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Riemann Hypothesis

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Introduction

The Riemann hypothesis is a fundamental mathematical conjecture that has huge implications for the rest of math. It forms the foundation for many other mathematical ideas — but no one knows if it's true. Its validity has become one of the most famous open questions in mathematics.

Where did this idea come from?

Back in 1859, a German mathematician named Bernhard Riemann proposed an answer to a particularly thorny math equation. His hypothesis goes like this: The real part of every non-trivial zero of the Riemann zeta function is 1/2. That's a pretty abstract mathematical statement, having to do with what numbers you can put into a particular mathematical function to make that function equal zero.

In math, a function is a relationship between different mathematical quantities. A simple one might look like this: y = 2x.

It's a sum of an infinite sequence, where each term — the first few are 1/1^s, 1/2^s and 1/3^s — is added to the previous terms. **What is a zero of the Riemann zeta function?**

A "zero" of the function is any number you can put in for x that causes the function to equal zero.

What's the "real part" of one of those zeros, and what does it mean that it equals 1/2?

The Riemann zeta function involves what mathematicians call "complex numbers." A complex number looks like this: a+b*i.

In that equation, "a" and "b" stand for any real numbers. A real number can be anything from minus 3, to zero, to 4.9234, pi, or 1 billion. But there's another kind of number: imaginary numbers. Imaginary numbers emerge when you take the square root of a negative number, and they're important, showing up in all kinds of mathematical contexts.

The simplest imaginary number is the square root of -1, which is written as "i." A complex number is a real number ("a") plus another real number ("b") times i. The "real part" of a complex number is that "a."

A few zeros of the Riemann zeta function, negative integers between -10 and 0, don't count for the Riemann hypothesis. These are considered "trivial" zeros because they're real numbers, not complex numbers. All the other zeros are "non-trivial" and complex numbers.

The Riemann hypothesis states that when the Riemann zeta functions crosses zero (except for those zeros between -10 and 0), the real part of the complex number has to equal to 1/2.

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Arguments for and against the Riemann hypothesis

Mathematical papers about the Riemann hypothesis tend to be cautiously noncommittal about its truth. Of authors who express an opinion, most of them, such as Riemann (1859) and Bombieri (2000), imply that they expect (or at least hope) that it is true. The few authors who express serious doubt about it include Ivić (2008), who lists some reasons for skepticism, and Littlewood (1962), who flatly states that he believes it false, that there is no evidence for it and no imaginable reason it would be true. The consensus of the survey articles (Bombieri 2000, Conrey 2003, and Sarnak 2005) is that the evidence for it is strong but not overwhelming, so that while it is probably true there is reasonable doubt.

Some of the arguments for and against the Riemann hypothesis are listed by Sarnak (2005), Conrey (2003), and Ivić (2008), and include the following:

Several analogues of the Riemann hypothesis have already been proved. The proof of the Riemann hypothesis for varieties over finite fields by Deligne (1974) is possibly the single strongest theoretical reason in favor of the Riemann hypothesis. This provides some evidence for the more general conjecture that all zeta functions associated with automorphic forms satisfy a Riemann hypothesis, which includes the classical Riemann hypothesis as a special case. Similarly Selberg zeta functions satisfy the analogue of the Riemann hypothesis, and are in some ways similar to the Riemann zeta function, having a functional equation and an infinite product expansion analogous to the Euler product expansion. But there are also some major differences; for example, they are not given by Dirichlet series. The Riemann hypothesis for the Goss zeta function was proved by Sheats (1998). In contrast to these positive examples, some Epstein zeta functions do not satisfy the Riemann hypothesis even though they have an infinite number of zeros on the critical line (Titchmarsh 1986). These functions are quite similar to the Riemann zeta function, and have a Dirichlet series expansion and a functional equation, but the ones known to fail the Riemann hypothesis do not have an Euler product and are not directly related to automorphic representations.

At first, the numerical verification that many zeros lie on the line seems strong evidence for it. But analytic number theory has had many conjectures supported by substantial numerical evidence that turned out to be false. See Skewes number for a notorious example, where the first exception to a plausible conjecture related to the Riemann hypothesis probably occurs around 10316; a counterexample to the Riemann hypothesis with imaginary part this size would be far beyond anything that can currently be computed using a direct approach. The problem is that the behavior is often influenced by very slowly increasing functions such as log log T, that tend to infinity, but do so so slowly that this cannot be detected by computation. Such functions occur in the theory of the zeta function controlling the behavior of its zeros; for example the function S(T) above has average size around (log log T)1/2. As S (T) jumps by at least 2 at any counterexample to the Riemann hypothesis, one might expect any counterexamples to the Riemann hypothesis to start appearing only when S (T) becomes large. It is never much more than 3 as far as it has been calculated, but is known to be unbounded, suggesting that calculations may not have yet reached the region of typical behavior of the zeta function.

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