$ for $\lambda=\overline{\lambda}$ is a solution of parametric general Wiener-Hopf equations (2.5). Again by Lemma 3.1 we see that $\overline{z}$ for $\lambda=\overline{\lambda}$ is a fixed point of $E_\lambda(z)$ and it is also a fixed point of $E_{\overline{\lambda}}(z)$ Consequently, we conclude that $z(\overline{\lambda})=\overline{z}=E_{\overline{\lambda}}(z(\overline{\lambda}))$.

Using Lemma 3.1 we can prove the continuity of the solution $z(\lambda)$ of parametric general Wiener-Hopf equations (2.5). However for the sake of completeness and to convey the idea of the techniques involved, we give the proof.

Lemma 3.3 Assume that the mappings $T_\lambda(.), F_\lambda(.)$ are D-Lipschitz continuous and $g_\lambda(.), h_\lambda(.)$ are Lipschitz continuous with respect to the parameter λ If the mapping N is Lipschitz continuous with first and second argument respectively, and the map $\lambda\to p_{kr}(z), \lambda\to T_\lambda(u), \lambda\to F_\lambda(u), _\lambda\to g_\lambda(u), \lambda\to h_\lambda(u)$ are continuous (or Lipschitz continuous), the function $z(\lambda)$ satisfying the (2.3) is Lipschitz continuous at $\lambda=\overline{\lambda}$

Proof. For all $\lambda\in\Omega$ invoking Lemma 3.1 and the triangle inequality, we have

$$\begin{aligned} \| z(\lambda)-z(\overline{\lambda})\| &\le \| E_\lambda(z(\lambda))-E_\lambda(z(\overline{\lambda}))\| + \| E_\lambda(z(\overline{\lambda}))-E_{\overline{\lambda}}(z(\overline{\lambda}))\| \\ &\le \theta\| z(\lambda)-z(\overline{\lambda})\| + \| E_\lambda(z(\overline{\lambda}))-E_{\overline{\lambda}}(z(\overline{\lambda}))\| \end{aligned} \tag{3.17}$$

From (3.1) and the fact that the mapping $N, T_\lambda, F_\lambda, h_\lambda$ and g_λ are Lipschitz continuous with respect to the parameter λ we have

$$\begin{aligned} &\| E_\lambda(z(\overline{\lambda}))-E_{\overline{\lambda}}(z(\overline{\lambda}))\| = \| g_{\overline{\lambda}}(u(\overline{\lambda})) - p(N(T_\lambda(u(\overline{\lambda})), F_\lambda(u(\overline{\lambda}))) - N(T_{\overline{\lambda}}(u(\overline{\lambda})), F_{\overline{\lambda}}(u(\overline{\lambda}))))\| \\ &\le \gamma\| \lambda-\overline{\lambda}\| + p(\alpha\| T_{\overline{\lambda}}(u(\overline{\lambda}))\| + \beta\| F_{\overline{\lambda}}(u(\overline{\lambda}))\|) \\ &\le \gamma\| \lambda-\overline{\lambda}\| + p(\alpha v\| \lambda-\overline{\lambda}\| + \beta x\| \lambda-\overline{\lambda}\|) \\ &\le (\gamma+p(\alpha v+\beta x))\| \lambda-\overline{\lambda}\| \end{aligned} \tag{3.18}$$

Combining (3.17) and (3.18), we obtain

$$\| z(\lambda)-z(\overline{\lambda})\| \le \frac{(\gamma+p(\alpha v+\beta x))}{1-\theta}\| \lambda-\overline{\lambda}\|, \forall\lambda,\overline{\lambda}\in\Omega$$

from which the required results follows.

We now state and prove the main result of this paper which is motivation of the next result.

Theorem 3.4 Let $\overline{u}$ be a solution of parametric general nonlinear nonconvex variational inequalities (2.4) and $\overline{n}$ be the solution of parametric general Wiener-Hopf equations (2.5) for $\lambda=\overline{\lambda}$. Let $h_\lambda(u)$ be ξ _-relaxed cocoercive mapping and Lipschitz continuous mapping, and $T_\lambda(u), F_\lambda(u)$ be the D-Lipschitz continuous mappings and N be (φ,ψ))-relaxed coco-ercive mapping with respect to first argument and g_λ and g_λ be the Lipschitz continuous for all $u,v\in x$ If the map $\lambda\to p_{kr}(z), \lambda\to T_\lambda(u), \lambda\to F_\lambda(u), \lambda\to g_\lambda(u), \lambda\to h_\lambda(u)$ are Lipschitz (continuous) mappings at $\lambda=\overline{\lambda}$ then there exists a neighbourhood M of Ω of $\overline{\lambda}$ such that for $\lambda\in M$ parametric general Wiener-Hopf equation (2.5) has a unique solution $z(\lambda)$ in the interior of $x, z(\overline{\lambda})=z(\lambda)$ and $z(\lambda)$ is (Lipschitz) continuous at $\lambda=\overline{\lambda}$.

Proof. Its proof follows from Lemma 3.1, 3.3 and Remark 3.2.

References

1. Stampacchia G (1964) An Formes bilineaires coercitives sur les ensembles convexes, C. R. Acad. Sci. Paris, 258: 4413-4416.
2. Agarwal RP, Cho YJ, Huang NJ (2000) Sensitivity analysis for strongly nonlinear quasi variational inclusions. Appl Math Lett 13: 19-24.
3. Anastassiou GA, Salahuddin, Ahmad MK (2013) Sensitivity analysis for generalized set valued variational inclusions. J Concrete Appl Math 11: 292-302.
4. Baiocchia C, Capelo A (1984) Variational Quasi Variational Inequalities, Wiley London.
5. Clarke FH, Ledyaw YS, Stern RJ, Wolenski PR (1998) Nonsmooth Analysis and Control Theory, Springer-Verlag, New York.
6. Dafermos S (1988) Sensitivity analysis in variational inequalities. Math Oper Res 13: 421-434.
7. Giannessi F, Maugeri A (1995) Variational Inequalities and Network Equilibrium Problems, Plenum Press, New York, NY, USA.
8. Kyparisis J (1987) Sensitivity analysis frame work for variational inequalities. Math Prog 38: 203-213.
9. Liu J (1995) Sensitivity analysis in nonlinear programs and variational inequalities via continuous selection. SIAM J Control Optim 33: 1040-1068.
10. Noor MA (1993) Wiener Hopf equations and variational inequalities. JOTA 79: 197-206.
11. Noor MA, Noor KI (2013) Sensitivity analysis of general nonconvex variational inequalities, JIA, 302.
12. Poliquin RA, Rockafellar RT, Thibault L (2000) Local differentiability of distance functions. Trans Amer Math Soc 352: 5231-5249.
13. Qiu Y, Magnanti TL (1989) Sensitivity analysis for variational inequalities defined on polyhedral sets. Math Oper Res 14: 410-432.
14. Shi P (1991) Equivalence of Wiener-Hopf equations with variational inequalities. Proc Amer Math Soc 111: 339-346.
15. Aubin JP, Ekeland I (1984) Applied Nonlinear Analysis, Wiley, New York.
16. Verma RU (2006) Generalized -resolvent operator technique and sensitivity analysis for relaxed cocoercive variational inclusions 7: 70-83.
17. Wen DJ (2010) Projection methods for a generalized system of nonconvex variational inequalities with di_erent nonlinear operators. Nonlinear Anal 73: 2292-2297.