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Scale-Free Networks in Economics

Orlando Gomes*

Lisbon Higher Institute of Accounting and Administration (ISCAL-IPL) and Business Research Unit of the Lisbon University Institute (BRU/LUI), Portugal

Complex Networks

Social and economic phenomena might be approached, in many different contexts, through the construction of networks that highlight the local interaction among heterogeneous agents. Real-world networks concerning human relations in the society or in the economy involve a large number of nodes, a large number of links connecting them and an evolving structure where both nodes and links may be generated or may disappear at each time period. They clearly are complex networks.

Complex networks are defined by Boccaletti et al. [1] as interaction structures that are irregular, that involve thousands or millions of nodes, and that change their shape as time elapses. High-dimensionality, heterogeneity and evolution are, thus, the main keywords that serve to characterize what a complex network truly is. Understanding the main regularities defining a complex network is vital to gain a comprehensive view on many important issues concerning the essence of collective human action.

At the mentioned level, an important discovery was made by Barabási et al., [2]. These authors claimed that many observable networks display power-law shaped degree distributions and, consequently, they can be designated scale-free networks. When the degree of a network follows a power-law distribution, this essentially signifies that a restrict number of nodes is strongly connected to the rest of the network and that a large percentage of nodes is poorly connected, i.e., they exhibit few links to other points in the network.

As Zschaler [3] emphasizes, such network configuration implies the existence of 'hubs', i.e., of nodes with a high degree of centrality, what allows for the presence of short average path lengths across the network (the small-world property highlighted by Zanette [4], to characterize real-world networks holds in this type of network structure).

The designation scale-free network originates on the properties of power-law distributions. This type of distribution is, as pointed out by Criado and Romance [5], scale invariant, in the sense that independently of the value of its argument (in the case, the connectivity degree), the distribution function preserves its shape. The power-law distribution function is a slow decreasing function (leptokurtic or fat-tailed) with no peak at its average value.

According to Barabási [6], the emergence of scale-free networks is essentially the outcome of two features that one often encounters in socio-economic relations: incremental growth and preferential attachment. Incremental growth relates to the idea that networks are not static structures; they evolve with the systematic addition of new nodes. Preferential attachment signifies that the new nodes that enter the network prefer to attach to the nodes that display a higher degree of connectivity; this is often described as a 'rich-gets-richer' process. Preferential attachment is the main reason for the generation of 'hubs' and for the power-law shape of the degree distribution of complex networks.

The interesting result is that the couple of properties mentioned in the previous paragraph are all that is required to describe the main features of a large number of network relations found in nature and in society. As highlighted by Barabási et al. [7], despite their diversity, most real networks apparently hold a similar architecture or a similar set of universal organizing principles; such organizing principles are associated essentially with their scale-free structure.

The Economy as a Scale-Free Network

Those who are acquainted with how business relations are organized in a decentralized economic environment will certainly encounter in the above description of scale-free networks some familiar features. A market for a given good, the financial system or the world economy, all display characteristics that allow to classify them as complex network structures and, more specifically, as scale-free networks. They are all frameworks involving thousands or millions of individual entities that have different degrees of connectivity inside the network; typically, a few economic agents have a dominant position, which might translate in a high degree of connectivity within the network, whereas the large majority of the agents are linked only with a small group of other agents.

Besides the mentioned facts, economic interaction structures are also characterized by an evolutionary process that might occur at multiple levels: agents eventually change status, as they are influenced by neighboring nodes, new agents can come into the network while others eventually abandon it, and links across vertices might be created or extinguished.

Most of the markets where goods and services are subject to transaction are imperfect competition markets that effectively show resemblance with the scale-free structure one has described. The notion of competition manifests itself on the atomistic nature of the network, when considering a large number of individual units; furthermore, the network's structure contemplates the possibility of free-entry and free-exit of agents.

On the other hand, the concept of network, as presented, renders necessarily the market environment as an environment where competition is not perfect. This occurs for two reasons: first, the existence of highly connected nodes indicates that there may exist agents who concentrate the power to dominate the market and influence transaction prices; second, when relations between agents occur through local interaction in a network, this implies that agents will not have the capacity to understand the functioning of the market as a whole and, therefore, their information set is not complete. In a certain sense, analyzing economic relations through the lenses of a

***Corresponding author:** Orlando Gomes, Lisbon Higher Institute of Accounting and Administration (ISCAL-IPL) and Business Research Unit of the Lisbon University Institute (BRU/LUI), Portugal, Tel: 351-933420915; E-mail: omgomes@iscal.ipl.pt

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network implies restricting the power of agents, who will be constrained in their ability to collect and process information about the whole of the economy, and thus they will circumscribe their decisions to the universe that are formed in turn of their neighboring contacts.

In the recent literature, there have been various attempts to characterize certain economic events resorting to the concept of scale-free network. Some of the most meaningful include: (i) the analysis of business networks and of the dynamics of mergers among banks and companies [8]; (ii) the inspection of the network topology of interbank payments [9]; (iii) the examination of the structure of transportation networks within cities [10]; (iv) the evaluation of the evolution of international trade in the global economy [11]; (v) the analysis of dynamic stochastic general equilibrium (DSGE) macroeconomic models in a network perspective [12]; (vi) the study of the diffusion of ideas and new technologies in the context of complex networks [13]; (vii) the debate on the asymmetric distribution of wealth [14].

State Transitions in Scale-Free Networks

An important subject on the analysis of scale-free networks in economics concerns how the transition of a node from a given state to another is processed. Imagine, for instance, an individual that is pessimistic about the future performance of the economy; this agent may remain on a pessimistic state or she can evolve to an optimistic state, depending with who she interacts and how this interaction is processed.

The probabilities of interaction in a scale-free network depend on two elements: first, the number of agents holding different types of beliefs and, second, the importance of each potential contact, with this importance measured by the number of links. It is more likely that a contact is established with someone possessing a large number of links than with those who are connected to the rest of the network through just a few links.

Suppose an individual in state x that establishes contact with an adjacent node in state y. When this occurs, assume that the first individual switches to state y with probability λ. According to Nekovee et al. [15], in the presence of a scale-free network this implies the following rule for the motion of nodes passing from state x to state y, for each connectivity class k and each time period t,

$$\dot{x}(k,t) = -k\lambda x(k,t)\sum_{k'} P(k'|k)\, y(k',t) \qquad (1)$$

In expression (1), the term $P(k'|k)$ respects to the degree-degree correlation function, i.e., it represents the weight of each connectivity class in the contacts established by the assumed node.

A scale-free network is an uncorrelated network for which

$$P(k'|k) = \frac{k'P(k')}{\langle k \rangle} \qquad (2)$$

with <k> the average degree and P(k) the degree distribution of k. In a scale-free network, the degree distribution is given by a power-law, i.e., $P(k) = Ak^{-\gamma}$, with A a scaling factor and γ the power-law exponent that is usually assumed as a value between 2 and 3 [1].

Consider an example. Let γ=2.5; for this value, it holds true that: P(1)=0.7455; P(2)=0.1318; P(3)=0.0478; P(4)=0.0233; P(5)=0.0133; … Notice that $\sum_{k'} P(k') = 1$, and thus, in a scale-free network with the specified power-law exponent, a large percentage of the nodes will belong to the lower connectivity classes; in the case, $\sum_{k=1}^{5} P(k) = 0.9617$, i.e., only 3.83% of the nodes have a connectivity degree larger than 5.

In this network, $\langle k \rangle = \sum_{k'} k'P(k') \approx 1.86$. Then, we can present equation (1) as a collection of dynamic equations, one for each connectivity degree,

$$\dot{x}(1,t) = -\lambda x(1,t)\left[0.4008y(1,t) + 0.1417y(2,t) + 0.0771y(3,t) + 0.0501y(4,t) + \ldots\right]$$
$$\dot{x}(2,t) = -2\lambda x(2,t)\left[0.4008y(1,t) + 0.1417y(2,t) + 0.0771y(3,t) + 0.0501y(4,t) + \ldots\right]$$
$$\dot{x}(3,t) = -3\lambda x(3,t)\left[0.4008y(1,t) + 0.1417y(2,t) + 0.0771y(3,t) + 0.0501y(4,t) + \ldots\right]$$
$$\dot{x}(4,t) = -4\lambda x(4,t)\left[0.4008y(1,t) + 0.1417y(2,t) + 0.0771y(3,t) + 0.0501y(4,t) + \ldots\right]$$

The above equations translate the idea that the shares of individuals in state x will diminish when entering in contact with nodes in state y. The dynamics are dependent on weights attributed to y for each k, with these weights given by the degree-degree correlations.

In synthesis, scale-free networks offer a substantive tool that the economic science can resort to in order to explain most observable phenomena. Their analytical tractability, associated to the fact that they accurately translate many aspects of the economic life, as the organization of markets, the functioning of the financial system, the distribution of wealth or the correlation of forces in the global economy, make them an indispensable tool to approach meaningful subjects in the field of economics.

References

1. Boccaletti S, Latora V, Moreno Y, Chavez M, Hwang DU (2006) Complex Networks: Structure and Dynamics. Physics Reports 424: 175-308.
2. Barabási AL, Albert R (1999) Emergence of Scaling in Random Networks. Science 286: 509-512.
3. Zschaler G (2012) Adaptive-Network Models of Collective Dynamics. European Physical Journal-Special Topics 211: 1-101.
4. Zanette DH (2002) Dynamics of Rumor Propagation on Small-World Networks. Physical Review E.
5. Criado R, Romance M (2012) Structural Vulnerability and Robustness in Complex Networks: Different Approaches and Relationships between them. Handbook of Optimization in Complex Networks, Springer Optimization and Its Applications New York: Springer 58: 3-36.
6. Barabási AL (2009) Scale-Free Networks: a Decade and Beyond. Science 325: 412-413.
7. Barabási AL, Ravasz E, Vicsek T (2001) Deterministic Scale-Free Networks. Physica A 299: 559-564.
8. Souma W, Fujiwara Y, Aoyama H (2003) Complex Networks and Economics. Physica A 324: 396-401.
9. Soramaki K, Bech ML, Arnold J, Glass RJ, Beyeler WE (2007) The Topology of Interbank Payment Flows. Physica A 379: 317-333.
10. Berliant M, Watanabe H (2011) A Scale-Free Transportation Network Explains the City-Size Distribution. European Regional Science Association working paper ersa.
11. Li X, Jin YY, Chen G (2003) Complexity and Synchronization of the World Trade Web. Physica A 328: 287-296.
12. Chang CL, Chen SH (2012) Interactions in the New Keynesian DSGE Models: The Boltzmann–Gibbs Machine and Social Networks Approach. Economics: The Open-Access, Open-Assessment E-Journal.
13. Ghiglino C (2012) Random Walk to Innovation: Why Productivity Follows a Power-Law. Journal of Economic Theory 147: 713-737.
14. Garlaschelli D, Loffredo MI (2008) Effects of Network Topology on Wealth Distributions. Journal of Physics A 41: 224018.
15. Nekovee M, Moreno Y, Bianconi G, Marsili M (2007) Theory of Rumor Spreading in Complex Social Networks. Physica A 374: 457-470.