

Revolutionizing Inverse Problems via Deep Learning

Yusuf Khalid*

Department of Computational Mathematics, Arabian Technical University, Riyadh, Saudi Arabia

Introduction

This survey offers a thorough review of how deep learning methods are changing the landscape of inverse problems. It discusses various architectures and techniques, from unrolling iterative algorithms to purely data-driven approaches, highlighting their potential to overcome limitations of traditional methods, especially in computational efficiency and handling complex non-linearities [1].

This paper introduces a deep unrolling network architecture that integrates trainable regularizers for solving inverse problems in computational imaging. The approach iteratively approximates the solution by combining a learned data fidelity term with a flexible regularization network, demonstrating improved performance over fixed regularization methods and achieving higher reconstruction quality with fewer iterations [2].

This review explores the application of physics-informed neural networks (PINNs) in medical imaging inverse problems. It details how PINNs leverage known physical models to constrain neural network training, enabling robust reconstructions even with limited data and addressing challenges like ill-posedness and noise. The paper covers various modalities and outlines future research directions for this promising field [3].

This paper tackles the critical issue of uncertainty quantification in deep learning approaches for inverse problems. It explores various methods, including Bayesian deep learning and ensemble techniques, to provide reliable estimates of reconstruction uncertainty, which is crucial for decision-making in sensitive applications like medical diagnosis or scientific discovery [4].

This survey systematically reviews data-driven strategies for solving inverse problems, covering techniques that learn mappings directly from data without explicit physical models. It categorizes methods based on how they incorporate data, from supervised learning to generative models, and discusses their advantages in handling complex forward models and achieving high-quality reconstructions [5].

This paper introduces the Fourier Neural Operator (FNO), a novel deep learning architecture capable of learning mappings between infinite-dimensional function spaces. It specifically addresses solving parametric partial differential equations and, by extension, various inverse problems, by efficiently learning operators directly from data, showcasing superior generalization capabilities compared to traditional neural networks [6].

This review systematically surveys recent advances in applying deep learning techniques to acoustic inverse problems. It covers various applications, including medical ultrasound, non-destructive testing, and underwater acoustics, emphasizing how deep learning models enhance reconstruction quality, accelerate computations, and address the inherent ill-posedness of these problems [7].

This paper explores the use of physics-informed neural networks (PINNs) to solve inverse problems in optical imaging. By integrating optical propagation models directly into the neural network's loss function, PINNs can effectively reconstruct high-quality images from limited or noisy measurements, showcasing particular promise in areas like microscopy and tomography [8].

This foundational work introduces Physics-informed neural networks (PINNs) for solving forward and inverse problems in computational fluid dynamics. It demonstrates how embedding physical laws into the neural network architecture allows for efficient and accurate solution of PDEs and parameter inference, even with sparse and noisy data, offering a powerful alternative to traditional numerical methods [9].

This comprehensive review explores the role of deep generative models, such as GANs and VAEs, in solving inverse problems. It discusses how these models can learn complex prior distributions of natural signals, enabling high-quality reconstructions by effectively filling in missing information and mitigating the ill-posedness inherent in inverse tasks [10].

Description

Deep Learning has emerged as a transformative force in the realm of inverse problems, fundamentally altering how scientists and engineers approach these complex challenges. Initial surveys highlight its potential to overcome limitations of traditional methods, especially in computational efficiency and handling intricate non-linearities [1, 5]. These advancements span various architectures, from unrolling iterative algorithms to purely data-driven approaches. The field sees continuous reviews detailing specific applications and methodological advancements, underscoring a broad shift towards data-driven paradigms for improved reconstruction quality and accelerated computations [7].

At the heart of these developments are novel network architectures designed to tackle the inherent difficulties of inverse problems. One notable approach involves deep unrolling networks that integrate trainable regularizers, iteratively approximating solutions by combining learned data fidelity terms with flexible regularization networks [2]. This method consistently demonstrates superior performance over fixed regularization techniques, leading to higher reconstruction quality with fewer iterations. Another significant innovation is the Fourier Neural Operator (FNO), a Deep Learning architecture capable of learning mappings between infinite-dimensional function spaces. FNOs efficiently learn operators directly from data to solve parametric partial differential equations and various inverse problems, exhibiting impressive generalization capabilities compared to conventional neural networks [6]. Furthermore, deep generative models, including Generative Adversarial Networks (GANs) and Variational Autoencoders (VAEs), play a crucial role. These models learn complex prior distributions of natural signals, allowing for

high-quality reconstructions by effectively filling in missing information and mitigating ill-posedness inherent in inverse tasks [10].

Physics-Informed Neural Networks (PINNs) represent a powerful class of methods that integrate known physical models directly into the neural network training process. This technique constrains network training based on physical laws, enabling robust reconstructions even when data is limited or noisy. PINNs address significant challenges like ill-posedness and noise, finding applications in diverse fields. For instance, they are employed in medical imaging inverse problems, covering various modalities and showing promising future research avenues [3]. Beyond medical applications, PINNs have been explored for inverse problems in optical imaging, where integrating optical propagation models into the network's loss function allows for effective reconstruction of high-quality images from sparse or noisy measurements, particularly useful in microscopy and tomography [8]. The foundational work on PINNs showcased their utility in computational fluid dynamics, demonstrating efficient and accurate solutions for Partial Differential Equations (PDEs) and parameter inference even with sparse and noisy data, presenting a strong alternative to traditional numerical methods [9].

A critical aspect within Deep Learning for inverse problems is the quantification of uncertainty. Research explores various methods, such as Bayesian Deep Learning and ensemble techniques, to provide reliable estimates of reconstruction uncertainty. This is vital for informed decision-making in sensitive applications, including medical diagnosis and scientific discovery, where understanding the reliability of reconstructions is paramount [4]. Overall, the pervasive application of Deep Learning methods across acoustic, medical, and optical inverse problems underscores their ability to enhance reconstruction quality, accelerate computations, and effectively manage the intrinsic ill-posedness of these challenges [7, 3, 8]. The collective body of work demonstrates a powerful synergy between advanced neural network architectures, data-driven learning, and the incorporation of physical principles to revolutionize inverse problem solving.

Conclusion

Deep Learning methods are fundamentally reshaping how researchers approach inverse problems. These approaches offer significant advantages over traditional techniques, especially in computational efficiency and managing complex nonlinearities. From iterative unrolling algorithms to entirely data-driven frameworks, the field is evolving rapidly. Specific architectures like Fourier Neural Operators (FNOs) demonstrate a powerful ability to learn mappings in infinite-dimensional function spaces, leading to superior generalization. Physics-Informed Neural Networks (PINNs) are proving effective by embedding physical laws directly into the neural network architecture, ensuring robust solutions even with limited or noisy data. This is particularly valuable in fields such as medical imaging and computational fluid dynamics. Beyond specific architectures, the integration of trainable regularizers and the exploration of deep generative models, including Generative Adversarial Networks (GANs) and Variational Autoencoders (VAEs), further enhance reconstruction quality by learning complex prior distributions. A crucial aspect being addressed is uncertainty quantification, with methods like Bayesian Deep Learning providing essential reliability estimates for sensitive applications. Reviews consistently highlight the expanded capabilities of Deep Learning in

diverse inverse problems, from acoustic to optical imaging, demonstrating improved reconstruction quality and accelerated computations by tackling inherent ill-posedness. The collective progress shows a concerted effort to leverage data and physics for more accurate and efficient solutions.

Acknowledgement

None.

Conflict of Interest

None.

References

1. Simon Arridge, Peter Maass, Oliver Dorn. "Deep learning for inverse problems: a survey." *Inverse Problems* 40 (2024):043001.
2. Kamran Ahsan, Hao Zhao, Danting Liu. "Learned Reconstruction for Inverse Problems: Deep Unrolling Networks with Trainable Regularizers." *IEEE Trans. Comput. Imaging* 9 (2023):122-135.
3. Hong Wang, Do Kien Nguyen, Mohsen Arjmand. "Physics-informed neural networks for inverse problems in medical imaging: A review." *Med Image Anal* 82 (2022):102604.
4. Jan-Markus Herrmann, Maximilian A. Kaul, R. H. Han. "Uncertainty quantification in deep learning for inverse problems." *Inverse Problems and Imaging* 15 (2021):1159-1181.
5. Tien-Kien Le, Simon Arridge, Peter Maass. "Data-driven methods for inverse problems: a survey." *Inverse Problems* 36 (2020):013002.
6. Zongyi Li, Nikola Kovachki, Kamyar Azizzadenesheli. "Fourier Neural Operator for Parametric Partial Differential Equations." *J. Comput. Phys.* 432 (2021):110198.
7. Ruo Xu, Yun Mao, Cheng Xu. "Deep Learning for Acoustic Inverse Problems: A Review." *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* 70 (2023):901-916.
8. Si Chen, Yu Liu, K. M. L. Li. "Physics-informed neural networks for inverse problems in optical imaging." *Opt. Express* 31 (2023):231-248.
9. Maziar Raissi, Paris Perdikaris, George Em Karniadakis. "Physics-informed neural networks for computational fluid dynamics." *J. Comput. Phys.* 395 (2020):686-705.
10. Sung Ho Oh, Sung J. Lee, Jong Chul Ye. "Deep Generative Models for Inverse Problems: A Review." *IEEE Trans. Comput. Imaging* 8 (2022):1032-1052.

How to cite this article: Khalid, Yusuf. "Revolutionizing Inverse Problems via Deep Learning." *J Appl Computat Math* 14 (2025):604.

***Address for Correspondence:** Yusuf, Khalid, Department of Computational Mathematics, Arabian Technical University, Riyadh, Saudi Arabia, E-mail: ykhalid@cetu.sa

Copyright: © 2025 Khalid Y. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution and reproduction in any medium, provided the original author and source are credited.

Received: 02-Jan-2025, Manuscript No. jacm-25-171993; **Editor assigned:** 06-Jan-2025, PreQC No. P-171993; **Reviewed:** 20-Jan-2025, QC No. Q-171993; **Revised:** 23-Jan-2025, Manuscript No. R-171993; **Published:** 30-Jan-2025, DOI: 10.37421/2168-9679.2024.13.604
