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Semi Analytical-Solution of Nonlinear Two Points Fuzzy Boundary Value Problems by Adomian Decomposition Method

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Abstract

In this paper the Adomian Decomposition Method (ADM) is employed to solve n^{th} order (n>2) non linear two point fuzzy boundary value problems (TPFBVP). The Adomian decomposition method can be used for solving nth order fuzzy differential equations directly without reduction to first order system. We illustrate the method in numerical experiment including fourth order nonlinear TPFBVP to show the capabilities of ADM.

Keywords: Fuzzy numbers; Fuzzy differential equations; Two point fuzzy boundary value problems; Adomian decomposition method

Introduction

Many dynamical real life problems may be formulated as a mathematical model. Many of them can be formulated either as a system of ordinary or partial differential equations. Fuzzy differential equations (FDEs) are a useful tool to model a dynamical system when information about its behavior is inadequate. FDE appears when the modeling of these problems was imperfect and its nature is under uncertainty. FDEs are suitable mathematical models to model dynamical systems in which there exist uncertainties or vagueness. These models are used in various applications including, population models [1-3], mathematical physics [4], and medicine [5,6]. In recent year's semi -analytical methods such as the Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM), Variational Iteration Method (VIM), Optimal Homotopy asymptotic method (OHAM) and Homotopy Analysis Method (HAM) have been used to solve fuzzy first and n^{th} order ordinary differential equations. For n^{th} order fuzzy initial value problems, The ADM was employed in [7] to solve second order linear fuzzy initial value problems. Abbasbandy et al. [8] used the VIM to solve linear system of first order fuzzy initial value problems. Moreover, some of these methods have been also used to obtain a semi-analytical solution of TPFBVP. VIM has been used in [9] to solve linear TPFBVP. Other method like undetermined fuzzy coefficients method has been introduced in [10] in order to obtain an approximate solution of second order linear TPFBVP.

The ADM have been introduced in [11,12] and has been applied to a wide class of deterministic and stochastic problems of mathematical and physical sciences [13-15]. This method provides the solution as a rapidly convergent series with components that are elegantly computed. This method can be used to solve all types of linear and nonlinear equations such as differential and integral equations, so it is known as a powerful method. Another important advantage of this method is that it can reduce the size of computations, while increases the accuracy of the approximate solutions so it is known as a powerful method

In this paper, our aim is to formulate ADM from crisp into fuzzy case in order to solve nonlinear n^{th} order TPFBVP directly. To the best of our knowledge, this is the first attempt at solving the n^{th} order TPFBVP using the ADM. The structure of this paper is as follows: In section 2, some basic definitions and notations are given about fuzzy numbers that will be used in other sections we discussed. In section 3, the structure of ADM is formulated for solving high order TPFBVP. In section 4, we present a numerical example and finally, in section 5, we give the conclusion of this study Figure 1.

Fuzzy Numbers

Fuzzy numbers are a subset of the real numbers set, and represent uncertain values. Fuzzy numbers are linked to degrees of membership which state how true it is to say if something belongs or not to a determined set Figure 2. A fuzzy number [16] μ is called a triangular fuzzy number if defined by three numbers $\alpha < \beta < \gamma$ where the graph of $\mu(x)$ is a triangle with the base on the interval $[\alpha, \beta]$ and vertex at $x = \beta$, and its membership function has the following form:

$$\mu(x;\alpha,\beta,\gamma) = \begin{cases} 0, \ if\, x < \alpha \\ \dfrac{x-\alpha}{\beta-\alpha}, \ if\, \alpha \le x \le \beta \\ \dfrac{\gamma - x}{\gamma-\beta}, \ if\, \beta \le x \le y \\ 1, \ if\, x > \gamma \end{cases}$$

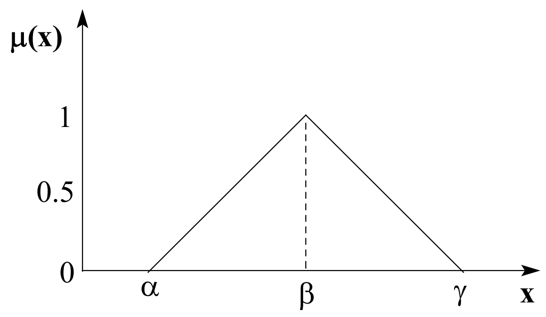

Figure 1: Triangular Fuzzy Number.

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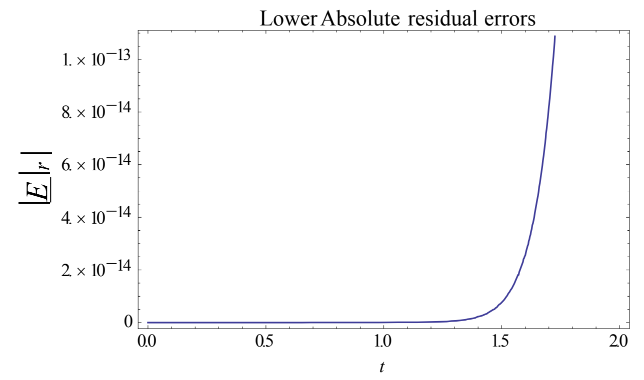

Figure 2: Absolute residual errors of the lower solution of Eq. (18) at r=0.8.

and its r-level is $[\mu]_r=[\alpha+r(\beta-\alpha),\ \gamma-r(\gamma-\beta)],\ r\in[0,1]$ In this paper the class of all fuzzy subsets of R will be denoted by RF and satisfy the following properties [16,17]:

1. $\mu(t)$ is normal, i.e $\exists t_0\in R$ with $\mu(t_0)=1$,
2. $\mu(t)$ is convex fuzzy set, i.e. $\mu(\lambda t+(1-\lambda)s)\ge\min\{\mu(t),\mu(s)\}\forall t,s\in\mathbb{R},\lambda\in[0,1]$,
3. μ upper semi-continuous on R, and $\overline{\{t\in\mathbb{R}:\mu(t)>0\}}$ is compact

RF is called the space of fuzzy numbers and R is a proper subset of RF.

Define the r-level set $x\in R,[\mu]_r=\{x\setminus\mu(x)\ge r\},0\le r\le1$ where $[\mu]_0=\{x\setminus\mu(x)>0\}$ is compact which is a closed bounded interval and denoted by $[\mu]_r=(\underline{\mu}(t),\overline{\mu}(t))$. In the parametric form, a fuzzy number is represented by an ordered pair of functions $(\underline{\mu}(t),\overline{\mu}(t))$, $r\in[0,1]$ which satisfies [18]:

1. $\underline{\mu}(t)$ is a bounded left continuous non-decreasing function over [0.1].
2. $\overline{\mu}(t)$ is a bounded left continuous non-increasing function over [0.1] .
3. $\underline{\mu}(t)\le\overline{\mu}(t)$, $r\in[0,1]$.

A crisp number r is simply represented by $\underline{\mu}(r)=\overline{\mu}(r)=r$, $r\in[0,1]$.

Fuzzification and Defuzzification of ADM

The general structure of ADM for solving crisp n^{th} order two point boundary value problems involving ordinary differential equations are mentioned in [18-20]. To solve n^{th} order TPFBVP, we need to fuzzify ADM and then defuzzify it Figure 3.

Consider the following general n^{th} order TPFBVP

$$\tilde{y}^{(n)}(t)=f\left(t,\tilde{y}(t),\tilde{y}^{'}(t),\tilde{y}^{''}(t),\ldots\tilde{y}^{(n-1)}(t)\right)+\tilde{w}(t),\ t\in[t_0,T]\ (1)$$

subject to two point boundary conditions

$$\begin{cases}\tilde{y}(t_0)=\tilde{\alpha}^{(0)},\tilde{y}^{'}(t_0)=\tilde{\alpha}^{(1)},\ldots,\tilde{y}^{(k)}(t_0)=\tilde{\alpha}^{(k)},\\ \tilde{y}(T)=\tilde{\beta}^{(0)},\tilde{y}^{'}(T)=\tilde{\beta}^{(1)},\ldots,\tilde{y}^{(n-k-r)}(T)=\tilde{\beta}^{(n-k-s)}\end{cases}\qquad(2)$$

where $0\le k\le n-2$ is an integer(For cases when n=4 and n=6, [21, 22]), $\tilde{y}(t)$ is a fuzzy function of the crisp variable t. Also, f is fuzzy function of the crisp variable t and the fuzzy variable $\tilde{y}$. Here $\tilde{y}^{(n)}$ is the n^{th} order fuzzy H-derivative [23] of $\tilde{y}(t)$ and the H-derivatives $\tilde{y}^{'}(t),\tilde{y}^{''}(t),\ldots\tilde{y}^{(n-1)}(t)$. Moreover the fuzzy boundary conditions $\tilde{y}(t_0),\tilde{y}^{'}(t_0),\ldots\tilde{y}^{(k)}(t_0),\tilde{y}(T),y',\ldots\tilde{y}^{(n-k-s)}(T)$ are convex fuzzy numbers as in section 2. We denote the fuzzy function y by $\tilde{y}=[\underline{y},\overline{y}]$, for $t\in[t_0,T]$ and $r\in[0,1]$ it means that the r-level set of $y(t)$ can be defined as:

$$[\tilde{y}(t)]_r=[\underline{y}(t;r),\overline{y}(t;r)],$$

$$[\tilde{y}^{'}(t)]_r=[\underline{y}^{'}(t;r),\overline{y}^{'}(t_0;r)],\ldots[\tilde{y}^{(n-1)}(t)]_r=[\underline{y}^{(n-1)}(t;r),\overline{y}^{(n-1)}(t;r)]$$

$$[\tilde{y}(t_0)]_r=[\underline{y}(t_0;r),\overline{y}(t_0;r)],$$

$$[\tilde{y}^{'}(t_0)]_r=[\underline{y}^{'}(t_0;r),\overline{y}^{'}(t_0;r)],\ldots[\tilde{y}^{(k)}(t_0)]_r=[\underline{y}^{(k)}(t_0;r),\overline{y}^{(k)}(t_0;r)],$$

$$[\tilde{y}^{'}(T)]_r=[\underline{y}^{'}(T;r),\overline{y}^{'}(T;r)],\ldots[\tilde{y}^{(k)}(T)]_r=[\underline{y}^{(k)}(T;r),\overline{y}^{(k)}(T;r)]$$

Where W(t) is crisp or fuzzy inhomogeneous term such that $[\tilde{w}(t)]_r=[\underline{w}(t;r),\overline{w}(t;r)]$.

Since $y^{(n)}(t)=f\left(t,y(t),y'(t),y''(t),\ldots y^{(n-1)}(t)\right)+w(t)$

If we let $(t)=y(t),y'(t),y''(t),\ldots y^{(n-1)}(t)$, such that

$\tilde{\mathcal{Y}}(t;r)=[\underline{\mathcal{Y}}(t;r),\overline{\mathcal{Y}}(t;r)]=[\underline{y}(t;r),\underline{y}^{'}(t;r),\ldots,\underline{y}^{(n-1)}(t;r),\overline{y}(t;r),\overline{y}^{'}(t;r),\ldots,\overline{y}^{(n-1)}(t;r)]$

Also, we can write

$$[\tilde{f}(t,\tilde{\mathcal{Y}})]_r=[\underline{f}(t,\tilde{\mathcal{Y}};r),\overline{f}(t,\tilde{\mathcal{Y}};r)]\qquad(3)$$

by using Zadeh extension principles [24,25] we have

$\tilde{f}(t,\tilde{\mathcal{Y}}(t;r))=[\underline{f}(t,\tilde{\mathcal{Y}}(t;r)),\underline{f}(t,\tilde{\mathcal{Y}}(t;r)),]$, such that

$$\underline{f}(t,\tilde{\mathcal{Y}}(t;r))=F(t,\underline{\mathcal{Y}}(t;r),\overline{\mathcal{Y}}(t;r))=F(t,\tilde{\mathcal{Y}}(t;r))$$

$$\overline{f}(t,\tilde{\mathcal{Y}}(t;r))=G(t,\underline{\mathcal{Y}}(t;r),\overline{\mathcal{Y}}(t;r))=G(t,\tilde{\mathcal{Y}}(t;r))$$

Then we have

$$\underline{y}^{(n)}(t;r)=\mathcal{F}(t,\tilde{\mathcal{Y}}(t;r))+\underline{w}(t;r)\qquad(4)$$

$$\overline{y}^{(n)}(t;r)=\mathcal{G}(t,\tilde{\mathcal{Y}}(t;r))+\overline{w}(t;r)\qquad(5)$$

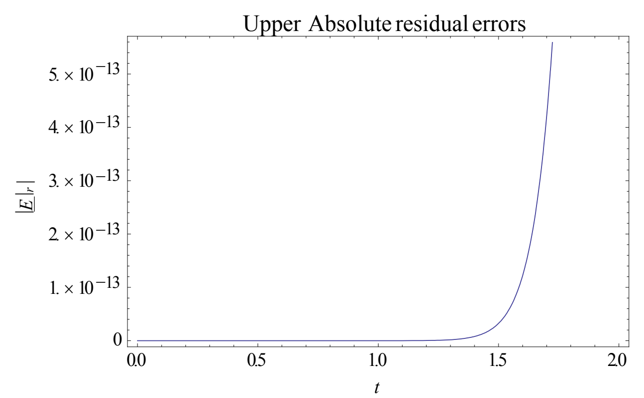

Figure 3: Absolute residual errors of the upper solution of Eq. (18) at r=0.8.

where the membership function of $\mathcal{F}\left(t,\tilde{\mathcal{Y}}(t;r)\right)+\underline{w}(t;r)$ and $\mathcal{G}\left(t,\tilde{\mathcal{Y}}(t;r)\right)+\overline{w}(t;r)$ can be defined as

$$\mathcal{F}\left(t,\tilde{\mathcal{Y}}(t;r)\right)+\underline{w}(t;r)=\min\{\tilde{y}^{(n)}\left(t,\tilde{\mu}(r)\right):\mu|\mu\in\left[\tilde{\mathcal{Y}}(t;r)\right]_r\},$$

$$\mathcal{G}\left(t,\tilde{\mathcal{Y}}(t;r)\right)+\overline{w}(t;r)=\max\{\tilde{y}^{(n)}\left(t,\tilde{\mu}(r)\right):\mu|\mu\in\left[\tilde{\mathcal{Y}}(t;r)\right]_r\},$$

for all r < [0,1], Esq.(4) –(5) can be written as follows

$$L_n\,\underline{y}(t;r)=\mathcal{F}\left(t,\tilde{\mathcal{Y}}(t;r)\right)+\underline{w}(t;r),\ t\in[t_0,T] \quad (6)$$

$$L_n\,\overline{y}(t;r)=\mathcal{G}\left(t,\tilde{\mathcal{Y}}(t;r)\right)+\overline{w}(t;r),\ t\in[t_0,T] \quad (7)$$

where $\tilde{\mathcal{L}}_n=\left[\underline{\mathcal{L}}_n,\overline{\mathcal{L}}_n\right]$ are the linear operators with $\tilde{\mathcal{L}}_n=\frac{d^{(n)}}{dt^{(n)}}$ and, F,G are nonlinear operators .define the inverse operators $\tilde{\mathcal{L}}_n^{-1}=\int_0^t\int_0^t\int_0^t\ldots[.]_r d\tau d\tau\ldots d\tau$ and applying it on Eqs . (6) and (7) we have

$$\underline{y}(t;r;\underline{a}_i(r))=\underline{y}_0(t;r;\underline{a}_i(r))+L_n^{-1}\underline{w}(t;r)+L_n^{-1}\mathcal{F}\left(t,\tilde{\mathcal{Y}}(t;r)\right) \quad (8)$$

$$\overline{y}(t;r;\overline{a}_i(r))=\overline{y}_0(t;r;\overline{a}_i(r))+L_n^{-1}\overline{w}(t;r)+L_n^{-1}\mathcal{G}\left(t,\tilde{\mathcal{Y}}(t;r)\right) \quad (9)$$

where $\underline{y}_0(t;r;\underline{a}_i(r))$, and $\overline{y}_0(t;r;\overline{a}_i(r))$ are initials guessing and can be defining as follows:

$$\underline{y}_0(t;r;\underline{a}_i(r))=\underline{a}_1(r)+\underline{a}_2(r)t+\frac{\underline{a}_3(r)}{2!}t^2+\ldots+\frac{\underline{a}_n(r)}{(n-1)!}t^{(n-1)} \quad (10)$$

$$\overline{y}_0(t;r;\overline{a}_i(r))=\overline{a}_1(r)+\overline{a}_2(r)t+\frac{\overline{a}_3(r)}{2!}t^2+\ldots+\frac{\overline{a}_n(r)}{(n-1)!}t^{(n-1)} \quad (11)$$

where $\underline{a}_i(r)$ and $\overline{a}_i(r)$ are fuzzy constants to be determined i=1,2…n. The ADM introduces the solution $\tilde{y}(t;r;\tilde{a}_i(r))$ and the nonlinear fuzzy functions F and G by infinite series

$$\tilde{y}(t;r;\tilde{a}_i(r))=\sum_{j=0}^{\infty}\tilde{y}_i(t;r;\tilde{a}_i(r)) \quad (12)$$

and

$$\begin{cases}F\left(t,\tilde{\mathcal{Y}}(t;r)\right)=\sum_{j=0}^{\infty}\underline{A}_j(t;r)\\ G\left(t,\tilde{\mathcal{Y}}(t;r)\right)=\sum_{j=0}^{\infty}\overline{A}_j(t;r)\end{cases} \quad (13)$$

where $\tilde{A}_i(t;r)$ are the so-called Adomian polynomials [19,20]. Now let

$$\begin{cases}\underline{\varnothing}(t;r;\underline{a}_i(r))=\underline{y}_0(t;r;\underline{a}_i(r))+\underline{\mathcal{L}}_n^{-1}\underline{w}(t;r)\\ \overline{\varnothing}(t;r;\overline{a}_i(r))=\overline{y}_0(t;r;\overline{a}_i(r))+\overline{\mathcal{L}}_n^{-1}\overline{w}(t;r)\end{cases} \quad (14)$$

then substituting (11) and (12) into (8) and (9) respectively we obtain

$$\sum_{i=1}^{\infty}\underline{y}_i(t;r;\tilde{a}_i(r))=\underline{\Psi}(t;r;\underline{a}_i(r))+\underline{\mathcal{L}}_n^{-1}\sum_{j=0}^{\infty}\underline{A}_j(t;r)\frac{1}{2} \quad (15)$$

$$\sum_{i=1}^{\infty}\overline{y}_i(t;r;\tilde{a}_i(r))=\underline{\Psi}(t;r;\underline{a}_i(r))+\overline{\mathcal{L}}_n^{-1}\sum_{j=0}^{\infty}\overline{A}_j(t;r) \quad (16)$$

According to the ADM, the components $\tilde{y}_i(t;r;\tilde{a}_i(r))$ can be determined as

$$\begin{cases}\underline{y}_0(t;r;\underline{a}_i(r))=\underline{\Psi}(t;r;\overline{a}_i(r))]\\ \underline{y}_1(t;r;\underline{a}_i(r))=2\,\underline{L}_4^{-1}\underline{A}_0(\underline{y}_0(t;r;\underline{a}_i(r)))\\ \underline{y}_{n+1}(t;r;\overline{a}_i(r))=\underline{L}_4^{-1}\underline{A}_n(\sum_{i=0}^{n}\underline{y}_i(t;r;\underline{a}_{i+1}(r)))\end{cases} \quad (17)$$

and for the upper bound

$$\begin{cases}\tilde{y}_0(t;r;\tilde{a}_i(r))=[\overline{\Psi}(t;r;\overline{a}_i(r))]^2\\ \tilde{y}_1(t;r;\tilde{a}_i(r))=2\,\overline{L}_4^{-1}\overline{A}_0(\underline{y}_0(t;r;\tilde{a}_i(r)))\\ \overline{y}_{n+1}(t;r;\overline{a}_i(r))=\overline{L}_4^{-1}\overline{A}_n(\sum_{i=0}^{n}\tilde{y}_i(t;r;\overline{a}_i(r)))\end{cases} \quad (18)$$

Since the approximate solution series $\tilde{y}(t;r;\tilde{a}_i(r))$ contain the constants $\tilde{a}_i(r)$ which can determine easily by using the boundary conditions (2) for all $r\in[0,1]$ and for i=1,2,…,n.

Numerical Example

In this section employ OHAM on two examples of a linear and non-linear fourth order TPFBVP and present their approximate results with absolute errors. These examples were presented in [26] for the crisp case (non- fuzzy).

Consider the fourth order nonlinear TPFBVP

$$y^{(4)}(t)=y(t)^2+1,\ 0\le t\le 2 \quad (18)$$

$$y(0)=[0.1r-0.1,0.1-0.1r],\ y(2)=[0.1r-0.1,0.1-0.1r]$$

$$y'(0)=[0.1r-0.1,0.1-0.1r],\ y'(2)=[0.1r-0.1,0.1-0.1r]$$

$$\forall\ r\in[0,1].$$

Define the linear operator of Eq. (18) $\tilde{\mathcal{L}}_4$ with the inverse operator $\tilde{\mathcal{L}}_4^{-1}$, then the initial approximation guesses

$$\tilde{y}(0;r;\tilde{a}_i(r))=\tilde{a}_1(r)+\tilde{a}_2(r)t+\frac{1}{2!}\tilde{a}_3(r)t^2+\frac{1}{3!}\tilde{a}_4(r)t^3 \quad (19)$$

such that

$$\tilde{\Psi}(t;r;\tilde{a}_i(r))=\tilde{y}_0(t;r;\tilde{a}_i(r))+\tilde{L}_4^{-1}(1) \quad (20)$$

for i=1,2,3,4 and $r\in[0,1]$.

According to section 3 the ADM approximate series solution of Eq. (18) is given by

$$\sum_{j=1}^{\infty}\tilde{y}_j(t;r;\tilde{a}_i(r))=\tilde{\Psi}(t;r;\tilde{a}_i(r))+\tilde{L}_4^{-1}\sum_{j=0}^{\infty}\tilde{A}_j(t;r) \quad (21)$$

Here $\tilde{A}_j(t;r)=\sum_{j=0}^{n-1}\tilde{y}_j(t;r;\tilde{a}_i(r))\tilde{y}_{n-1-j}(t;r;\tilde{a}_i(r))$ for i=1,2,3,4>1.

According to the ADM in section 3, the n components of $\tilde{y}_j(t;r;\tilde{a}_i(r))$ can be determined as

$$\begin{cases}\tilde{y}_0(t;r;\tilde{a}_i(r))=[\tilde{\Psi}(t;r;\tilde{a}_i(r))]^2\\ \tilde{y}_1(t;r;\tilde{a}_i(r))=2\,\tilde{L}_4^{-1}[\tilde{y}_0(t;r;\tilde{a}_i(r))\tilde{y}_1(t;r;\tilde{a}_i(r))]\\ \tilde{y}_n(t;r;\tilde{a}_i(r))=\tilde{L}_4^{-1}\tilde{A}_n(\sum_{i=0}^{n}\tilde{y}_i(t;r;\tilde{a}_i(r)))\end{cases} \quad (22)$$

After ten terms of the ADM approximate solution series we have

$$\tilde{y}(t;r;\tilde{a}_i(r))=\sum_{j=0}^{10}\tilde{y}_j(t;r;\tilde{a}_i(r)) \quad (23)$$

for i=1,2,3,4 and $r \in [0,1]$. Now to determine the values of $\tilde{a}_i(r)$, we substitute the boundary conditions of Eq.(4.1) with the ADM solution series (23) in (18), we obtained

Substitute these values in Eq. (12) to get 10- order ADM approximate solution series. The following tables show ADM approximate solution series $\sum_{j=0}^{5} \tilde{y}_j(t;r;\tilde{a}_i(r))$ at t=0.8 and for all $\in [0,1]$. Since Eq. (18) is without exact analytical solution, so to show the accuracy of 10-order ADM absolute errors $[\underline{E}]_r$ and $[\overline{E}]_r$ of solutions $\underline{y}(0.8;r)$ and $\overline{y}(0.8;r)$, we define residual absolute error [27] as follows

r	$\underline{a}_1(r)$	$\underline{a}_2(r)$	$\underline{a}_3(r)$	$\underline{a}_4(r)$
0	−0.5	−0.5	1.916150	−2.76970
0.25	−0.375	−0.375	1.503530	−2.27339
0.5	−0.25	−0.25	1.102100	−1.81276
0.75	−0.125	−0.125	0.712063	−1.38835
1	0	0	0.333655	−1.00071

Table 1: Lower bound $\sum_{i=1}^{4} \underline{a}_i(r)$ values.

r	$\overline{a}_1(r)$	$\overline{a}_2(r)$	$\overline{a}_3(r)$	$\overline{a}_4(r)$
0	0.5	0.5	−1.058770	0.1702920
0.25	0.375	0.375	−0.7293630	−0.0642764
0.5	0.25	0.25	−0.3873120	−0.3380610
0.75	0.125	0.125	−0.0328861	−0.6504120
1	0	0	0.3336550	−1.000710

Table 2: Upper bound $\sum_{i=1}^{4} \overline{a}_i(r)$ values.

r	$\underline{y}\left(0.8;r;\sum_{i=1}^{4}\underline{a}_i(r)\right)$	$[\underline{E}]_r$	$\overline{y}\left(0.8;r;\sum_{i=1}^{4}\overline{a}_i(r)\right)$	$[\overline{E}]_r$
0	−0.501024	3.77476×10^{-15}	0.5981310	4.88498×10^{-16}
0.25	−0.367949	6.66134×10^{-16}	0.4562040	4.44089×10^{-16}
0.5	−0.233698	1.11022×10^{-16}	0.3156360	0
0.75	−0.0439827	0	0.1763930	2.22045×10^{-16}
1	0.0384436	0	0.0384436	2.22045×10^{-16}

Table 3: Numerical results by 10-order ADM for the lower bound of Eq. (18) at *t*=0.8 for all *r* ∈ [0,1].

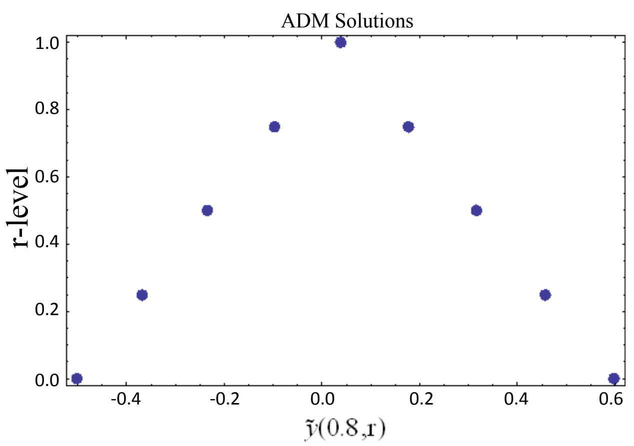

Figure 4: ADM solution of Eq. (18) when t = 0.8 for all 0 ≤ *r* ≤ *1*.

$$[\tilde{E}]_r = \left| \left(\sum_{j=0}^{10} \tilde{y}_j(t;r;\tilde{a}_i(r)) \right)^{(4)} + e^{-t} \left[\sum_{j=0}^{10} \tilde{y}_j(t;r;\tilde{a}_i(r)) \right]^2 - 1 \right| \quad (24)$$

From Tables 1-3 and Figure 4 one can see that the numerical results are satisfies the convex symmetric triangular fuzzy number and we have the same results for all $0 \le t \le 2$.

Conclusions

In this study, a semi- analytical method ADM was applied to obtain an approximate solution n[th] order nonlinear TPFBVP. A scheme based ADM approximate the solution of n[th] order TPFBVP has been formulated to obtain the approximate solution directly with our reduced in to first order system. From the nonlinear TPFIVP the accuracy of ADM can be determined even these equations without exact analytical solution. ADM give more accurate solution when the number of terms of the series solution was increased. Numerical examples which included nonlinear TPFBVP show the efficiency of the implemented of this semi- analytical method. The problem results satisfied the fuzzy numbers properties by taking the triangular fuzzy numbers shape.

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